Imperfect information and the business cycle *

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Abstract

Imperfect information has played a prominent role in modern business cycle theory. We assess its importance by estimating the New Keynesian (NK) model under alternative informational assumptions. One version focuses on confusion between temporary and persistent disturbances. Another, on unobserved variation in the inflation target of the central bank. A third on persistent misperceptions of the state of the economy (measurement error). And a fourth assumes perfect information (the standard NK–DSGE version). We find that models with imperfect information contain additional explanatory power for business fluctuations relative to models without it. Signal extraction thus seems to provide an empirically plausible and quantitatively important business cycle mechanism.

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Introduction

The role of imperfect information in business fluctuations has been a prominent theme in modern business cycle theory. In the monetary, rational expectations model of Lucas, 1972, misperceptions of nominal aggregates give rise to confusion between nominal and relative price movements and constitute the main source of economic fluctuations[1]. Confusion about the shocks afflicting the economy is also present in the original version of the RBC model (Kydland and Prescott, 1982). In this model, the confusion arises from the agents' inability to distinguish between temporary and permanent changes in multi-factor productivity.

In spite of its presence in two of the most influential modern macroeconomic models, imperfect information is not considered an important element for understanding business cycles. There is a presumption that imperfect knowledge of the true values of nominal aggregates is not important enough to generate large and persistent movements in real economic activity. And the success of the Kydland-Prescott RBC model in accounting for business cycle owes little, if anything, to informational problems.

Recent work, however, has questioned the unimportance of imperfect information/signal extraction problems in influencing macroeconomic outcomes. For instance, Orphanides (2002, 2003) has forcefully argued that misperceptions of potential output may have played the key role behind the excessively loose monetary policy and the great inflation of the 70s. Collard and Dellas, 2007, claim that the failure of the rational expectations literature to find significant effects of mis-perceived money may be due to its use of inappropriate measures of misperceptions. In particular, using the proper measure leads to the uncovering of significant effects on economic activity. Moreover, they demonstrate that under –empirically plausible levels of– imperfect information the NK model can deliver considerable inertia. Collard and Dellas, 2006, argue that confusion regarding the types of shocks afflicting the economy plays a crucial role in allowing the RBC model to fit a number of conditional, dynamic, stylized facts.

Why should one expect that imperfect information about the shocks and the true state of the economy could play an important role in business cycles? The reason is to be found in the response of the agents to perceived shocks. First, the agents react to shocks when they should not and this generates "excessive" volatility[2]. And second, and this is the flip side of the first one, imperfect information imparts caution on economic behavior. The reaction to a particular shock may be initially mooted as the agents try to identify the true underlying source of the shock. And it may pick up gradually as the agents learn and update their perceptions of the state of the economy. Imperfect information thus provides an endogenous propagation mechanism that

[1] Such mis-perceptions may arise either from the complete unavailability of up-to-date information. Or, from the fact that the available information is contaminated by measurement error (King, 1982)

[2] For instance, they respond to changes in monetary aggregates while such changes would have been otherwise neutral.

can generate inertia, persistence and reversals, and it can often lead to hump shaped dynamics (see, for instance, Dellas, 2006).

The objective of this paper is to undertake a comprehensive study of the role of imperfect information in macroeconomic fluctuations. Comprehensive in the sense that alternative informational setups are considered. And that the alternative versions of imperfect information are compared not only among themselves but also to versions that assume perfect information.

Of the latter class of models we consider the two most widely used ones. Namely, the purely forward looking version of the NK model(Woodford, 2002). And its "hybrid" version with both forward and backward looking elements (Christiano et al, 2005, Galí and Gertler, 1999, Smets and Wouters, 2003). Of the class of models with imperfect information we consider three versions, each one emphasizing a different source of misperception. The first one is motivated by Lucas, 1972, Kydland and Prescott, 1982, and Orphanides, 2002, and involves confusion between temporary and permanent shocks. The second is motivated by Cogley and Sbordone, 2006, and relies on –unobserved– variation in the inflation target of the central bank. These two versions share in common the assumption that the agents observe perfectly all endogenous variables (inflation, output, interest rate) but not the shocks. The third and final version draws on Collard and Dellas', 2007, argument that, as revealed by the process of data revisions, very few aggregate variables are observed accurately. Consequently, unlike the two versions presented above, it assumes that in addition to the shocks some of the endogenous variables are also observed with noise (due to measurement error). This version thus represents a more severe case of information imperfection than the other two. Nevertheless, as we will establish later, the degree of mis-perception required by the model in order to fit the data is well within the range observed in the real world.

The models are estimated on US data over the 1966-2002 period using Bayesian methods. They are then evaluated and compared in terms of various criteria: Overall fit (the log-likelihood), unconditional second moments, and IRFs.[3]

The main results can be summarized as follows: First, imperfect information has considerable explanatory power for the US business cycle. In particular, the imperfect information models improve the fit of the NK model according to the log-likelihood. The models that contain the most "severe" informational problems (the Collard and Dellas version and, to a smaller degree, the Cogley-Sbordone one) fare the best[4]. Second, while the hybrid NK model (perfect

[3]These comparisons constitute one of the main differences between this paper and other work in the literature that tests the empirical validity of the *perfect information* NK model under alternative specifications. For instance, Eichenbaum and Fisher, 2004, find that an estimated version of the NK model with backward indexation is consistent with the data (as judged by the J–statistic in the context of GMM estimation). De Walque, Smets and Wouters, 2004, find that the Smets and Wouters model performs well even when the parameter of backward indexation is close to zero. But the fact that the model without indexation is not rejected by the data does not mean that it performs the best within a particular class of models.

[4]The reason that the version with confusion between transitory and persistent shocks does not perform as well

information and backward indexation) exhibits inertia in its dynamics of inflation and output so do the imperfect information versions. This confirms earlier findings by Collard and Dellas, 2007, that imperfect information can provide an alternative and equivalent propagation mechanism to backward indexation (or rule-of-thump-agents). Third, while there is no clear winner regarding unconditional moments (and version 1 performs quite well), the measurement error version tends to capture quite adequately the behavior of nominal interest rates (volatility and procylicality), a encouraging finding given the shortcomings of the NK model along this dimension.

The remaining of the paper is organized as follows. Sections 1 and 2 present the model with and without a signal extraction problem (what we call perfect and imperfect information, respectively). Section 3 discusses the econometric methodology. Section 4 presents the main results. The last section offers some concluding remarks.

1 The model without signal extraction

1.1 The Household

There exists an infinite number of households distributed over the unit interval and indexed by $j \in [0,1]$. The preferences of household j are given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \chi_{t+\tau} \left[\frac{(c_{t+\tau} - g_{t+\tau})^{1-\sigma_c}}{1-\sigma_c} - \nu^h \frac{h_{t+\tau}^{1+\sigma_h}}{1+\sigma_h} \right] \tag{1}$$

where $0 < \beta < 1$ is a constant discount factor, c_t denotes consumption in period t, and h_{jt} is the quantity of labor supplied by the representative household of type j. χ_t is a preference shock that is assumed to follow an AR(1) process of the form

$$\log(\chi_t) = \rho_\chi \log(\chi_{t-1}) + (1-\rho_\chi) \log(\overline{\chi}) + \varepsilon_{\chi,t}$$

where $|\rho_\chi| < 1$ and $\varepsilon_{\chi,t} \rightsquigarrow \mathscr{N}(0, \sigma_\chi^2))$.

g_t denotes an external habit stock which is assumed to be proportional to past aggregate consumption:

$$g_t = \vartheta \overline{c}_{t-1} \text{ with } \vartheta \in (0,1).$$

In each period, household j faces the budget constraint

$$B_t + P_t c_t = R_{t-1} B_{t-1} + W_t h_t + \Pi_t \tag{2}$$

where B_t is nominal bonds. P_t, the nominal price of goods. c_t denotes consumption expenditures. W_t is the nominal wage. Ω_t is a nominal lump-sum transfer received from the monetary authority and Π_t denotes the profits distributed to the household by the firms.

is because, due to the structure of the model, the agents manage to quickly solve the signal extraction problem. In the absence of persistence in the signal extraction process this version behaves similarly to the standard forward looking NK model.

This yields the following set of first order conditions

$$\nu_h h_t^{\sigma_h} = (c_t - \theta \overline{c}_{t-1})^{-\sigma_c} w_t \tag{3}$$

$$\chi_t (c_t - \theta \overline{c}_{t-1})^{-\sigma_c} = \beta R_t E_t \frac{\chi_{t+1}(c_{t+1} - \theta \overline{c}_t)^{-\sigma_c}}{\pi_{t+1}} \tag{4}$$

1.2 The firms

1.2.1 Final Good Producers

The final good, y is produced by combining intermediate goods, y_i, by perfectly competitive firms. The production function is given by

$$y_t = \left(\int_0^1 y_{it}^{\theta} \mathrm{d}i \right)^{\frac{1}{\theta}} \tag{5}$$

where $\theta \in (-\infty, 1)$. Profit maximization and free entry lead to the general price index

$$P_t = \left(\int_0^1 P_{it}^{\frac{\theta}{\theta-1}} \mathrm{d}i \right)^{\frac{\theta-1}{\theta}} \tag{6}$$

Profit maximization gives rise to the following demand function for good i

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{\frac{1}{\theta-1}} y_t \tag{7}$$

The final good may be used for consumption.

1.2.2 Intermediate goods producers

Each firm i, $i \in (0,1)$, produces an intermediate good by means of labor according to a constant returns–to–scale technology, represented by the Cobb–Douglas production function

$$y_{it} = a_t h_{it} \tag{8}$$

where h_{it} denotes the labor input used by firm i in the production process. a_t is an exogenous technology shock which is assumed to follow an AR(1) process of the form

$$\log(a_t) = \rho_a \log(a_{t-1}) + (1 - \rho_a) \log(\overline{a}) + \varepsilon_{a,t}$$

where $|\rho_a| < 1$ and $\varepsilon_{a,t} \leadsto \mathcal{N}(0, \sigma_a^2)$.

Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. We follow Calvo in assuming that firms set their prices for a stochastic number of periods. In each and every period, a firm either gets the chance to adjust its price (an event occurring with probability $(1-\xi)$) or it does not. If it does not get the chance, then it is assumed to set prices according to

$$P_{it} = \pi_{t-1}^{\gamma}\overline{\pi}^{1-\gamma}P_{it-1} \tag{9}$$

where $\gamma \in (0,1)$ determines the price indexation scheme. Note that by setting $\gamma = 1$ we retrieve the lagged indexation specification used by Christiano et al., 2005, while when $\gamma = 0$ the prices are indexed to steady state inflation.

If a firm i sets its price optimally in period t then it chooses a price, $P_t^\star$, in order to maximize:

$$\max_{P_t^\star} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Phi_{t+\tau}(1-\xi)^{\tau}\left(P_t^\star \Xi_{t,\tau} - P_{t+\tau}\Psi_{t+\tau}\right) y_{it+\tau}$$

subject to the total demand (7) and

$$\Xi_{t,\tau} = \begin{cases} \pi_t^{\gamma}\overline{\pi}^{1-\gamma} \times \ldots \times \pi_{t+\tau-1}^{\gamma}\overline{\pi}^{1-\gamma} & \text{if } \tau \geqslant 1 \\ 1 & \text{otherwise} \end{cases}$$

Ψ_t denotes the real marginal cost of the firm.[5] Note that we have $\Xi_{t,\tau+1} = \pi_t^{\gamma}\overline{\pi}^{1-\gamma}\Xi_{t+1,\tau}$.

$\Phi_{t+\tau}$ is an appropriate discount factor derived from the household's evaluation of future relative to current consumption. This leads to the price setting equation

$$P_t^\star = \frac{1}{\theta} \frac{\mathbb{E}_t \sum_{\tau=0}^{\infty}(1-\xi)^{\tau}\Phi_{t+\tau}P_{t+\tau}^{\frac{\theta-2}{\theta-1}}\Xi_{t,\tau}^{\frac{1}{\theta-1}} s_{t+\tau}y_{t+\tau}}{\mathbb{E}_t \sum_{\tau=0}^{\infty}(1-\xi)^{\tau}\Phi_{t+\tau}\Xi_{t,\tau}^{\frac{\theta}{\theta-1}} P_{t+\tau}^{\frac{1}{1-\theta}} y_{t+\tau}} \tag{10}$$

Since the price setting scheme is independent of any firm specific characteristic, all firms that reset their prices will choose the same price.

In each period, a fraction ξ of contracts ends and $(1-\xi)$ survives. Hence, from (6) and the price mechanism, the aggregate intermediate price index writes

$$P_t = \left(\xi P_t^{\star\frac{\theta}{\theta-1}} + (1-\xi)(\pi_{t-1}^{\gamma}\overline{\pi}^{1-\gamma}P_{t-1})^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}} \tag{11}$$

1.3 Monetary Policy

Monetary policy is conducted according to

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho_r)[\alpha_y(\log(y_t) - \log(y_t^{\mathrm{N}})) + \alpha_\pi(\log(\pi_t) - \log(\pi_t^\star)] + \epsilon_{r,t}$$

where $\pi_t^\star$ represents the inflation target of the central bank and y_t^{N} is the natural rate of output, that is the output level that would prevail in a flexible price economy. $\epsilon_{r,t}$ is a monetary policy shock. It follows a gaussian iid process ($\varepsilon_{r,t} \rightsquigarrow \mathscr{N}(0,\sigma_r^2)$).

[5]Since there is only one common technology shock and labor is homogenous, the real marginal cost is the same across firms.

We will consider two alternative cases for the inflation target. In the first one, the central bank targets the constant steady state inflation, hence $\pi_t^\star = \overline{\pi}$. In the second one, following Cogley and Sbordone, 2006, we assume that the inflation target varies over time. In particular, it follows an AR(1) process of the form

$$\log(\pi_t^\star) + \rho_\pi \log(\pi_{t-1}) + (1-\rho_\pi)\log(\overline{\pi}) + \varepsilon_{\pi,t}$$

where $|\rho_\pi| < 1$ and $\varepsilon_{\pi,t} \rightsquigarrow \mathscr{N}(0, \sigma_\pi^2)$.

1.4 Equilibrium

In equilibrium, we have $y_t = c_t$. Log-linearization of the model around the deterministic steady state leads to the standard IS-AS-MP representation of the NK model.

$$\widehat{y}_t = \frac{\vartheta}{1+\vartheta}\widehat{y}_{t-1} + \frac{1}{1+\vartheta}\mathbb{E}_t\widehat{y}_{t+1} - \frac{1-\vartheta}{\sigma_c(1+\vartheta)}(\widehat{R}_t - \mathbb{E}_t\widehat{\pi}_{t+1}) + \widehat{x}_t + \tag{12}$$

$$\widehat{\pi}_t = \frac{\gamma}{1+\beta\gamma}\widehat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma}\mathbb{E}_t\widehat{\pi}_{t+1} + \frac{\xi(1-\beta(1-\xi))}{(1-\xi)(1+\beta\gamma)}\left(\frac{\sigma_c + \sigma_h(1-\vartheta)}{1-\vartheta}\right)\widehat{y}_t - \frac{\xi(1-\beta(1-\xi))}{(1-\xi)(1+\beta\gamma)}\frac{\vartheta\sigma_c}{1-\vartheta}\widehat{y}_{t-1} - \widehat{z}_t + \widehat{\nu}_t \tag{13}$$

$$\widehat{R}_t = \rho_r\widehat{R}_{t-1} + (1-\rho_r)(\alpha_y(\widehat{y}_t - \widehat{y}_t^{\mathrm{N}}) + \alpha_\pi(\widehat{\pi}_t - \widehat{\pi}_t^\star) + \epsilon_{r,t} \tag{14}$$

$$\widehat{y}_t^{\mathrm{N}} = \frac{\vartheta}{\sigma_c + \sigma_h(1-\vartheta)}\widehat{y}_{t-1}^{\mathrm{N}} + \frac{(1-\vartheta)(1-\xi)(1+\beta\gamma)}{(\sigma_c+\sigma_h(1-\vartheta))\xi(1-\beta(1-\xi))}\widehat{z}_t \tag{15}$$

where $\widehat{x}_t = \frac{1-\vartheta}{1+\vartheta}(\widehat{\chi}_t - \mathbb{E}_t\widehat{\chi}_{t+1})$, $\widehat{z}_t = \frac{\xi(1-\beta(1-\xi))(1+\sigma_h)}{(1-\xi)(1+\beta\gamma)}\widehat{a}_t$.[6] $\widehat{\nu}_t$ is a cost push shock which is assumed to be iid and normally distributed with mean 0 and standard deviation σ_ν.

We will study five versions of the model, two with perfect and three with imperfect information. In particular, in the first version we assume that the agents have perfect information about all shocks. Moreover, we assume that the Central Bank targets a constant inflation rate ($\sigma_\pi = 0$, $\rho_\pi = 0$) and that there is no indexation to lagged inflation in price setting ($\gamma = 0$). This version, which we will call No- Indexation-Perfect-Information (NIPI) thus corresponds to the so called baseline New Keynesian (NK) model augmented with real rigidities (habit persistence). The second version only differs from the first regarding the price setting scheme. In particular, here we assume that firms that cannot reset their prices optimally adopt a backward indexation scheme, such that $\gamma \in (0,1)$. This version is denoted by IPI (Indexation-Perfect-Information).

The remaining versions bring in imperfect information.[7] The third version of the model is similar to the NIPI version ($\gamma = \sigma_\pi = \rho_\pi = 0$) with the exception that the agents do not observe the shocks directly and can only infer them through their –perfect– observation of output, inflation

[6]Note that both $\widehat{x}_t$ and $\widehat{z}_t$ follow the same AR(1) process as their underlying fundamentals, χ_t and a_t, but the volatility of the innovations are simple re–scaling of the original innovations. In order to save on notations, we will keep σ_χ and σ_a to denote the volatilities of the transformed shocks.

[7]We make the assumption that all agents in the model, including the central bank, have symmetric information.

and the nominal interest rate. While the observation of the nominal interest rate and the level of output enables them to infer the preference and monetary policy shock, the observation of inflation is not sufficient to help them distinguish perfectly the persistent technology shocks from the transient cost push shock. This version, denoted by NIII (No Indexation, Imperfect Information), is –loosely– motivated by Lucas, 1972, Kydland and Prescott, 1982, and Orphanides, 2002, in the sense that it involves confusion between temporary and persistent shocks. Under these circumstance, the Central Bank therefore is assumed to use the best possible estimates of the output gap and the inflation gap it can produce. This leads us to consider a Taylor rule of the type[8]

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1-\rho_r)(\alpha_y(\widehat{y}_t - \widehat{y}^{\text{N}}_{t|t}) + \alpha_\pi \widehat{\pi}_t) + \epsilon_{r,t} \tag{16}$$

where $x_{t|t}$ denotes the expected value of variable x conditional on the information set of the Central Bank, which now consists of output, inflation and the nominal interest rate. In other words, the Central Bank uses its best predictor of the natural rate of output.

The fourth version of the model is like the previous one with an important difference. Namely, it emphasizes a different source of misperception. The agents still observe the same variables, but the imperfect information problem does not regard distinguishing between persistent technology shocks and transient cost push shock, but rather distinguishing between persistent inflation target shocks and transient monetary policy shocks. This version –which abstracts from the cost push shocks ($\sigma_\nu = 0$) in order to be differentiated from version three above– is motivated by the work of Cogley and Sbordone, 2006, who emphasize the role of –unobserved– variation in the inflation target of the central bank as a driving force of fluctuations in output and inflation. The central bank is now assumed to follow an interest rate rule of the form:[9]

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1-\rho_r)(\alpha_y(\widehat{y}_t - \widehat{y}^{\text{N}}_{t|t}) + \alpha_\pi(\widehat{\pi}_t - \widehat{\pi}^\star_t) + \epsilon_{r,t} \tag{17}$$

This version is denoted by NIII–CS ((No Indexation, Imperfect Information – Cogley Sbordone). Admittedly, the assumption of symmetric information between the private agents and the monetary authorities is rather tenuous under these circumstances. But the technical complications arising from dropping this assumption are rather prohibitive in this context.

The last –fifth– version of the model is like the third version, that is, it re-introduces the cost–push shocks and drops the inflation target shocks. But it adopts a different information structure that makes the informational problem more severe. In particular, here we assume that some of the variables of the model (shocks and endogenous variables) are measured imprecisely. We assume that the shocks, output and the inflation rate are measured with error, that is, for

[8]Note that since output and inflation are perfectly observed, their perceived value is identical to their actual value.

[9]Note that since output and inflation are perfectly observed, their perceived values coincide with their actual values.

variable ω_i we have that

$$\omega_t^\star = \omega_t^{\mathrm{T}} + \upsilon_{\omega,t}$$

where ω_t^{T} denotes the true value of the variable and $\upsilon_{\omega,t}$ is a noisy process that satisfies $E(\upsilon_{\omega,t}) = 0$ for all t; $E(\upsilon_{\omega,t}\varepsilon_{z,t}) = E(\upsilon_{\omega,t}\varepsilon_{x,t}) = E(\upsilon_{\omega,t}\varepsilon_{\pi,t}) = E(\upsilon_{\omega,t}\varepsilon_{\nu,t}) = 0$; and

$$E(\upsilon_{\omega,t}\eta_{\omega,k}) = \begin{cases} \eta_\omega^2 & \text{if } t = k \\ 0 & \text{Otherwise} \end{cases}$$

The nominal interest rate is still observed perfectly. In this version, the Central Bank follows the rule

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1-\rho_r)(\alpha_y(\widehat{y}_{t|t} - \widehat{y}^{\mathrm{N}}_{t|t}) + \alpha_\pi \widehat{\pi}_{t|t} + \epsilon_{r,t} \tag{18}$$

The motivation for this specification [10] comes from Orphanides, 2002, who argues that a large fraction of the output gap misperception during the great inflation of the 70s can be attributed to the mis-measurement of *actual* output. Using the real time data constructed by the Philadelphia FED, Collard and Dellas, 2007, have argued that such mis–measurement is present in all macroeconomic series (including inflation) and that it is quantitatively substantial. The existence of real time data can help assess the plausibility of the estimated amount of noise in the model by comparing it to that present, say, in data revisions. Or, by comparing perceived values to the corresponding real time data.[11]

In this fifth version of the model, the agents face a more severe –and hence, potentially more consequential– signal extraction problem than in the other imperfect information version as misperceptions apply to *all* shocks in the model rather than only to the transitory cost–push and the persistent technology shock for instance. So even if one were not convinced that measurement errors complicate decision making, this model could still provide a useful vehicle for thinking about how widespread confusion about a multitude of shocks may matter for the business cycle.

2 Econometric Methodology

2.1 Data

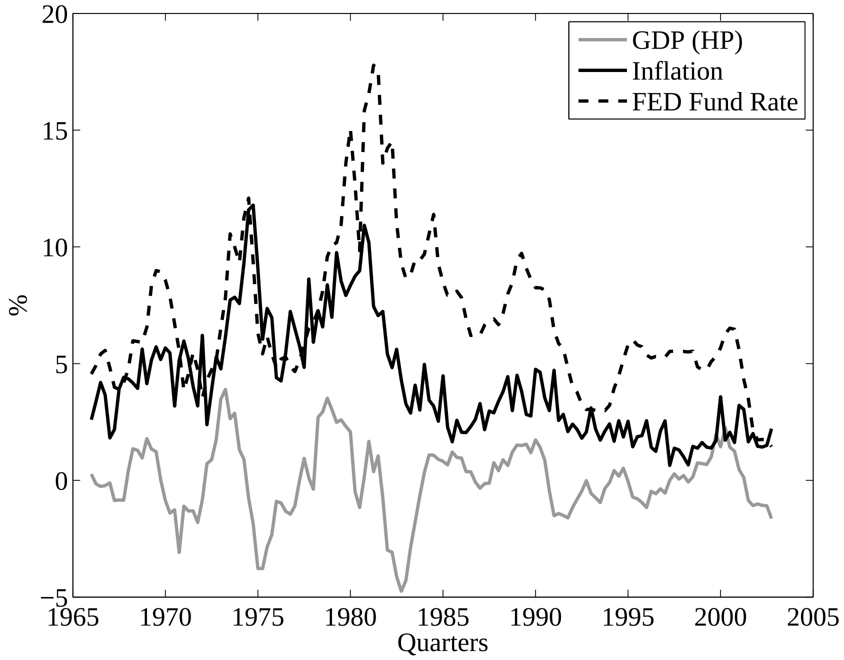
The model is estimated on US quarterly data for the period 1966:I–2002:IV. The data are taken from the Federal Reserve Economic Database. Output is measured by real GDP, inflation is the

[10] It should be noted that we are not the first one to estimate a NK model with signal extraction due to measurement errors. Lippi and Neri, 2006, estimate such a model but, having a different objective, they do not examine the stochastic properties of their model. Moreover, they do not compare the performance of their model to that of alternative NK specifications, so one cannot judge its *relative* success.

[11] In addition to the measurement error specification of the signal extraction problem presented above there exists an alternative specification that relies on the distinction between idiosyncratic and aggregate shocks. We have opted for the former approach because it is easier to implement and can be evaluated using available extraneous information on real time data. The latter approach would require knowledge of (or assumptions about) the relative variance of idiosyncratic and aggregate shocks and would not be as readily testable as the former.

annualized quarterly change in the GDP deflator, while the nominal interest rate is the Federal Funds Rate. The output series is detrended using the H–P filter.

Figure 1: US Data



2.2 Estimation method

We do not estimate all the parameters of the model as some of them cannot be identified in the steady state and do not enter the log–linear representation of the economy. This is the case for the demand elasticity, θ, and the weight on leisure, ν_h, in the utility function. All the other structural parameters are estimated, including the steady state level of annualized inflation, $\pi^\star$, and real interest rate, $r^\star$. The discount factor, β, is then computed using the estimated level of the real interest rate.[12] We therefore estimate the vector of parameters $\Psi = \{\vartheta, \sigma_h, \xi, \gamma, \pi^\star, r^\star, \rho_a, \rho_\chi, \rho_\pi, \sigma_a, \sigma_\chi, \sigma_r, \sigma_\nu, \sigma_\pi, \eta_y, \eta_\pi, \rho_r, \alpha_\pi, \alpha_y\}$. Ψ is estimated relying on a Bayesian maximum likelihood procedure. As a first step of the procedure, the log–linear system (12)–(15) is solved using the Blanchard-Khan method. In the specification with the signal extraction, the model is solved according to the method outlined in the Appendix. The Kalman filter is then used on the solution of the model to form the log–likelihood, $\mathcal{L}_m(\{\mathscr{Y}_t\}_{t=1}^T, \Psi)$, of each model, m. Once the posterior mode is obtained by maximizing the likelihood function, we obtain the posterior density function using the Metropolis–Hastings algorithm (see Lubik and Schorfheide, 2004).

Table 1 presents the prior distribution of the parameters. The habit persistence parameter, ϑ, is

[12] More precisely, we have $\beta = (1 + r^\star/400)^{-1}$.

beta distributed as it is restricted to belong to the [0,1) interval. The average of the distribution is set to 0.50, which is in line with the prior distribution used by Smets and Wouters, 2004. The intertemporal elasticity parameter, σ_c, is assigned a Gamma distribution centered around 1.5, while the parameter of the labor supply elasticity, σ_h, has a Gamma prior distribution with mean 1. These two values are commonly used in the literature. The steady state inflation rate and real interest rate have a Γ-distribution with means 4% and 2% per year respectively.

Table 1: Priors

Param.	Type	Mean	Median	Std. Dev.	95% HPDI
ϑ	Beta	0.50	0.50	0.10	[0.33;0.67]
σ_c	Gamma	1.50	1.45	0.50	[0.78;2.41]
σ_h	Gamma	1.00	0.92	0.50	[0.34;1.94]
ξ	Beta	0.25	0.25	0.025	[0.21;0.29]
γ	Beta	0.50	0.50	0.20	[0.17;0.83]
$r^\star$	Gamma	2.00	1.84	1.00	[0.68;3.86]
$\pi^\star$	Gamma	4.00	3.92	1.00	[2.51;5.78]
ρ_r	Beta	0.50	0.50	0.20	[0.17;0.83]
α_π	Gamma	1.50	1.49	0.20	[1.19;1.84]
α_y	Gamma	0.125	0.12	0.05	[0.05;0.21]
ρ_a	Beta	0.50	0.50	0.20	[0.17;0.83]
ρ_χ	Beta	0.50	0.50	0.20	[0.17;0.83]
ρ_π	Beta	0.50	0.50	0.20	[0.17;0.83]
σ_a	Invgamma	0.20	0.17	4.00	[0.10;0.38]
σ_χ	Invgamma	0.20	0.17	4.00	[0.10;0.38]
σ_r	Invgamma	0.20	0.17	4.00	[0.10;0.38]
σ_π	Invgamma	0.20	0.17	4.00	[0.10;0.38]
σ_ν	Invgamma	0.20	0.17	4.00	[0.10;0.38]
η_y	Invgamma	0.20	0.17	4.00	[0.10;0.38]
η_π	Invgamma	0.20	0.17	4.00	[0.10;0.38]

The parameters pertaining to the nominal rigidities are distributed according to a beta distribution as they belong to the [0,1) interval. The average probability of price resetting is set to 0.25, implying that a firm expects to reset prices on average every four quarters. The average lagged price indexation parameter, γ, is set to 0.50.

The persistence parameter of the Taylor rule, ρ_r, has a beta prior over [0,1) so as to guarantee the stationarity of the rule. The prior distribution is centered on 0.5. The reaction to inflation, α_π, and output, α_y, is assumed to be positive, and with a gamma distribution centered on 1.5 and 0.125 respectively. These values correspond to values commonly used in the literature.

We have little knowledge of the processes that describe the forcing variables. We assume a beta distribution for the persistence parameter in order to guarantee the stationarity of the process. Each distribution is centered on 0.5. Volatility is assumed to follow an inverse gamma distribution (to guarantee positiveness). However, in order to take into account the limited knowledge we have regarding these process we impose non informative priors. The same strategy is applied for the noise process in the signal extraction model.

3 Estimation Results

Tables 3–7 report the posterior estimates for the five model versions. Figures 2–6 record the corresponding IRFs to the shocks in the models and Table 8 provides information on unconditional moments. The main findings are summarized below.

First, the estimated parameters are within the range typically found in the literature. Moreover, there is information in the sample that helps identify the parameters of the models as the comparison of the prior to posterior densities reveals. That is, the data are informative[13].

Second, there is a clear ranking of model fit as measured by the log-likelihood. As expected, within the models with perfect information, the model with backward indexation does better than the model without it. Within the models of imperfect information, the model with the most severe signal extraction problem (version 5) does best. Comparing across the two classes of models one can see that the latter class fares better with the exception of model 3 whose performance is identical to that of the baseline NK model (version 1). The main reason for this exception can be found in the fact that this model lacks persistence in the signal extraction problem. The complete absence of serial correlation in the cost-push shock allows the agents to quickly discriminate between this and the persistent technology shock, leaving little room for inertial dynamics. Inspection of the corresponding IRFs confirms this. A more direct confirmation can be found in Figure 11 which compares the paths of actual and perceived shocks in version 3. As can be seen, misperceptions are resolved fairly fast.

The comparisons of the IRFs and moments across models offers some clues regarding not only the reasons for the differences in their relative performance but also about the driving forces in each model. The most successful models (2,4 and 5) tend to exhibit inertia in inflation dynamics while the less successful ones (1 and 3) do not. Good performance at this front is the key source of strength of the model with indexation (model 2), as its performance regarding unconditional moments falls considerably short of that of the standard NK model[14], (version 1). To a slightly lower degree, the same can be said about the imperfect information model with variable inflation target (version 3). On the other hand, the strength of the model with measurement errors (version 5) is not as dependent on its good dynamic properties for inflation.

Inspection of the unconditional moments thus can shed some light on the possible reasons for

[13]The main exception to this regards rge second model in the identification of the indexation parameter and the parameter of persistence in the technology shock.

[14]Compare the three first columns of Table 8.

why the fifth model (measurement errors) outperforms the second one (perfect information, indexation). Namely, it matches better the behavior of inflation as well as that of the nominal interest rate, in particular, their volatility and procyclicality. But of particular interest is the fact that both of these models –and especially the model with imperfect information– do well in matching the *procyclicality* of the nominal interest rate. One of the best known and most embarrassing weaknesses of existing monetary models is their implication of a strongly countercyclical nominal rate. Our results thus provide some clues about what features a model should possess in order to overcome this problem.

Note that the main contributor to inflation inertia differs across the three specifications. In the perfect information model with indexation, inflation inertia comes from the two demand disturbances (the preference and the monetary policy shock). In the model with the variable inflation target, it arises exclusively from the inflation target shock while in the model with measurement errors, from the preference shock. Hence the three models differ regarding their identification of the cause of inertia between –direct– policy and non-policy factors.

The mechanism for inflation inertia in the model of indexation is well known so let us examine the mechanisms in the models with imperfect information. Figures 9–10 plot the actual and perceived paths of the shocks in the fourth and fifth models. As can be seen, the agents underreact to the shocks. It also takes them considerable time to recognize the true nature of the shock that has afflicted the economy. For instance, consider a positive inflation target shock (a higher inflation target). If the shock were perfectly observable, everybody's expectation and hence actual inflation would jump up and then, given the AR(1) assumption on the shock, it would monotonically decline. But if the shock is not observed then the agents cannot tell whether the lower nominal interest rate and higher inflation should be expected to persist in the future or not (it would not if the shock represented a purely transient monetary disturbance ($\epsilon_{t,t}$). As time goes one and the agents continue observing a lower nominal interest rate they become more convinced that n inflation target shock has taken place and inflation increase relative to the initial period. Due to the AR(1) nature of the shock, it reaches a peak after a few periods and then starts declining towards the initial steady steady state.

Judging the misperceptions story

Models with measurement errors and signal extraction are often criticized as requiring implausibly large amounts of imperfect information in order to deliver good results. In model 5 we have modelled the misperception problem as arising from the measurement error associated with preliminary data releases. Consequently, the plausibility of this assumption can be evaluated by comparing the properties of the noise of the model to that present in real time data. One way of doing so is by computing the volatility of the data revisions in the real world (for instance, initial minus final release) and the volatility between the actual and the perceived values in the model. The assumption here is that the actual value corresponds to the final revision while the perceived value to the preliminary release. Table 2 presents information on these volatilities for output and inflation. As can be seen, the amount of noise the model needs in trying to fit the data is not excessive. It is actually below the values observed in the real world. This is as it should be because some of the revisions in the data may contain predictable elements and thus the real time data volatilities represent an upper bound on the degree of misperception.

Table 2: Volatility of Revisions

	Data	NIII–NS
y	1.55 (0.91)	0.20
		[0.10,0.35]
Δy	0.73	0.21
		[0.12,0.31]
π	1.95	1.62
		[1.35,1.91]

Note: Standard deviation of revisions in the data and in the model. The first entry for output corresponds to linear detrending while the second one (in parenthesis) to HP filtering.

4 Conclusions

The concepts of imperfect information and misperceptions have a distinguished presence in macroeconomic theory going back to Lucas', 1972, and Kydland and Prescott's 1982, seminal work. Notwithstanding this history and also in spite of Orphanides' influential work on the role of misperceptions in the great inflation of the 70s, macroeconomic theory has more or less stayed clear of them. In this paper we have undertaken a comprehensive analysis of its role in US business cycles during the last 40 years using the New Keynesian model as the vehicle for the analysis. In particular, we have examined the performance of the model under different structures of information which are motivated by influential suggestions in the literature, such as unobserved variation in inflation target (Cogley and Sbordone, 2006) or, confusion between transitory and persistent shocks (Kydland and Prescott, 1982, Orphanides, 2002). We found that imperfect information plays an important role in the business cycles and can help the NK model match key dynamic properties of the data (inertia, persistence). Moreover, its success is not confined to providing an alternative mechanism – to other inertia inducing mechanisms such as inflation indexation or rule-of thump-agents– for persistence. It also helps the model better capture the behavior of nominal interest rate, since long the Achilles heel of monetary models. And this good performance does not hinge on implausibly large amount of informational frictions. Even the version with the most severe information problems (the one with measurement errors) requires a degree of misperception that is well within the range observed in real time data.

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A Tables

Table 3: Posterior Results – NIPI

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.38	0.38	0.38	0.06	[0.26, 0.50]
ξ	0.19	0.19	0.19	0.02	[0.15, 0.23]
$r^{\star}$	2.49	2.47	2.47	0.42	[1.65, 3.30]
$\pi^{\star}$	3.92	3.92	3.93	0.38	[3.16, 4.67]
ρ_r	0.77	0.77	0.77	0.03	[0.72, 0.82]
α_π	1.49	1.53	1.53	0.16	[1.20, 1.84]
α_y	0.11	0.13	0.12	0.05	[0.04, 0.22]
ρ_a	0.98	0.97	0.98	0.02	[0.93, 1.00]
ρ_χ	0.84	0.83	0.83	0.05	[0.74, 0.90]
σ_a	0.16	0.16	0.16	0.04	[0.09, 0.23]
σ_χ	0.11	0.14	0.12	0.05	[0.08, 0.23]
σ_r	0.29	0.30	0.30	0.02	[0.26, 0.34]
σ_ν	0.13	0.13	0.13	0.02	[0.09, 0.18]
σ_c	2.18	2.19	2.16	0.41	[1.41, 2.99]
σ_h	0.43	0.56	0.51	0.27	[0.11, 1.09]
Average log marginal density:					-702.976309

Table 4: Posterior Results – IPI

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.71	0.71	0.71	0.06	[0.58, 0.82]
ξ	0.17	0.17	0.17	0.02	[0.14, 0.21]
γ	0.95	0.93	0.94	0.04	[0.86, 0.99]
$r^\star$	2.58	2.57	2.57	0.40	[1.78, 3.34]
$\pi^\star$	4.04	4.05	4.05	0.39	[3.28, 4.81]
ρ_r	0.71	0.71	0.71	0.04	[0.63, 0.79]
α_π	1.15	1.19	1.17	0.11	[1.00, 1.38]
α_y	0.33	0.35	0.34	0.06	[0.23, 0.47]
ρ_a	0.15	0.19	0.18	0.10	[0.03, 0.38]
ρ_χ	0.52	0.51	0.51	0.09	[0.33, 0.67]
σ_a	0.09	0.09	0.09	0.01	[0.07, 0.11]
σ_χ	0.49	0.50	0.50	0.07	[0.36, 0.65]
σ_r	0.22	0.22	0.22	0.02	[0.19, 0.26]
σ_ν	0.20	0.21	0.20	0.02	[0.18, 0.24]
σ_c	1.07	1.15	1.12	0.29	[0.62, 1.71]
σ_h	0.21	0.29	0.26	0.15	[0.06, 0.60]
Average log marginal density:					-692.790527

Table 5: Posterior Results – NIII

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.38	0.37	0.37	0.06	[0.26, 0.49]
ξ	0.20	0.20	0.20	0.02	[0.16, 0.23]
$r^\star$	2.48	2.46	2.47	0.42	[1.66, 3.30]
$\pi^\star$	3.92	3.93	3.93	0.38	[3.18, 4.68]
ρ_r	0.77	0.77	0.77	0.03	[0.72, 0.82]
α_π	1.51	1.56	1.56	0.15	[1.26, 1.87]
α_y	0.10	0.12	0.12	0.05	[0.04, 0.22]
ρ_a	0.99	0.98	0.99	0.01	[0.96, 1.00]
ρ_χ	0.84	0.84	0.84	0.03	[0.78, 0.90]
σ_a	0.16	0.16	0.16	0.03	[0.10, 0.23]
σ_χ	0.11	0.13	0.12	0.03	[0.08, 0.18]
σ_r	0.29	0.30	0.30	0.02	[0.26, 0.34]
σ_ν	0.14	0.14	0.14	0.02	[0.10, 0.19]
σ_c	2.20	2.20	2.17	0.42	[1.41, 3.01]
σ_h	0.39	0.53	0.49	0.27	[0.10, 1.05]
Average log marginal density:					-701.549770

Table 6: Posterior Results – NIII–CS

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.49	0.44	0.44	0.09	[0.27, 0.61]
ξ	0.19	0.20	0.20	0.03	[0.16, 0.25]
$r^\star$	2.56	2.53	2.53	0.44	[1.69, 3.39]
$\pi^\star$	4.00	4.01	4.00	0.49	[3.02, 4.96]
ρ_r	0.57	0.56	0.57	0.06	[0.44, 0.68]
α_π	1.25	1.64	1.64	0.30	[1.08, 2.19]
α_y	0.34	0.24	0.24	0.11	[0.06, 0.43]
ρ_a	0.93	0.94	0.96	0.05	[0.83, 1.00]
ρ_χ	0.64	0.70	0.72	0.10	[0.50, 0.86]
ρ_π	0.90	0.83	0.87	0.09	[0.63, 0.94]
σ_a	0.12	0.18	0.16	0.06	[0.10, 0.29]
σ_χ	0.31	0.24	0.22	0.11	[0.09, 0.45]
σ_r	0.25	0.25	0.25	0.04	[0.17, 0.33]
σ_π	0.16	0.21	0.19	0.07	[0.11, 0.36]
σ_c	1.74	1.93	1.89	0.43	[1.15, 2.80]
σ_h	0.30	0.58	0.52	0.32	[0.09, 1.20]
Average log marginal density:					-684.148410

Table 7: Posterior Results – NIII–NS

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.36	0.37	0.37	0.06	[0.25, 0.50]
ξ	0.19	0.19	0.19	0.02	[0.15, 0.23]
$r^\star$	2.50	2.47	2.46	0.51	[1.48, 3.49]
$\pi^\star$	4.00	4.02	4.02	0.45	[3.16, 4.93]
ρ_r	0.41	0.43	0.43	0.07	[0.28, 0.57]
α_π	1.63	1.64	1.63	0.21	[1.23, 2.06]
α_y	0.19	0.21	0.20	0.06	[0.10, 0.32]
ρ_a	0.89	0.89	0.89	0.03	[0.84, 0.94]
ρ_χ	0.79	0.78	0.78	0.05	[0.68, 0.87]
ρ_π	0.15	0.16	0.16	0.02	[0.12, 0.21]
σ_a	0.36	0.39	0.39	0.06	[0.28, 0.51]
σ_χ	0.14	0.14	0.14	0.02	[0.10, 0.18]
σ_r	0.21	0.21	0.21	0.02	[0.17, 0.25]
σ_π	0.18	0.26	0.22	0.13	[0.09, 0.52]
η_y	2.73	3.36	3.11	1.21	[1.58, 5.81]
η_π	0.72	0.79	0.75	0.25	[0.36, 1.27]
σ_c	0.23	0.32	0.29	0.17	[0.07, 0.66]
Average log marginal density:					-669.201964

Table 8: Moments

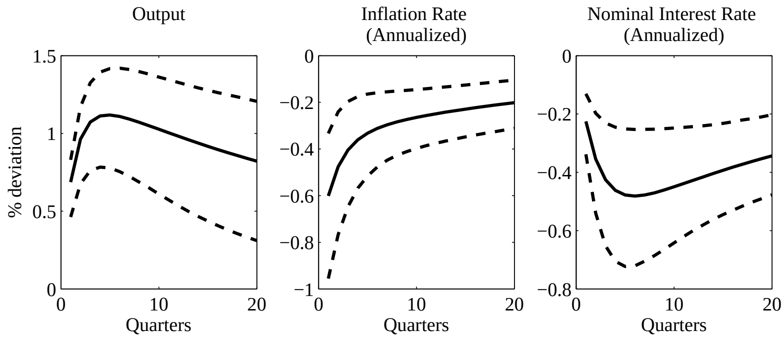
	Data	NIPI	IPI	NIII	NIII CS	NIII Noise
σ_y	1.57	1.56	1.26	1.52	1.53	1.16
		[1.29,1.88]	[1.06,1.47]	[1.27,1.80]	[1.23,1.85]	[1.00,1.34]
σ_π	1.14	1.59	1.86	1.56	1.57	1.34
		[1.37,1.82]	[1.62,2.11]	[1.37,1.76]	[1.37,1.77]	[1.18,1.51]
σ_R	1.59	1.62	2.15	1.59	1.94	1.57
		[1.44,1.82]	[1.87,2.46]	[1.41,1.77]	[1.60,2.27]	[1.37,1.77]
$\rho(\pi, y)$	0.14	0.10	0.33	0.20	0.07	0.31
		[-0.07,0.26]	[0.20,0.47]	[0.04,0.35]	[-0.15,0.24]	[0.21,0.41]
$\rho(R, y)$	0.36	-0.23	0.13	-0.16	-0.16	0.36
		[-0.36,-0.09]	[-0.04,0.31]	[-0.29,-0.04]	[-0.33,0.02]	[0.19,0.51]
$\rho_y(1)$	0.87	0.83	0.75	0.82	0.82	0.76
		[0.78,0.87]	[0.70,0.80]	[0.78,0.86]	[0.77,0.87]	[0.71,0.81]
$\rho_\pi(1)$	0.48	0.49	0.62	0.48	0.55	0.38
		[0.44,0.54]	[0.57,0.66]	[0.43,0.53]	[0.50,0.60]	[0.27,0.49]
$\rho_R(1)$	0.82	0.71	0.79	0.70	0.77	0.74
		[0.66,0.76]	[0.75,0.82]	[0.66,0.75]	[0.72,0.81]	[0.69,0.79]
$\rho_y(2)$	0.69	0.58	0.40	0.57	0.57	0.46
		[0.51,0.65]	[0.32,0.47]	[0.51,0.63]	[0.46,0.65]	[0.39,0.53]
$\rho_\pi(2)$	0.30	0.22	0.28	0.21	0.27	0.21
		[0.17,0.27]	[0.22,0.35]	[0.16,0.25]	[0.21,0.33]	[0.14,0.28]
$\rho_R(2)$	0.58	0.46	0.49	0.44	0.49	0.45
		[0.39,0.52]	[0.43,0.55]	[0.39,0.50]	[0.42,0.55]	[0.38,0.51]
$\rho_y(4)$	0.26	0.15	-0.11	0.14	0.13	0.02
		[0.07,0.22]	[-0.18,-0.05]	[0.09,0.19]	[0.01,0.21]	[-0.03,0.08]
$\rho_\pi(4)$	0.25	-0.06	-0.11	-0.07	-0.03	-0.01
		[-0.09,-0.03]	[-0.15,-0.06]	[-0.10,-0.04]	[-0.07,0.02]	[-0.03,0.02]
$\rho_R(4)$	0.25	0.07	-0.04	0.06	0.03	0.01
		[0.02,0.12]	[-0.10,0.03]	[0.02,0.11]	[-0.02,0.09]	[-0.05,0.07]

Note: NIPI: No indexation, Perfect Info., IPI: Indexation, Perfect. Info, NIII: No Indexation, Imperfect Info., NIII CS: No indexation, Imperfect Info., Cogley–Sbordone, NIII noise: No indexation, Imperfect Info., noisy signals.

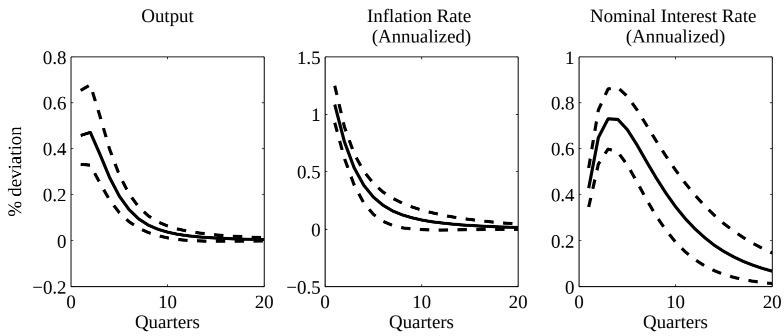
B Figures

Figure 2: Impulse Response Functions – NIPI

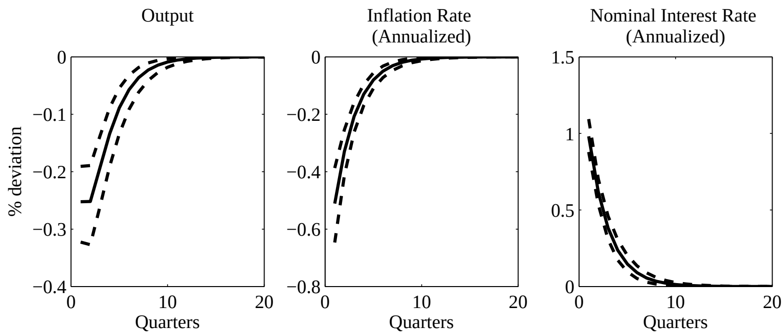
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Cost Push Shock

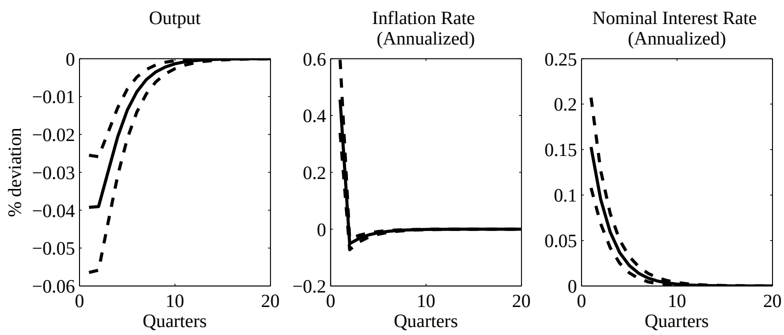


Figure 3: Impulse Response Functions – IPI

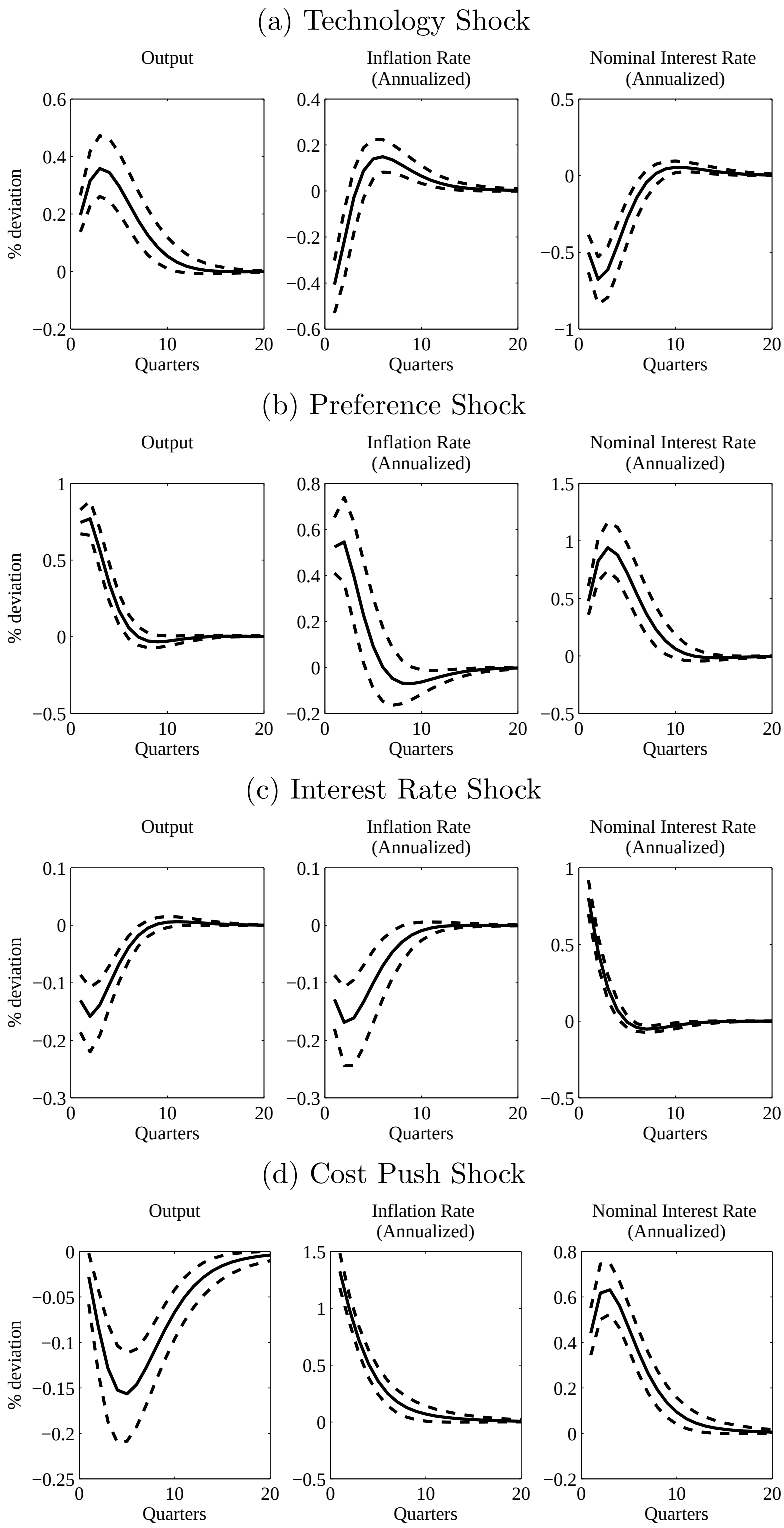


Figure 4: Impulse Response Functions – NIII

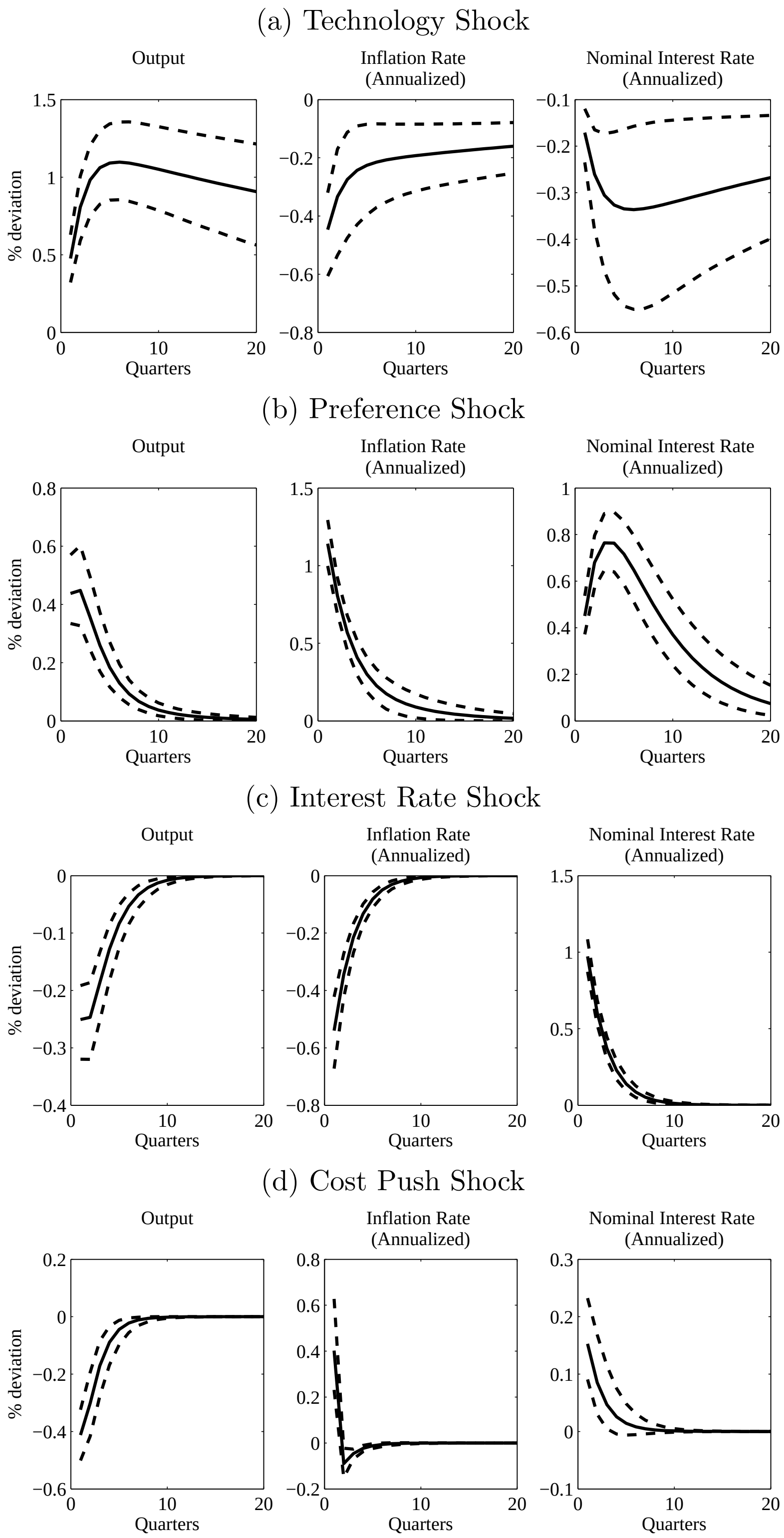
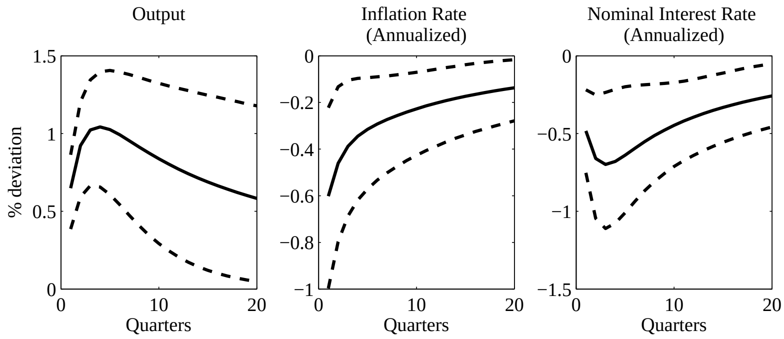
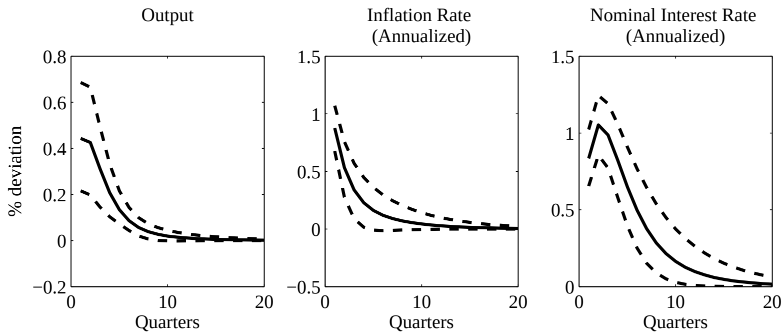


Figure 5: Impulse Response Functions – NIII–CS

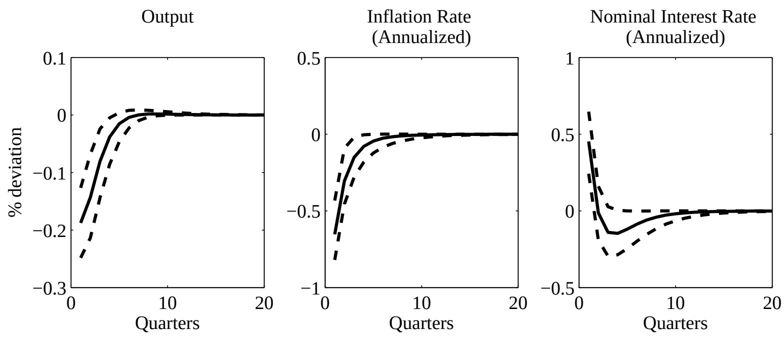
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Inflation Target Shock

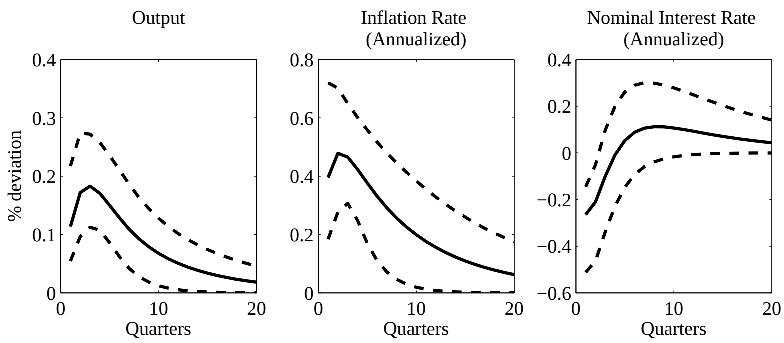


Figure 6: Impulse Response Functions – NIII–NS

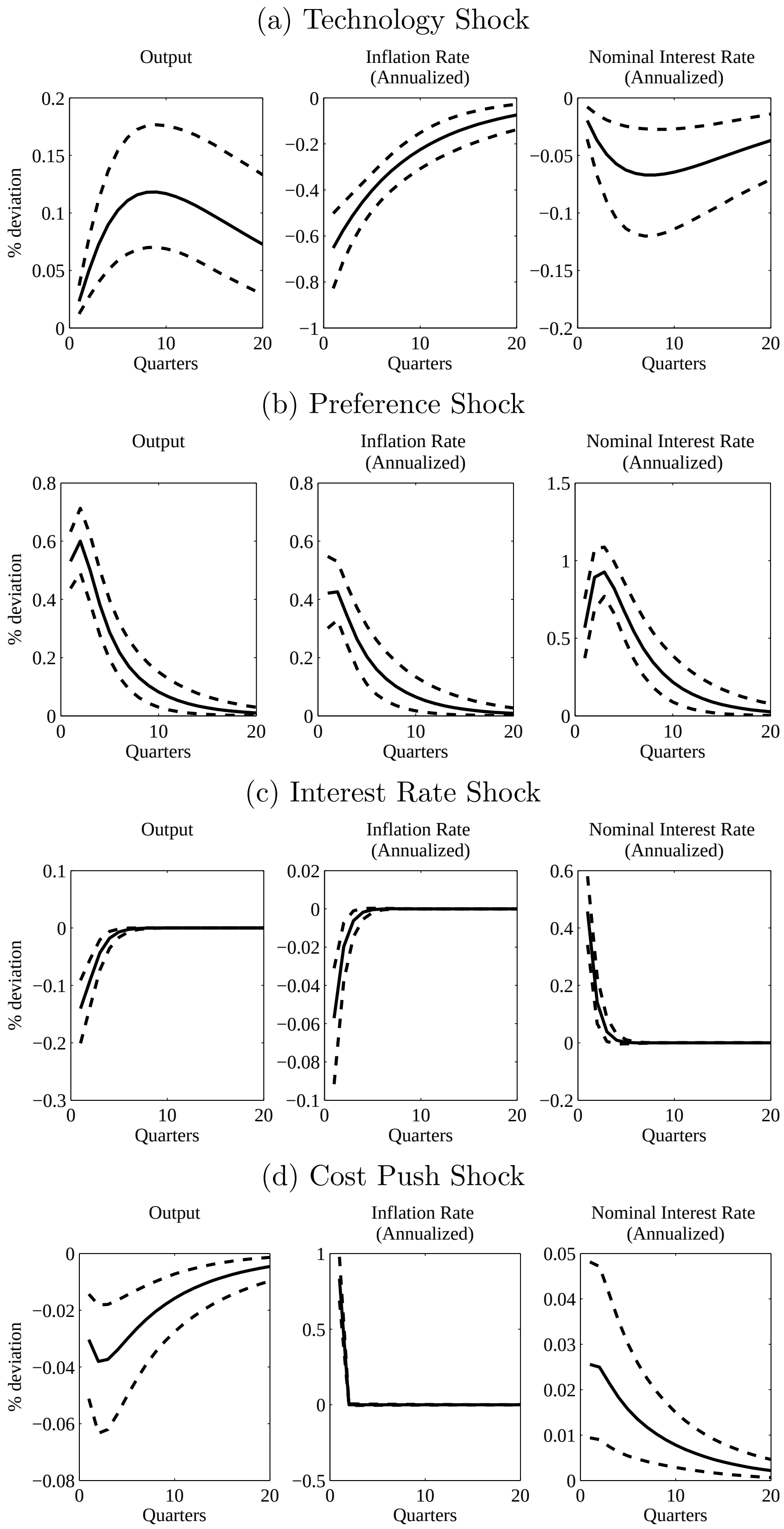
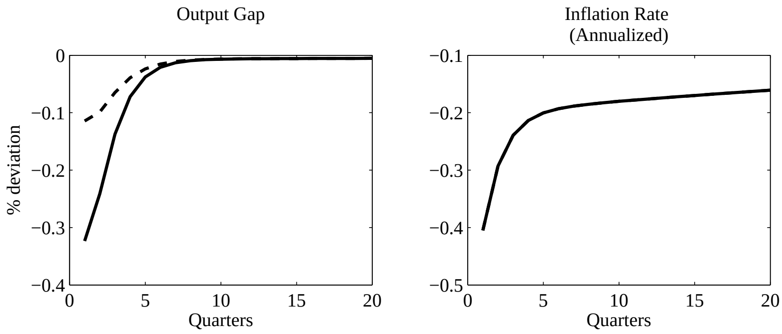


Figure 7: IRF: Actual vs Perceived — NIII

(a) Technology Shock



(b) Cost Push Shock

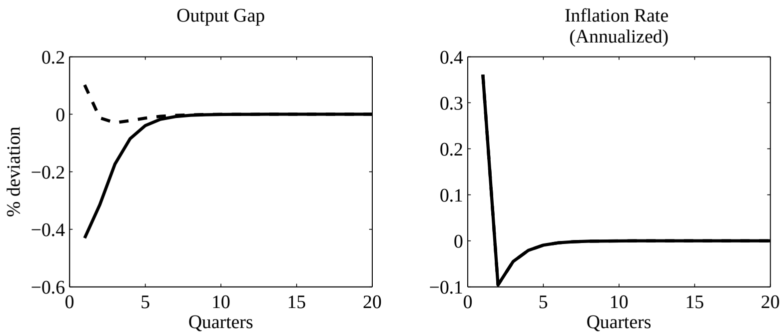
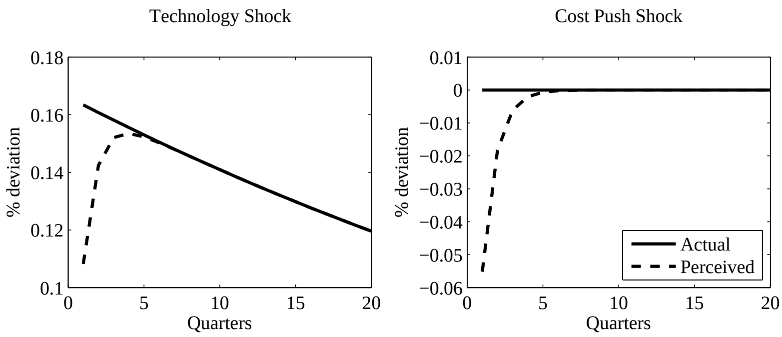


Figure 8: IRF: Actual vs Perceived — NIII

(a) Technology Shock



(b) Cost Push Shock

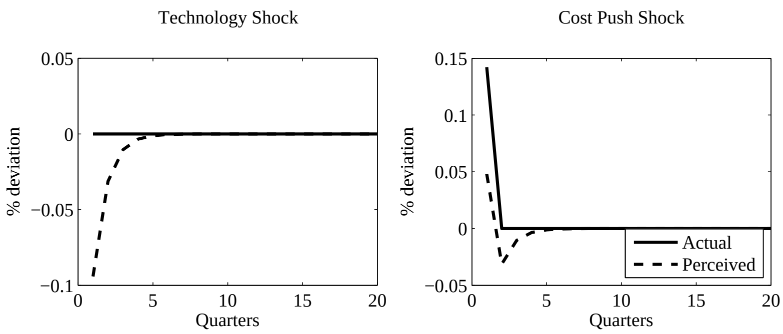
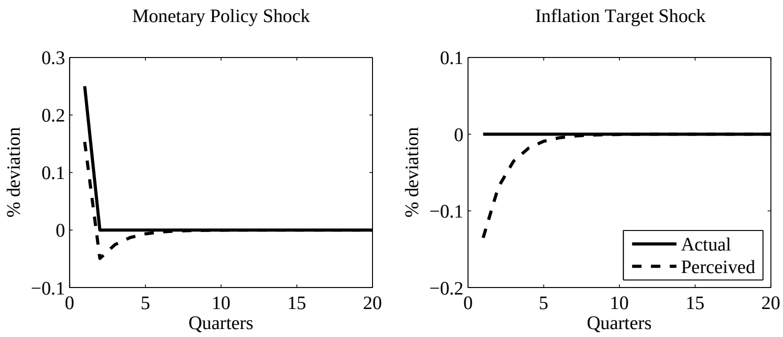


Figure 9: IRF: Actual vs Perceived — NIII–CS

(a) Monetary Policy Shock



(b) Inflation Target Shock

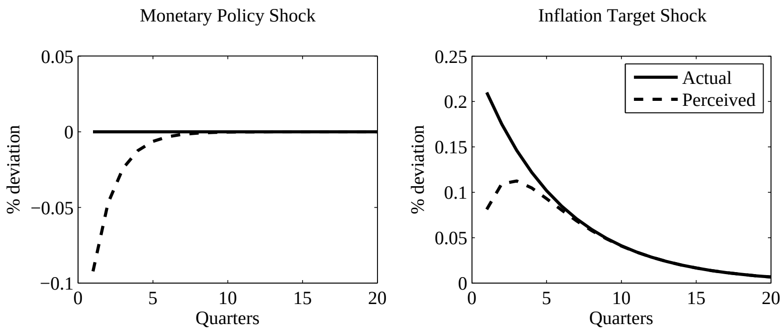
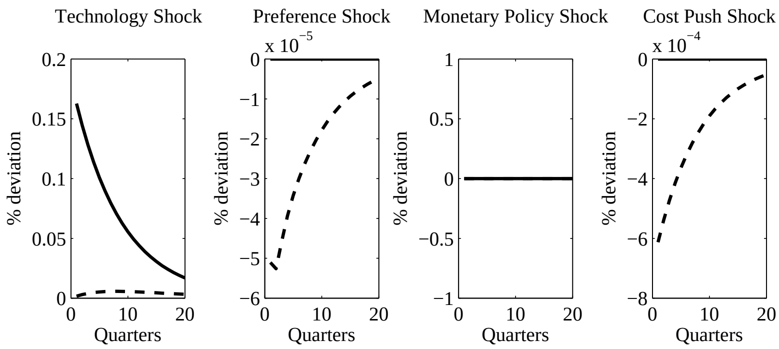
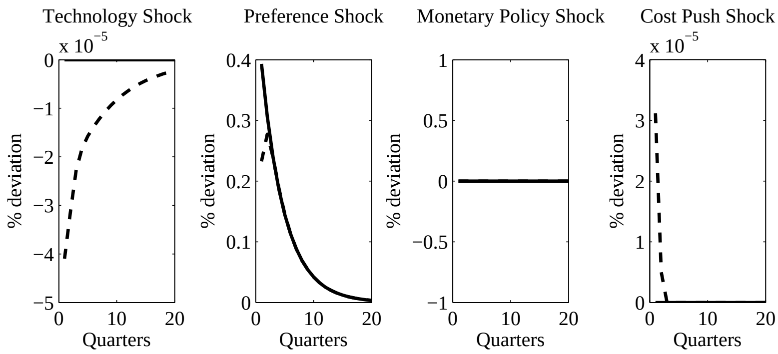


Figure 10: IRF: Actual vs Perceived — NIII–NS

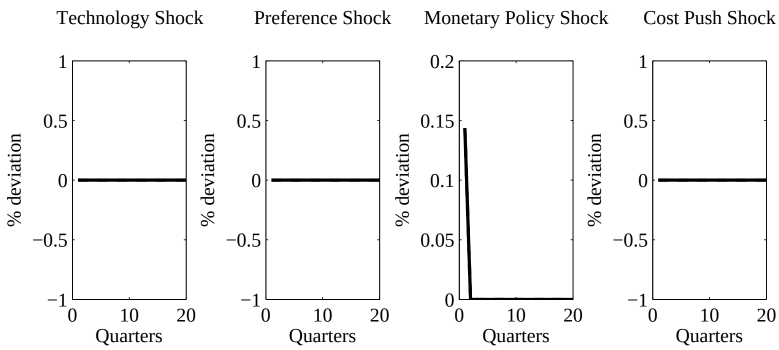
(a) Technology Shock



(b) Preference Shock



(c) Monetary Policy Shock



(d) Cost Push Shock

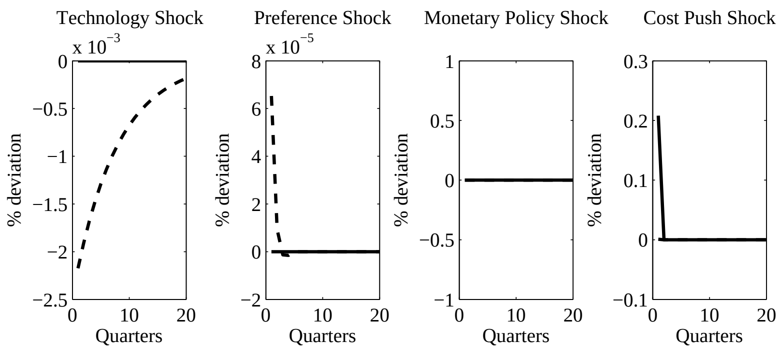
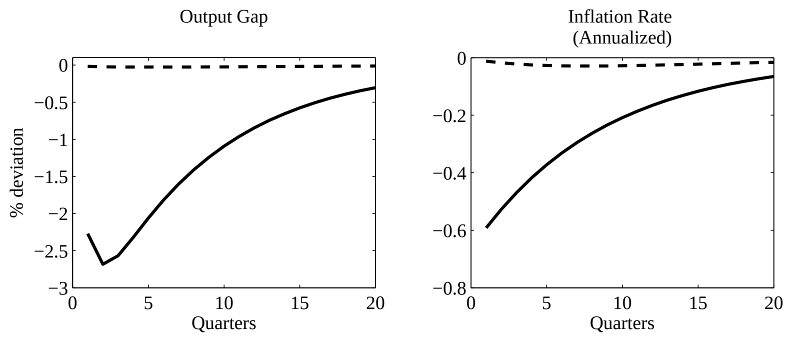
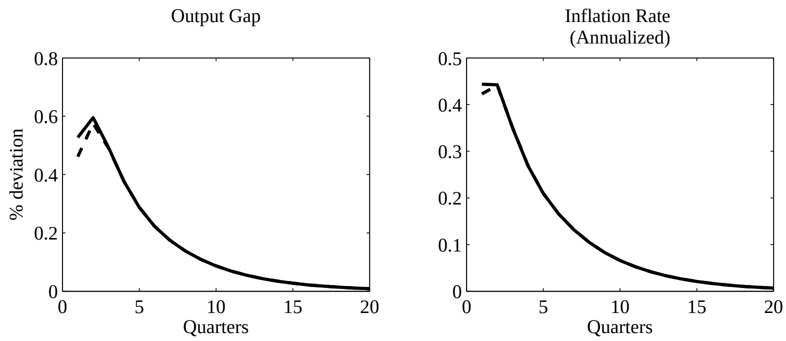


Figure 11: IRF: Actual vs Perceived — NIII–NS

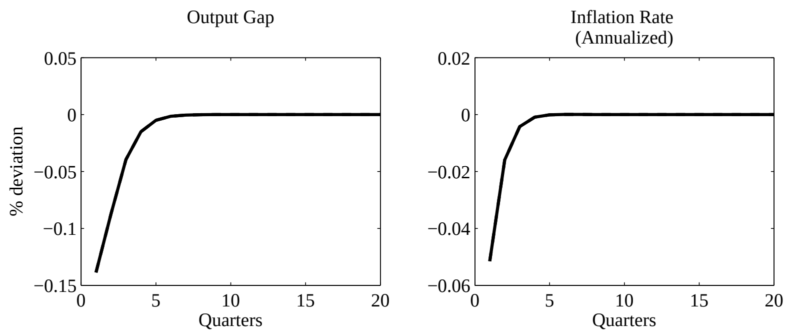
(a) Technology Shock



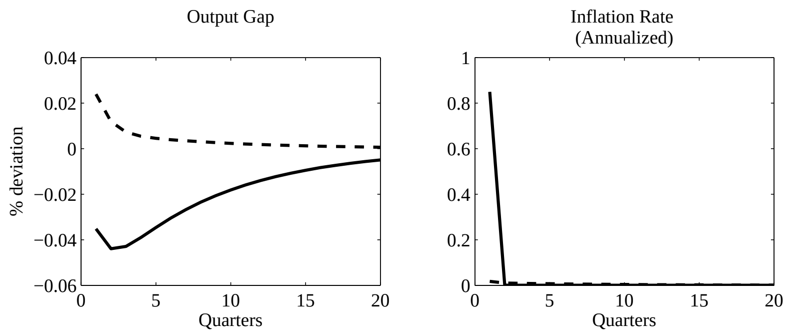
(b) Preference Shock



(c) Monetary Policy Shock



(d) Cost Push Shock



C Appendix: Solution Method

Let the state of the economy be represented by two vectors $\widetilde{X}_t^b$ and $\widetilde{X}_t^f$. The first one includes the predetermined (backward looking) state variables, i.e. $\widetilde{X}_t^b = (R_{t-1}, \widetilde{z}_t, \widetilde{g}_t, \widetilde{\varepsilon}_t^R)'$, whereas the second one consists of the forward looking state variables, i.e. $\widetilde{X}_t^f = (\widetilde{y}_t, \widetilde{\pi}_t)'$. The model admits the following representation

$$M_0 \begin{pmatrix} \widetilde{X}_{t+1}^b \\ \mathbb{E}_t \widetilde{X}_{t+1}^f \end{pmatrix} + M_1 \begin{pmatrix} \widetilde{X}_t^b \\ \widetilde{X}_t^f \end{pmatrix} = M_2 \varepsilon_{t+1} \tag{19}$$

Let us denote the signal process by $\{S_t\}$. The measurement equation relates the state of the economy to the signal:

$$S_t = C \begin{pmatrix} \widetilde{X}_t^b \\ \widetilde{X}_t^f \end{pmatrix} + v_t. \tag{20}$$

Finally u and v are assumed to be normally distributed covariance matrices Σ_{uu} and Σ_{vv} respectively and $E(uv') = 0$.

$X_{t+i|t} = E(X_{t+i}|\mathscr{I}_t)$ for $i \geqslant 0$ and where $\mathscr{I}_t$ denotes the information set available to the agents at the beginning of period t. The information set available to the agents consists of *i)* the structure of the model and *ii)* the history of the observable signals they are given in each period:

$$\mathscr{I}_t = \{S_{t-j}, j \geqslant 0, M_0, M_1, M_2, C, \Sigma_{uu}, \Sigma_{vv}\}$$

The information structure of the agents is described fully by the specification of the signals.

C.0.1 Solving the system

Step 1: We first solve for the expected system:

$$M_0 \begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} + M_1) \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} = \tag{21}$$

which rewrites as

$$\begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} = W \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \tag{22}$$

where

$$W = -M_0^{-1} M_1$$

After getting the Jordan form associated to (22) and applying standard methods for eliminating bubbles, we get

$$X_{t|t}^f = G X_{t|t}^b$$

From which we get

$$X_{t+1|t}^b = (W_{bb} + W_{bf} G) X_{t|t}^b = W^b X_{t|t}^b \tag{23}$$

$$X_{t+1|t}^f = (W_{fb} + W_{ff} G) X_{t|t}^b = W^f X_{t|t}^b \tag{24}$$

Step 2: We have

$$M_0 \begin{pmatrix} X^b_{t+1} \\ X^f_{t+1|t} \end{pmatrix} + M_1 \begin{pmatrix} X^b_t \\ X^f_t \end{pmatrix} = M_2 u_{t+1}$$

Taking expectations, we have

$$M_0 \begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + M_1 \begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} = 0$$

Subtracting, we get

$$M_0 \begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} + M_1 \begin{pmatrix} X^b_t - X^b_{t|t} \\ X^f_t - X^f_{t|t} \end{pmatrix} = M_2 u_{t+1} \tag{25}$$

which rewrites

$$\begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} = W^c \begin{pmatrix} X^b_t - X^b_{t|t} \\ X^f_t - X^f_{t|t} \end{pmatrix} + M_0^{-1} M_2 u_{t+1} \tag{26}$$

where, $W^c = -M_0^{-1} M_1$. Hence, considering the second block of the above matrix equation, we get

$$W^c_{fb}(X^b_t - X^b_{t|t}) + W^c_{ff}(X^f_t - X^f_{t|t}) = 0$$

which gives

$$X^f_t = F^0 X^b_t + F^1 X^b_{t|t}$$

with $F^0 = -W^c_{ff}{}^{-1} W^c_{fb}$ and $F^1 = G - F^0$.

Now considering the first block, we have

$$X^b_{t+1} = X^b_{t+1|t} + W^c_{bb}(X^b_t - X^b_{t|t}) + W^c_{bf}(X^f_t - X^f_{t|t}) + M^2 u_{t+1}$$

from which we get, using (23)

$$X^b_{t+1} = M^0 X^b_t + M^1 X^b_{t|t} + M^2 u_{t+1}$$

with $M^0 = W^c_{bb} + W^c_{bf} F^0$, $M^1 = W^b - M^0$ and $M^2 = M_0^{-1} M_2$.

We also have

$$S_t = C_b X^b_t + C_f X^f_t + v_t$$

from which we get

$$S_t = S^0 X^b_t + S^1 X^b_{t|t} + v_t$$

where $S^0 = C_b + C_f F^0$ and $S^1 = C_f F^1$

C.0.2 Filtering

Since our solution involves terms in $X^b_{t|t}$, we would like to compute this quantity. However, the only information we can exploit is a signal S_t that was described previously. We therefore use a Kalman filter approach to compute the optimal prediction of $X^b_{t|t}$.

In order to recover the Kalman filter, it is a good idea to think in terms of expectation errors. Therefore, let us define

$$\widetilde{X}^b_t = X^b_t - X^b_{t|t-1}$$

and

$$\widetilde{S}_t = S_t - S_{t|t-1}$$

Note that since S_t depends on $X^b_{t|t}$, only the signal relying on $\widetilde{S}_t = S_t - S^1 X^b_{t|t}$ can be used to infer anything on $X^b_{t|t}$. Therefore, the policy maker revises its expectations using a linear rule depending on $\widetilde{S}^e_t = S_t - S^1 X^b_{t|t}$. The filtering equation then writes

$$X^b_{t|t} = X^b_{t|t-1} + K(\widetilde{S}^e_t - \widetilde{S}^e_{t|t-1}) = X^b_{t|t-1} + K(S^0 \widetilde{X}^b_t + v_t)$$

where K is the filter gain matrix, that we would like to compute.

The first thing we have to do is to rewrite the system in terms of state–space representation. Since $S_{t|t-1} = (S^0 + S^1)X^b_{t|t-1}$, we have

$$\begin{aligned}
\widetilde{S}_t &= S^0(X^b_t - X^b_{t|t}) + S^1(X^b_{t|t} - X^b_{t|t-1}) + v_t \\
&= S^0 \widetilde{X}^b_t + S^1 K(S^0 \widetilde{X}^b_t + v_t) + v_t \\
&= S^\star \widetilde{X}^b_t + \nu_t
\end{aligned}$$

where $S^\star = (I + S^1 K)S^0$ and $\nu_t = (I + S^1 K)v_t$.

Now, consider the law of motion of backward state variables, we get

$$\begin{aligned}
\widetilde{X}^b_{t+1} &= M^0(X^b_t - X^b_{t|t}) + M^2 u_{t+1} \\
&= M^0(X^b_t - X^b_{t|t-1} - X^b_{t|t} + X^b_{t|t-1}) + M^2 u_{t+1} \\
&= M^0 \widetilde{X}^b_t - M^0(X^b_{t|t} + X^b_{t|t-1}) + M^2 u_{t+1} \\
&= M^0 \widetilde{X}^b_t - M^0 K(S^0 \widetilde{X}^b_t + v_t) + M^2 u_{t+1} \\
&= M^\star \widetilde{X}^b_t + \omega_{t+1}
\end{aligned}$$

where $M^\star = M^0(I - KS^0)$ and $\omega_{t+1} = M^2 u_{t+1} - M^0 K v_t$.

We therefore end–up with the following state–space representation

$$\widetilde{X}^b_{t+1} = M^\star \widetilde{X}^b_t + \omega_{t+1} \tag{27}$$

$$\widetilde{S}_t = S^\star \widetilde{X}^b_t + \nu_t \tag{28}$$

For which the Kalman filter is given by

$$\widetilde{X}^b_{t|t} = \widetilde{X}^b_{t|t-1} + PS^{\star\prime}(S^\star PS^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widetilde{X}^b_t + \nu_t)$$

But since $\widetilde{X}^b_{t|t}$ is an expectation error, it is not correlated with the information set in $t-1$, such that $\widetilde{X}^b_{t|t-1} = 0$. The prediction formula for $\widetilde{X}^b_{t|t}$ therefore reduces to

$$\widetilde{X}^b_{t|t} = PS^{\star\prime}(S^\star PS^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widetilde{X}^b_t + \nu_t) \tag{29}$$

where P solves

$$P = M^\star PM^{\star\prime} + \Sigma_{\omega\omega}$$

and $\Sigma_{\nu\nu} = (I + S^1K)\Sigma_{vv}(I + S^1K)'$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime}$

Note however that the above solution is obtained for a given K matrix that remains to be computed. We can do that by using the basic equation of the Kalman filter:

$$\begin{aligned} X^b_{t|t} &= X^b_{t|t-1} + K(\widetilde{S}^e_t - \widetilde{S}^e_{t|t-1}) \\ &= X^b_{t|t-1} + K(S_t - S^1X^b_{t|t} - (S_{t|t-1} - S^1X^b_{t|t-1})) \\ &= X^b_{t|t-1} + K(S_t - S^1X^b_{t|t} - S^0X^b_{t|t-1}) \end{aligned}$$

Solving for $X^b_{t|t}$, we get

$$\begin{aligned} X^b_{t|t} &= (I + KS^1)^{-1}(X^b_{t|t-1} + K(S_t - S^0X^b_{t|t-1})) \\ &= (I + KS^1)^{-1}(X^b_{t|t-1} + KS^1X^b_{t|t-1} - KS^1X^b_{t|t-1} + K(S_t - S^0X^b_{t|t-1})) \\ &= (I + KS^1)^{-1}(I + KS^1)X^b_{t|t-1} + (I + KS^1)^{-1}K(S_t - (S^0 + S^1)X^b_{t|t-1})) \\ &= X^b_{t|t-1} + (I + KS^1)^{-1}K\widetilde{S}_t \\ &= X^b_{t|t-1} + K(I + S^1K)^{-1}\widetilde{S}_t \\ &= X^b_{t|t-1} + K(I + S^1K)^{-1}(S^\star\widetilde{X}^b_t + \nu_t) \end{aligned}$$

where we made use of the identity $(I + KS^1)^{-1}K \equiv K(I + S^1K)^{-1}$. Hence, identifying to (29), we have

$$K(I + S^1K)^{-1} = PS^{\star\prime}(S^\star PS^{\star\prime} + \Sigma_{\nu\nu})^{-1}$$

remembering that $S^\star = (I + S^1K)S^0$ and $\Sigma_{\nu\nu} = (I + S^1K)\Sigma_{vv}(I + S^1K)'$, we have

$$K(I{+}S^1K)^{-1} = PS^{0\prime}(I{+}S^1K)'((I{+}S^1K)S^0PS^{0\prime}(I{+}S^1K)'{+}(I{+}S^1K)\Sigma_{vv}(I{+}S^1K)')^{-1}(I{+}S^1K)S^0$$

which rewrites as

$$\begin{aligned} K(I + S^1K)^{-1} &= PS^{0\prime}(I + S^1K)'\left[(I + S^1K)(S^0PS^{0\prime} + \Sigma_{vv})(I + S^1K)'\right]^{-1} \\ K(I + S^1K)^{-1} &= PS^{0\prime}(I + S^1K)'(I + S^1K)'^{-1}(S^0PS^{0\prime} + \Sigma_{vv})^{-1}(I + S^1K)^{-1} \end{aligned}$$

Hence, we obtain

$$K = PS^{0\prime}(S^0PS^{0\prime} + \Sigma_{vv})^{-1} \tag{30}$$

Now, recall that

$$P = M^{\star}PM^{\star\prime} + \Sigma_{\omega\omega}$$

Remembering that $M^{\star} = M^0(I + KS^0)$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime}$, we have

$$\begin{aligned} P &= M^0(I - KS^0)P\left[M^0(I - KS^0)\right]' + M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \\ &= M^0\left[(I - KS^0)P(I - S^{0\prime}K') + K\Sigma_{vv}K'\right]M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \end{aligned}$$

Plugging the definition of K in the latter equation, we obtain

$$P = M^0\left[P - PS^{0\prime}(S^0PS^{0\prime} + \Sigma_{vv})^{-1}S^0P\right]M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \tag{31}$$

D Summary

We end–up with the system of equations:

$$\begin{aligned} X^b_{t+1} &= M^0X^b_t + M^1X^b_{t|t} + M^2u_{t+1} && (32) \\ S_t &= S^0_bX^b_t + S^1_bX^b_{t|t} + v_t && (33) \\ X^f_t &= F^0X^b_t + F^1X^b_{t|t} && (34) \\ X^b_{t|t} &= X^b_{t|t-1} + K(S^0(X^b_t - X^b_{t|t-1}) + v_t) && (35) \\ X^b_{t+1|t} &= (M^0 + M^1)X^b_{t|t} && (36) \end{aligned}$$

which fully describe the dynamics of our economy.

This may be recast as a standard state–space problem

$$\begin{aligned} X^b_{t+1|t+1} &= X^b_{t+1|t} + K(S^0(X^b_{t+1} - X^b_{t+1|t}) + v_{t+1}) \\ &= (M^0 + M^1)X^b_{t|t} + K(S^0(M^0X^b_t + M^1X^b_{t|t} + M^2u_{t+1} - (M^0 + M^1)X^b_{t|t}) + v_{t+1}) \\ &= KS^0M^0X^b_t + ((I - KS^0)M^0 + M^1)X^b_{t|t} + KS^0M^2u_{t+1} + Kv_{t+1} \end{aligned}$$

Then

$$\begin{pmatrix} X^b_{t+1} \\ X^b_{t+1|t+1} \end{pmatrix} = M_{\text{X}} \begin{pmatrix} X^b_t \\ X^b_{t|t} \end{pmatrix} + M_{\text{E}} \begin{pmatrix} u_{t+1} \\ v_{t+1} \end{pmatrix}$$

where

$$M_{\text{X}} = \begin{pmatrix} M^0 & M^1 \\ KS^0M^0 & ((I - KS^0)M^0 + M^1) \end{pmatrix} \text{ and } M_{\text{E}} = \begin{pmatrix} M^2 & 0 \\ KS^0M^2 & K \end{pmatrix}$$

and

$$X^f_t = M_{\text{F}} \begin{pmatrix} X^b_t \\ X^b_{t|t} \end{pmatrix}$$

where

$$M_{\mathrm{F}} = \begin{pmatrix} F^0 & F^1 \end{pmatrix}$$