Friday Lecture 1
Liquidity Hoarding

August 10, 2012

Motivation

- As early as August 2007, European banks reported difficulty borrowing in the interbank market (Acharya and Merrouche, 2009; Heider, Hoerova and Holthausen, 2008; Ashcraft, McAndrews and Skeie, 2009).
- Central banks in Europe and US forced to provide liquidity to financial system
- Two explanations for interbank "freeze"
  - counterparty risk
  - liquidity hoarding
- These explanations are not unrelated, of course

Model

- A large number of bankers hold two types of assets, liquid assets ("cash") and illiquid assets ("assets")
- Bankers are subject to stochastic liquidity shocks, requiring payment of one unit of cash
- Illiquid bankers sell assets to obtain cash
- If asset prices are too low (cost of cash is too high), bankers may choose to default
- Default results in bankruptcy and liquidation
- Bankers weigh the opportunity cost of holding cash against the risk of costly bankruptcy

Time, goods and assets

- *Time*: Time is divided into four dates, $t = 0, 1, 2, 3$
- *Goods:* There is a single consumption good at each date
- *Assets*: There are two assets, a liquid asset ('cash') and an illiquid asset ('the asset')
- *Returns*:
  - one unit of cash can be turned into one unit of consumption at any date;
  - one unit of the asset pays a return of $R > 1$ units of cash (consumption) at date 3

Bankers

- *Bankers*: There is a continuum of ex ante identical, risk-neutral bankers, $i \in [0, 1]$
- *Endowments:* each banker is endowed with one unit of cash and one unit of the asset at date 0
- *Preferences:* a banker values consumption only at date 0 and date 3

$$U(c_0, c_3) = \rho c_0 + c_3, \qquad (\rho > 1)$$

- *Activities:*
  - *at date* 0: bankers choose the level of liquidity in their portfolios
  - *at dates* 1 *and* 2: bankers receive liquidity shocks and trade assets to obtain liquidity
  - *at date* 3: asset returns are realized

Debt and default

- *Liquidity shocks*: A banker receives a liquidity shock at date $t = 1, 2, 3$ with probability

$$\begin{array}{ll} \theta_1 & \text{if } t = 1 \\ (1-\theta_1)\,\theta_2 & \text{if } t = 2 \\ (1-\theta_1)\,(1-\theta_2) & \text{if } t = 3 \end{array}$$

  where $\theta_1 \sim F_1(\theta_1)$, $\theta_2 \sim F_2(\theta_2)$, and θ_1 and θ_2 have support $[0, 1]$.

- *Default*: On receiving a shock, a banker must either pay one unit of cash to discharge a senior claim or default and suffer a loss of 100% of the value of his portfolio

The planner's problem

- The planner controls the economy in two ways:
  - he accumulates and distributes liquidity
  - and he reallocates payoffs at date 3
- Formally, the planner chooses
  - an initial cash balance m_0
  - an amount $x_1(\theta_1)$ to distribute in state θ_1 at date 1 and a balance $m_1(\theta_1)$ to carry forward, where

$$x_1(\theta_1) = m_0 - m_1(\theta_1)$$

  - an amount $x_2(\theta_1, \theta_2)$ to distribute to bankers in state (θ_1, θ_1) at date 2 and a balance $m_1(\theta_1, \theta_2)$ to carry forward, where

$$x_2(\theta_1, \theta_2) = m_1(\theta_1) - m_2(\theta_1, \theta_2)$$

The planner's solution

- Suppose the planner has $m_1(\theta_1)$ units of cash at date 2 and the state is (θ_1, θ_2); the optimal policy is to choose

$$x_2(\theta_1, \theta_2) = \min\{m_1(\theta_1), (1-\theta_1)\theta_2\}$$

  and $m_2(\theta_1, \theta_2) = m_1(\theta_1) - x_2(\theta_1, \theta_2)$
- Suppose the planner has m_0 units of cash at date 1 and the state is θ_1; the optimal policy is to choose

$$x_1(\theta_1) = \min\{m_0, \theta_1\}$$

  and $m_1(\theta_1) = m_0 - x_1(\theta_1)$
- At date 0, the planner holds m_0 units of cash, where m_0 satisfies

$$(R-1)\Pr[\theta_1 + (1-\theta_1)\theta_2 > m_0] + 1 = \rho$$

The constrained-efficient allocation

Theorem

The planner's optimal strategy is characterized by an array $(m_0, m_1(\theta_1), m_2(\theta_1, \theta_2))$ *defined by the following conditions:*

$$m_2(\theta_1, \theta_2) = \max\{m_1(\theta_1) - (1-\theta_1)\theta_2, 0\};$$

$$m_1(\theta_1) = \max\{m_0 - \theta_1, 0\}$$

and

$$R\left(1 - \int_0^{m_0} F_2\left(\frac{m_0 - \theta_1}{1-\theta_1}\right) f_1(\theta_1)\, d\theta_1\right) + 1 = \rho.$$

The market economy

- *Date* 0: a fraction α of bankers choose to consume their cash and $1 - \alpha$ decide to hold it
- *Date* 1: A fraction θ_1 of bankers receive a demand for payment; assets can be sold on the spot market to raise cash; failure to pay leads to default and liquidation
- *Date* 2: A fraction θ_2 of bankers receive a demand for payment; assets can be sold on the spot market to raise cash; failure to pay leads to default and liquidation
- At date 3, solvent bankers receive the returns from the assets they hold; remaining debts are due and paid

Figure 1: Timeline

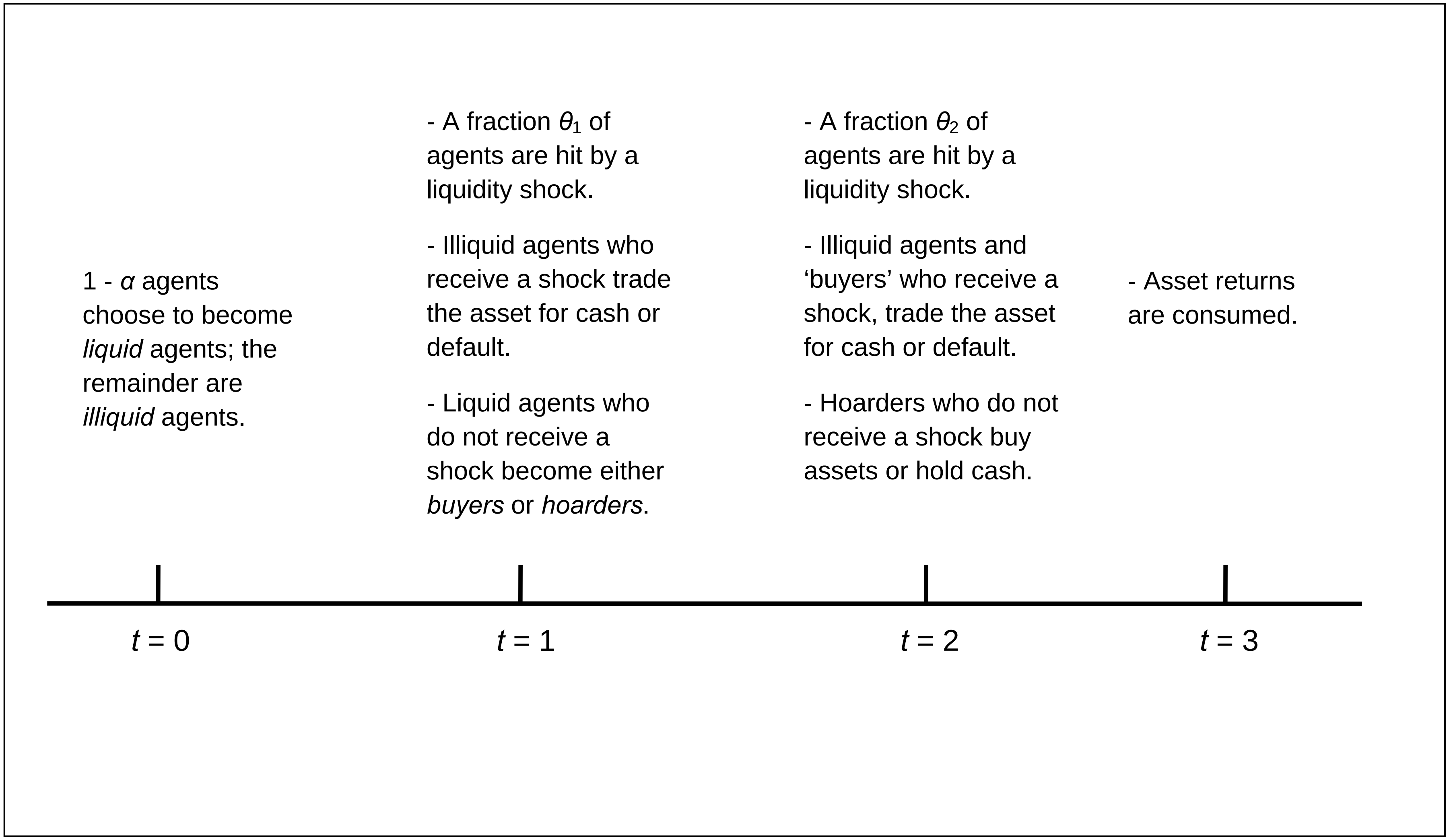


Figure 2: Allocations at dates 0 and 1

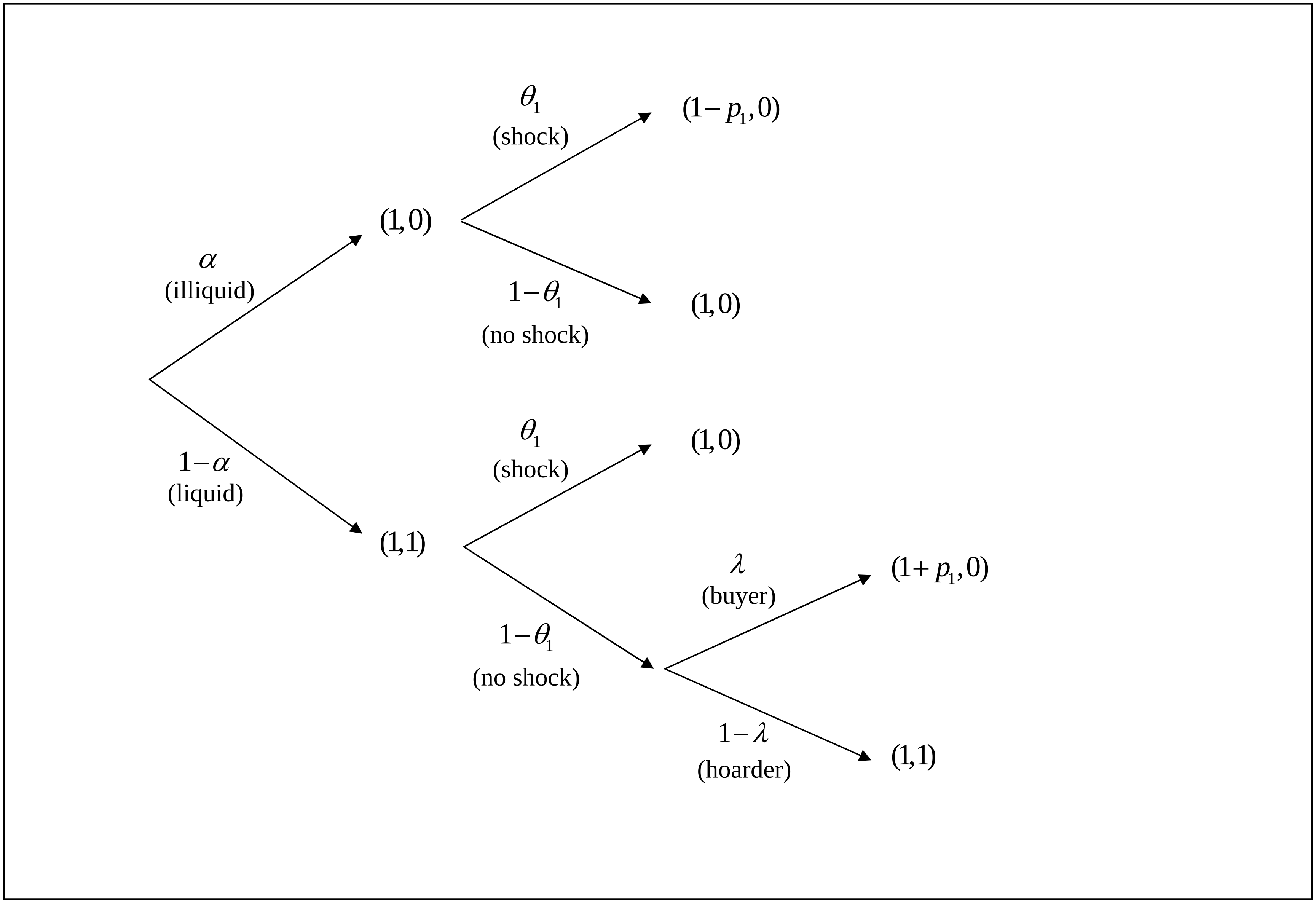


Figure 3a: Allocations at date 2

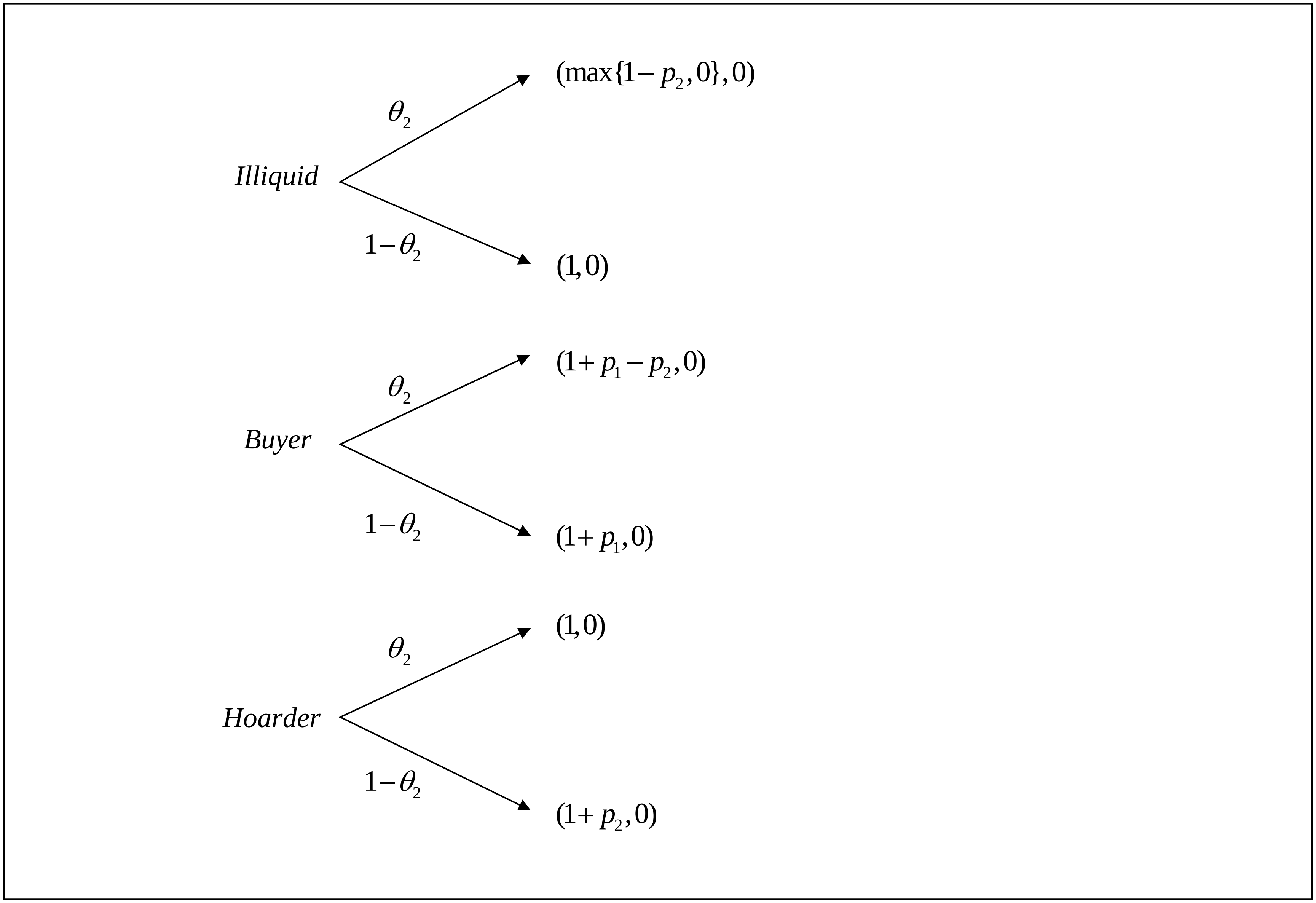
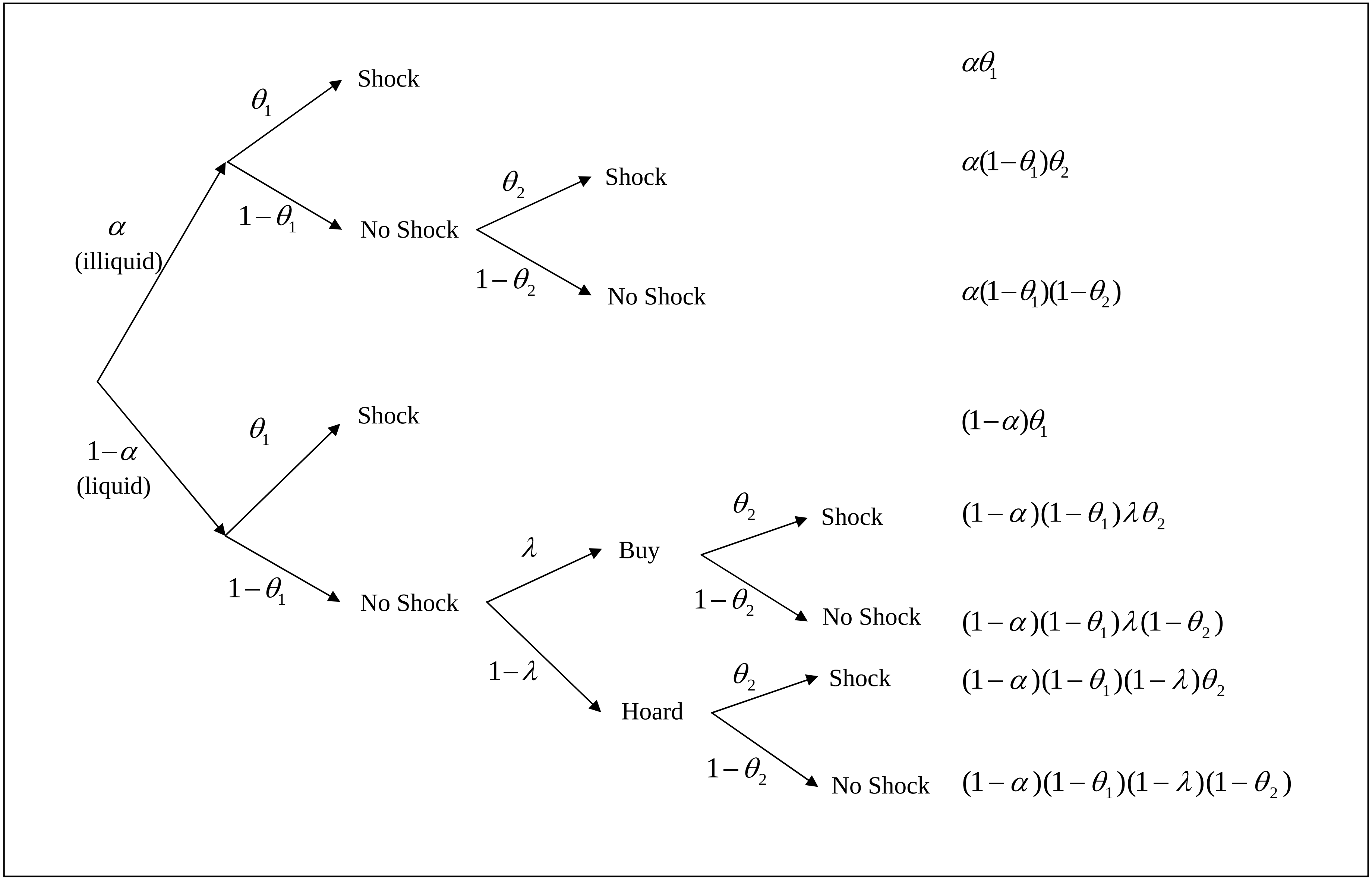


Figure 3b: Allocations at date 2



Demand and supply for cash at date 2

- Demand for cash comes from
  - the "buyers" hit by a liquidity shock

$$(1-\alpha)(1-\theta_1)\,\theta_2\lambda$$

  - and the illiquid bankers hit by a liquidity shock

$$\alpha\,(1-\theta_1)\,\theta_2$$

- The supply of cash comes
  - from hoarders who do not receive a liquidity shock

$$(1-\alpha)(1-\theta_1)(1-\theta_2)(1-\lambda)$$

Market clearing at date 2

- Supply is greater than demand if

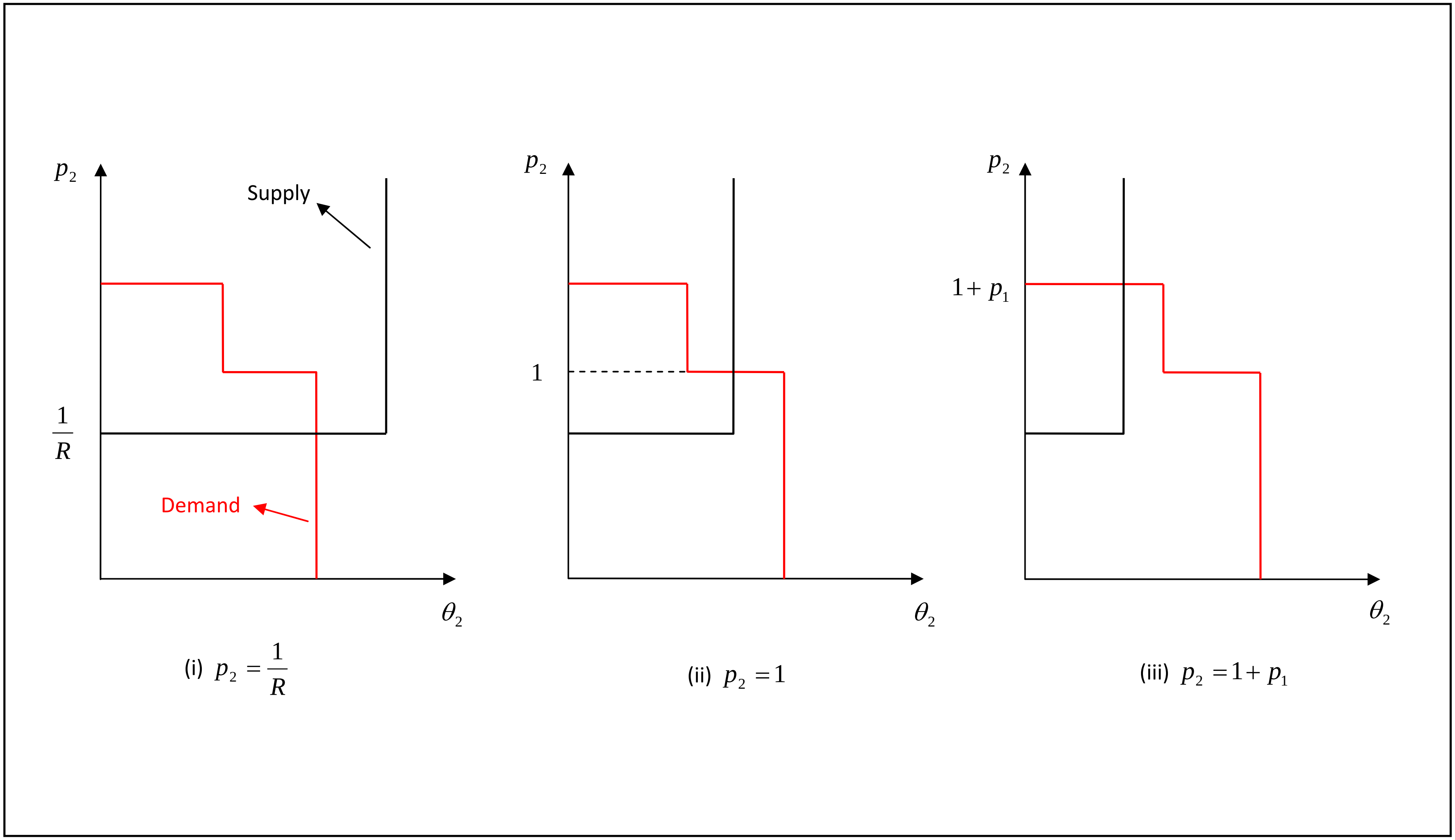
$$(1-\alpha)(1-\theta_1)(1-\theta_2)(1-\lambda) > (1-\alpha)(1-\theta_1)\theta_2\lambda + \alpha(1-\theta_1)\theta_2$$

$$\iff (1-\alpha)(1-\theta_2)(1-\lambda) > (1-\alpha)\theta_2\lambda + \alpha\theta_2$$
$$\iff \theta_2 < \theta_2^* = (1-\alpha)(1-\lambda)$$

- Demand from "buyers" is greater than supply if

$$(1-\alpha)(1-\theta_1)\theta_2\lambda > (1-\alpha)(1-\theta_1)(1-\theta_2)(1-\lambda)$$
$$\iff \theta_2\lambda > (1-\theta_2)(1-\lambda)$$
$$\iff \theta_2 > \theta_2^{**} = 1-\lambda$$

Figure 5C: Different demand and supply regimes as functions of θ_2



Equilibrium prices at date 2

There are three possible regimes:

1. Supply of cash is *high* relative to demand

$$\theta_2 < \theta_2^* \text{ and } p_2 = \frac{1}{R}$$

2. Supply of cash is *intermediate* relative to demand

$$\theta_2^* < \theta_2 < \theta_2^{**} \text{ and } p_2 = 1$$

3. Supply of cash is *low* relative to demand

$$\theta_2 > \theta_2^{**} \text{ and } p_2 = 1 + p_1$$

Buying and hoarding at date 1

- *Buying* is optimal if

$$p_1(\theta_1) \geq E[p_2(\theta_1, \theta_2) | \theta_1]$$

- *Hoarding* is optimal if

$$p_1(\theta_1) \leq E[p_2(\theta_1, \theta_2) | \theta_1])$$

- Let $\lambda(\theta_1)$ denote the fraction of liquid bankers who choose to buy assets in state θ_1 at date 1. Equilibrium requires

$$0 < \lambda(\theta_1) < 1$$

  for every value of θ_1

- Hence,

$$p_1(\theta_1) = E[p_2(\theta_1, \theta_2) | \theta_1].$$

Market clearing at date 1

- From the equilibrium distribution of $\tilde{p}_2$, we can calculate

$$E\left[\tilde{p}_2\right] = F_2\left(\theta_2^*\right)R^{-1} + F_2\left(\theta_2^{**}\right) - F_2\left(\theta_2^*\right) + \left(1 - F_2\left(\theta_2^{**}\right)\left(1 + p_1\right)\right)$$

so the equilibrium condition $p_1 = E\left[\tilde{p}_2\right]$ implies that

$$p_1 = \tilde{p}\left(\lambda\right) \equiv \frac{1 - F_2\left(\left(1-\alpha\right)\left(1-\lambda\right)\right)\left(1 - R^{-1}\right)}{F_2\left(1-\lambda\right)}$$

- By inspection, $\tilde{p}\left(\lambda\right)$ is increasing and

$$p\left(0\right) < 1 \text{ and } p\left(1\right) > 1$$

- Hence, there is a unique value of λ, call it $\bar{\lambda} \in \left(0,1\right)$, such that $\tilde{p}\left(\bar{\lambda}\right) = 1$ and $\tilde{p}\left(\lambda\right) < 1$ if and only if $\lambda < \bar{\lambda}$.

Market clearing at date 1

- If $p_1 < 1$, market-clearing requires

$$\lambda\left(\theta_1\right)\left(1-\alpha\right)\left(1-\theta_1\right) = \alpha\theta_1$$

or

$$\lambda\left(\theta_1\right) = \frac{\alpha\theta_1}{\left(1-\alpha\right)\left(1-\theta_1\right)}$$

- The equilibrium value of $\lambda\left(\theta_1\right)$ is given by

$$\lambda\left(\theta_1\right) = \min\left\{\frac{\alpha\theta_1}{\left(1-\alpha\right)\left(1-\theta_1\right)}, \bar{\lambda}\right\}, \text{ for any } \theta_1$$

- The equilibrium value of $p\left(\theta_1\right)$ is given by

$$p_1\left(\theta_1\right) = \min\left\{\tilde{p}\left(\frac{\alpha\theta_1}{\left(1-\alpha\right)\left(1-\theta_1\right)}\right), 1\right\}, \text{ for any } \theta_1$$

Market clearing at date 0

- In equilibrium at date 0, $0 < \alpha < 1$, which implies that bankers must be indifferent between acquiring liquidity and not acquiring it.

- Agents are indifferent if and only if

$$\int_0^1 p_1 \left\{1 + (1-\theta_1)(1 - F_2(\theta_2^{**}))E\left[\theta_2 \left|\theta_2 > \theta_2^{**}\right.\right]\right\} \; f_1(\theta_1)d\theta_1 = \frac{\rho}{R}.$$

Equilibrium

An *equilibrium* is described by the endogenous variables α, $\lambda(\theta_1)$, $p_1(\theta_1)$, and $p_2(\theta_1, \theta_2)$ satisfying the following conditions:

- at date 2, for every value of (θ_1, θ_2), $p_2(\theta_1, \theta_2)$ is the market clearing price, given the values of α, $\lambda(\theta_1)$ and $p_1(\theta)$
- at date 1, for every value of θ_1, $\lambda(\theta_1)$ and $p_1(\theta)$ satisfy the market clearing conditions, given the value of α
- at date 0, bankers are indifferent between acquiring liquidity and not acquiring it

The Lender of Last Resort

- Suppose that $\alpha = 1$ and that the Bank pursues the socially optimal
- At date 2, the market-clearing price is denoted by $p_2(\theta_1, \theta_2)$ and defined by

$$p_2(\theta_1, \theta_2) = \begin{cases} 1 & \text{if } (1-\theta_1)\theta_2 > \max\{m_0^* - \theta_1, 0\} \\ R^{-1} & \text{if } (1-\theta_1)\theta_2 < \max\{m_0^* - \theta_1, 0\} \end{cases}$$

- At date 1, the market clearing price is *assumed* to be

$$p_1(\theta_1) = \begin{cases} 1 & \text{if } \theta_1 > m_0^* \\ E[p_2(\theta_1, \theta_2) \mid \theta_1] & \text{if } \theta_1 < m_0^* \end{cases}$$

Optimality I

- An illiquid banker's payoff is

$$\begin{aligned} E\left[\theta_1 R\left(1-p_1\left(\theta_1\right)\right)+\left(1-\theta_1\right)\theta_2 R\left(1-p_2\left(\theta_1,\theta_2\right)\right)\right. \\ \left.+\left(1-\theta_1\right)\left(1-\theta_2\right)R\right] \\ = E\left[R-\left(\theta_1+\left(1-\theta_1\right)\theta_2\right)p_2\left(\theta_1,\theta_2\right)R\right] \end{aligned}$$

- A liquid banker's payoff is

$$E\left[R+\left(1-\theta_1\right)\left(1-\theta_2\right)p_2\left(\theta_1,\theta_2\right)R\right]-\rho$$

- Then it is optimal to be illiquid if and only if

$$E\left[p_2\left(\theta_1,\theta_2\right)R\right]\leq\rho$$

Optimality II

- The first-order condition for the planner's problem is

$$R + 1 - R \int_0^{m_0} F_2 \left(\frac{m_0 - \theta_1}{1 - \theta_1} \right) f_1 (\theta_1) \, d\theta_1 = \rho.$$

- From the definition of $p_2 (\theta_1, \theta_2)$,

$$\begin{aligned} E \left[p_2 (\theta_1, \theta_2) R \right] &= R - (R - 1) \int_0^{m_0^*} F_2 \left(\frac{m_0^* - \theta_1}{1 - \theta_1} \right) f_1 (\theta_1) \, d\theta_1 \\ &\leq R + 1 - R \int_0^{m_0^*} F_2 \left(\frac{m_0^* - \theta_1}{1 - \theta_1} \right) f_1 (\theta_1) \, d\theta_1 \\ &\leq \rho \end{aligned}$$

Incomplete information

- The decentralization theorem has an important corollary: the solution to the planner's problem can be extended to an economy with private information
- Suppose that the aggregate shocks θ_1 and θ_2 are public information, but a banker's liquidity shock is private information
- The planner chooses an incentive compatible direct mechanism: at each date $t = 1, 2$, bankers announce whether they have received a liquidity shock
- At date $t = 1, 2$, the planner allocates one unit of cash with probability $\mu_1(\theta_1)$ (resp. $\mu_2(\theta_1, \theta_2)$) to a banker who claims to have received a liquidity shock
- In exchange, the banker supplies $p_1(\theta_1)$ (resp. $p_2(\theta_1, \theta_2)$) units of the asset
- The mechanism $(\mu_1(\theta_1), p_1(\theta_1), \mu_2(\theta_1, \theta_2),)$ is incentive compatible in the sense that truth telling is an optimal strategy

Incomplete information

- The equilibrium in which the Lender of Last Resort supplies all the liquidity is equivalent to an incentive compatible direct mechanism
- Set

$$\mu_1(\theta_1) = \frac{x_1(\theta_1)}{\theta_1} \text{ and } \mu_2(\theta_1, \theta_2) = \frac{x_2(\theta_1, \theta_2)}{(1-\theta_1)\theta_2}$$

  and let $p_1(\theta_1)$ and $p_2(\theta_1, \theta_2)$ be the equilibrium prices
- The optimality of the bankers' behavior implies incentive compatibility

Theorem

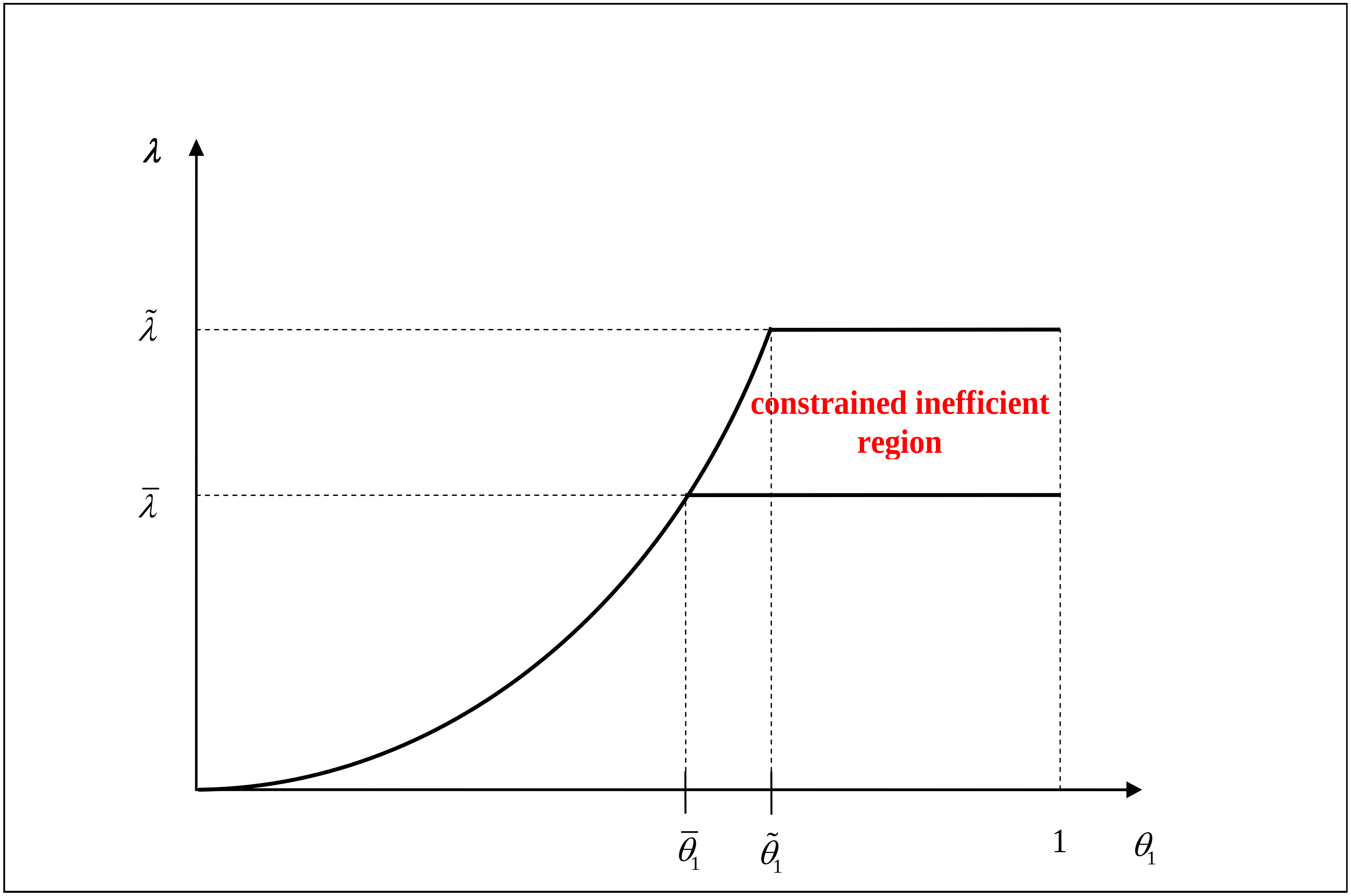
The allocation that solves the planner's problem can be implemented by an incentive-compatible direct mechanism

Market regulation I

- Suppose the CB can control λ while allowing markets to clear at other dates
- The optimal level of λ^{soc} has the same structure as the equilibrium λ but is larger:

$$\lambda^{soc} = \min\left\{\frac{\alpha\theta_1}{(1-\alpha)(1-\theta_1)}, \widetilde{\lambda}\right\}, \text{ where } \widetilde{\lambda} > \bar{\lambda}$$

- It is not optimal to set $\bar{\lambda} = 1$: because bankers are not allowed to make the optimal hoarding decision, the value of holding cash is reduced, other things being equal
- The CB faces a tradeoff between efficient allocation of aggregate liquidity at date 0 and the amount of aggregate liquidity at date 1



λ
$\tilde{\lambda}$
$\bar{\lambda}$
constrained inefficient region
$\bar{\theta}_1$
$\tilde{\theta}_1$
1
θ_1

Market regulation II

- Now suppose the CB can only control the quantity of aggregate liquidity α at date 0 while allowing markets to clear at other dates
- The welfare maximizing value α^{soc} is smaller than the equilibrium level of α
- *Intuition*: by "envelope theorem" argument, increased aggregate liquidity lowers cost of liquidity at date 2
- It is never optimal to set $\alpha^{soc} = 0$ unless $\rho = 1$

A model without hoarding I

- To show that
- Three dates, $t = 0, 1, 2$; liquidity shock θ_1 at date 1; returns at date 3
- $1 - \alpha$ hold cash at date 0, $(1 - \alpha)\,\theta_1$ supply their own cash needs and $(1 - \alpha)\,(1 - \theta_1)$ have spare cash to lend:

$$p_1(\theta_1) = \begin{cases} 1 & \text{if } \theta_1 > 1 - \alpha, \\ R^{-1} & \text{if } \theta_1 < 1 - \alpha. \end{cases}$$

- The allocation of cash at date 1 is efficient, but the equilibrium allocation is not efficient: α too low because utility of lenders not taken into account

Model without hoarding II

- Bankers are indifferent between being liquid or illiquid if $E[p_1] = \rho / R$ or

$$F_1(1-\alpha) = \frac{R-\rho}{R-1}.$$

- The planner's FOC is

$$(R-1)(1-F_1(m_0)) + 1 = \rho,$$

or

$$F_1(m_0) = 1 - \frac{\rho-1}{R-1} = \frac{R-\rho}{R-1}$$

- Thus, $m_0 = 1-\alpha$ and the equilibrium allocation is constrained efficient

Asset price volatility and fire sales I

- When large bankers default at date 2, they raise the price of liquidity to

$$p_2(\theta_1, \theta_2) = 1 + p_1(\theta_1)$$

  creating a fire sale of assets
- The anticipation of this asset price volatility increases both the precautionary and speculative motives for hoarding liquidity
- In fact, if we remove the excess volatility, we can show that inefficient hoarding disappears
- Suppose that the liquidity shock experience by the bank is the demand for payment of a non-recourse loan and that the collateral for this loan is the initial endowment of one unit of the asset
- When the banker receives a liquidity shock, only one unit of the asset is at risk of being liquidated

Asset price volatility and fire sales II

- The maximum amount that an illiquid banker is willing to pay for cash at date 2 is one unit of the asset, since he will lose only one unit if he defaults
- Then the equilibrium price at date 2 is given by

$$p_2(\theta_1, \theta_2) = \begin{cases} R^{-1} & \text{w. pr. } F_2((1-\alpha)(1-\lambda(\theta_1))), \\ 1 & \text{w. pr. } 1 - F_2((1-\alpha)(1-\lambda(\theta_1))). \end{cases}$$

- As before, we can show that market-clearing at date 1 requires

$$p_1(\theta_1) = E[p_2(\theta_1, \theta_2) \mid \theta_1]$$

- Inefficient hoarding at date 1 implies $p_1(\theta_1) = 1$, so $p_2(\theta_1, \theta_2) = 1$ with probability one, but this is impossible if there is a positive amount of hoarding
- So there is no inefficient hoarding in equilibrium

Incomplete markets

- The fundamental cause of inefficiency in this model is the incompleteness of markets, i.e., the absence of markets for insuring liquidity shocks
- Under symmetric information, trading contingent claims (to cash and assets) could achieve the first best
- Under asymmetric information, it may be impossible to improve on the allocation achieved through spot markets
- Suppose that individual liquidity shocks are private information; then a market mechanism must give bankers incentives to reveal their information truthfully
- We show that the equilibrium cannot be improved on by the introduction of an incentive compatible market mechanism when there is asymmetric information

Direct mechanism I

- By the revelation principle, we can restrict our attention to direct mechanisms
- Let $\{\alpha, \lambda(\theta_1), p_1(\theta_1), p_2(\theta_1, \theta_2)\}$ be an equilibrium and consider the effect of opening a market for liquidity insurance at date 0
- At date 0, bankers enter into contracts to deliver or receive liquidity under specified conditions
- Suppliers acquire one unit of liquidity at date 0; demanders do not
- At dates $t = 1, 2$, each banker is required to report his type, that is, whether or not he has received a liquidity shock
- Suppliers who report "shock" and demanders who report "no shock" do not trade

Direct mechanism II

- At date 1,
  - a supplier who reports "no shock" receives $(-1, \hat{p}_1(\theta_1))$ with probability $\nu_1(\theta_1)$
  - a demander who reports "shock" receives $(1, -\hat{p}(\theta_1))$ with probability $\mu_1(\theta_1)$
- At date 2,
  - a supplier who reports "no shock" for the second time and has not traded receives $(-1, \hat{p}_2(\theta_1, \theta_2))$ with probability $\nu_2(\theta_1, \theta_2)$
  - a demander who reports "shock" for the first time receives $(1, -\hat{p}_2(\theta_1, \theta_2))$ with probability $\mu_2(\theta_1, \theta_2)$

The impossibility of insurance

- If $\hat{p}_1(\theta_1) > p_1(\theta_1)$, a demander who receives a shock will report "no shock" and buy on the spot market; if $\hat{p}_1(\theta_1) < p_1(\theta_1)$, a supplier who did receive a shock will report "shock" and sell on the spot market

- Thus, incentive compatibility at date 1 requires

$$\hat{p}_1(\theta_1) = p_1(\theta_1), \text{ for every } \theta_1$$

- Similarly, incentive compatibility at date 2 requires

$$\hat{p}_2(\theta_1, \theta_2) = p_2(\theta_1, \theta_2), \text{ for every } (\theta_1, \theta_2)$$

Conclusion

- Goodfriend and King argued that it is sufficient to provide adequate liquidity to the system as a whole ...
- ... but should the Central Bank become the sole provider of liquidity?
- Limits of the Lender of Last Resort
  - inflation
  - counterparty risk
  - asset risk
  - moral hazard
  - the unwind problem