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Optimal Cyclical Monetary Policy: Does Steady-State Inflation Matter?

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Abstract

In general equilibrium models, optimal cyclical monetary policy is usually derived around an optimal steady-state inflation level, which in most cases is zero or equal to the negative of the real interest rate. This paper examines whether and how different steady-state inflation levels and other steady-state distortions affect the optimal monetary policy response to shocks. This issue is first discussed in general terms. Then, a simple example is presented, where optimal policy can be procyclical or countercyclical depending on the steady-state inflation level. This paper suggests that both issues of the choice of inflation target and optimal cyclical monetary policy should be addressed simultaneously, as steady-state distortions influence the optimal reaction of monetary policy to shocks. More generally, the paper shows that assumptions about steady-state distortions affect the derived optimal cyclical policy.

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1. Introduction

In general equilibrium models, optimal cyclical monetary policy is usually derived after log-linearizing structural equations around a first-best steady-state inflation level, which in most cases is zero[1] or equal to the negative of the real interest rate[2] (Friedman rule). However, those inflation levels do not correspond to what we observe empirically. This paper examines whether and how deviating from those first-best steady-state inflation levels affects the optimal monetary policy response to shocks, or in other words, how different inflation targets affect the optimal cyclical monetary policy.

The choice of a higher inflation target can be motivated by different reasons. First, inflation is potentially able to offset or dampen some distortions, like e.g. imperfect competition or downward wage rigidities. Then, it is sometimes argued that low steady-state interest rates provide central bank with less flexibility to react to strong negative shocks and increase deflation risks. I will argue that the choice of steady-state inflation level affects the optimal monetary policy response to shocks, whether the inflation target has been chosen optimally to ease distortions or is sub-optimally high.

Some studies[3] have examined the merit of different monetary policy rules in environments that are not Pareto optimal, and analyzed second-best policies where

[1] See e.g. Gali (2001), King and Wolman (1999), and Woodford (1999). Often, in models featuring sticky prices, distortions arising from imperfect competition or from the opportunity cost of holding money are assumed away, and monetary authorities choose to stabilize the price level in order to avoid different relative prices induced by the lack of synchronization in price adjustments. Another approach was adopted by King and Wolman, who consider a dynamic optimization problem that also leads to a zero inflation optimum.

[2] See e.g. Carlstrom and Fuerst (1998), Fuerst (1994), and Ireland (1996).

[3] See e.g. Aiyagari and Braun (1998), Carlstrom and Fuerst (1995, 1996), and Fuerst (1994, 1998).

nominal interest rates were strictly positive in order to avoid indeterminacy issues. In this paper I examine the mechanisms at work that affect optimal cyclical policy when monetary policy deviates from the standard first-best inflation levels of zero or the negative of the real interest rate, whether it does so sub-optimally or to ease other steady-state distortions, and the consequences of varying steady-state inflation levels for optimal cyclical monetary policy.

The main idea is that in a distorted economy with a striclty positive steady-state inflation, the choice faced by the monetary authority is whether and how to let the distortion fluctuate when the economy is subject to shocks. In the example studied below, a varying distortion on one hand lowers average consumption as it does not allow firms to let labor covary optimally with productivity, but on the other hand a distortion relatively high in booms and low in downturns implies a relatively high distortion when the marginal utility of consumption is low and thus the utility cost of the distortion is relatively low. Different steady-state distortion levels affect those margins and lead to different optimal cyclical monetary policy prescriptions. In other terms, with different levels of steady-state distortion, fluctuations affect households more or less strongly as the curvature of the utility function differs. The optimal monetary policy fluctuations smoothing should thus also vary.

In section 2, a general discussion is provided on the mechanisms that affect cyclical policy when the degree of distortion changes. Then, in section 3, a simple example is presented, based on a setup proposed by Carlstrom and Fuerst (1996), in order to illustrate the effect of different steady-state inflation levels and of the role of risk aversion on optimal cyclical monetary policy. Section 4 concludes.

2. A Discussion of the Mechanisms at Work

This section presents some intuition on the way steady-state inflation can affect the optimal monetary policy response to shocks.

In general equilibrium models, optimal monetary policy is derived by maximizing the expected utility of a representative household. Let us write its utility as

$$U\left[Y\left(\theta,R\right),L\left(\theta,R\right)\right]. \tag{1}$$

The representative household consumes what its produces, thus consumption equals production Y, and it supplies labor L to the firm. $Y\left(\theta,R\right)$ represents the equilibrium behavior of output, where the factors of production are already expressed in terms of the monetary policy instrument R and the underlying shocks θ. R is the gross nominal interest rate, but it could also represent money growth. $L\left(\theta,R\right)$ represents the equilibrium behavior of labor.

There is no consensus on the way to model monetary non-neutrality, but the function $Y\left(\theta,R\right)$ can be interpreted in the lights of the different theories. First, monetary policy affects the real economy through steady-state inflation. Steady-state inflation, which determines the steady-state nominal interest rate $\overline{R}$, can affect real output through shoe-leather costs, through a tax on labor (Carlstrom and Fuerst, 1996), or through relative price distortion (Galí, 2001), for example, depending on the model. Then, monetary policy can have short-run effects through imperfect information or nominal rigidities, like e.g. limited participation constraints or stickiness in prices or wages. In those cases, deviations from steady-state policy, $R-\overline{R}$, can generate short-term non-neutrality.

The question of interest for this paper is then how would optimal cyclical monetary policy, represented by $R - \overline{R}$, be affected by the steady-state inflation level, which determines $\overline{R}$. There are several potential reasons that could justify deviating from a first-best zero inflation level or from the Friedman rule. Distortions, like e.g. imperfect competition or downward wage rigidities, could potentially be offset or dampened by a strictly positive inflation level. In this paper however, I do not model such kinds of distortions. A positive inflation level should thus be seen here as sub-optimal and exogenously imposed. Optimal monetary policy becomes thus the solution of a second-best problem, which is to maximize expected utility given a sub-optimal steady-state inflation level.

One way to motivate this approach would be to think of a government imposing an inflation target to an independent central bank, which would then be responsible for the conduct of an optimal cyclical monetary policy. An inflation target that is too high can be thought of as a consequence of misjudgment on the optimal inflation target level, or arising from the fear that low interest rate levels could prevent a central bank of reacting to strong negative shocks and increase deflation risks.

However, the arguments developed in this paper also apply to models where distortions justifying a strictly positive inflation level are explicitly modeled and where a strictly positive steady-state inflation level is optimal, as long as the steady-state inflation level differs from the first-best case that would be characterized in the absence of those distortions. As soon as distortions are present, whether they are generated by a strictly positive inflation level or by other factors, the resulting allocations should induce a different cyclical monetary policy. In other words, this paper tries to characterize the consequences of assuming away distortions that justify a strictly positive inflation rate in models that examine optimal monetary policy, or the consequences of choosing a sub-optimal inflation target.

2.1. Optimal Cyclical Monetary Policy

We can represent the monetary policy reaction function as

$$R = \overline{R} + \delta (\theta - 1), \tag{2}$$

where δ is the coefficient of reaction, and θ is an i.i.d. shock with mean 1 and variance σ_θ^2.

Monetary authorities are maximizing the representative agent's expected utility. Taking the expectation of the second-order Taylor approximation of equation (1) around the steady-state values $R = \overline{R}$ and $\theta = 1$, we obtain

$$E(U) = \overline{U} + \overline{U_{\theta R}}\sigma_{\theta R} + \frac{1}{2}\overline{U_{RR}}\sigma_R^2 + \frac{1}{2}\overline{U_{\theta\theta}}\sigma_\theta^2, \tag{3}$$

where the upper-bar means that these derivatives are evaluated at their steady-state values. Under certain conditions, $\overline{U_{RR}}$ and $\overline{U_{\theta\theta}}$ are negative[4]. The central bank cannot influence σ_θ^2, thus the last term does not affect the second-best maximization problem.

Monetary authorities face a trade-off. First, depending on the sign and amplitude of $\overline{U_{\theta R}}$, it will be optimal to let the nominal interest rate covary more or less positively or negatively with the shock. In other words, the optimal monetary policy reaction depends on how monetary actions affect the marginal influence of shocks on utility

[4] $\overline{U_{RR}}$ needs to be negative for the problem to be well defined. In this setup, a positive value for that variable could potentially happen if monetary policy can increase average production by increasing the variance of the interest rate per se, being more (less) effective at low (high) interest rates, which does not seem plausible.

in equilibrium. On the other hand, interest rate fluctuations per se lower welfare, i.e. $\overline{U_{RR}} < 0$. The structure of the model will thus determine the optimal degree of monetary policy reaction to shocks. The influence of different steady-state inflation levels can then be studied through the effect of varying $\overline{R}$ on the derivatives $\overline{U_{\theta R}}$ and $\overline{U_{RR}}$.

The optimal monetary policy reaction coefficient δ^* can be determined as follows. From the policy function (2), we can obtain

$$\sigma_R^2 = \delta^2 \sigma_\theta^2, \tag{4}$$

and

$$\sigma_{\theta R} = \delta \sigma_\theta^2, \tag{5}$$

thus the maximization problem can be written as

$$\max_\delta \left(\overline{U_{\theta R}} \delta + \frac{1}{2} \overline{U_{RR}} \delta^2 \right), \tag{6}$$

which leads to the optimal reaction coefficient

$$\delta^* = \frac{\overline{U_{\theta R}}}{-\overline{U_{RR}}}. \tag{7}$$

In order to gain intuition on the optimal covariance between the monetary policy instrument and the productivity shock, as well as on the effect of a change in steady-state inflation on that covariance, it is useful to examine the numerator of (7). From equation (1), we can write

$$\overline{U_{\theta R}} = U_{YY}Y_\theta Y_R + U_Y Y_{\theta R} + U_{YL}Y_\theta L_R + U_{YL}Y_R L_\theta + U_{LL}L_\theta L_R + U_L L_{\theta R}. \qquad (8)$$

We can see that the sign and amplitude of the optimal monetary policy response to shocks depend on the curvature of the utility function as well as on the equilibrium effects of shocks and monetary policy actions on labor and output. To find out the determinants of optimal cyclical policy and the influence of different steady-state inflation on those determinants, we need to be able to interpret the sign, amplitude and sensitivity with respect to changes in $\overline{R}$ of each term, including those in the denominator, in light of economic theory or empirical evidence. I will focus here on only a few terms, in order to gain some intuition.

If θ represents a productivity shock, the first term at the right-hand side of (8), $U_{YY}Y_\theta Y_R$, should be positive, implying a positive covariance between the nominal interest rate and productivity shocks. This first term captures the fact that a positive covariance between the policy instrument and productivity shocks increases welfare as it dampens the negative effect of consumption fluctuations.

The second term of equation (8), $U_Y Y_{\theta R}$, should be negative and thus push the covariance between interest rates and productivity shocks in the other direction than the determinants discussed in the previous paragraph. The main idea behind this result, which again depends of the specific model used, is that the marginal productivity of labor increases with a positive productivity shock, and a decrease in interest rate increases labor. Thus a positive covariance between interest rates and productivity shock will increase average output and thus welfare, as the marginal utility of consumption U_Y is positive.

The two first terms of equation (8) already show us two forces that pull the optimal covariance between the policy instrument and productivity shocks in opposite directions. Consumption smoothing concerns would call for a positive covariance, while a negative covariance would increase average consumption. The sign of the optimal covariance will also be influenced by other model specifications, like e.g. the way the disutility of labor is modeled, and the amplitude of that covariance will depend on the denominator of equation (7) as well, which accounts for the effect of interest rate variations on utility.

2.2. Effects of Varying Steady-State Inflation/Distortions

Concerning the main question addressed in this paper, we see that different steady-state inflation levels or other distortions will affect the optimal cyclical policy by influencing the sign and amplitude of the different determinants of the optimal covariance between the policy instrument and the shocks. A higher steady-state inflation will cause the output level to decrease, due either to additional resources spent carrying out transactions (shoe-leather costs), to a higher tax on labor (Carlstrom and Fuerst, 1996), or to a higher relative price distortion (Galí, 2001), depending of the model. Lower equilibrium output and labor levels will influence the different derivatives.

For example, households will find themselves on a point on the utility curve with more curvature, i.e. where the absolute value of U_{YY} is larger, and will thus suffer more from output fluctuations. From the first term of equation (8), this will push towards a more positive covariance between the interest rate and shocks. On the other hand, U_Y will be larger, thus households will appreciate more the higher average output generated by a negative covariance between the interest rate and shocks. It could also be that at new allocations, the effect of shocks or interest rate changes on output, Y_θ and Y_R, are different, calling for a different monetary policy reaction to shocks. The net effect will also depend of the other terms of the numerator, and the amplitude of the change will be determined by the denominator as well.

We can already see that certain model parameters, like e.g. the coefficient of risk aversion, may play a crucial role. Higher degrees of risk aversion will make households more averse to fluctuations and thus induce a more positive optimal covariance between the interest rate and productivity shocks. This fact will be illustrated in section 3, with the specific model considered.

We can also see that what applies to a higher distortion generated by a higher inflation target also applies to any other distortion. We can use the analysis presented above to assess how results derived when distortions are assumed away would be affected if those distortions were taken into account. For example, if a distortion like imperfect competition justifies a higher inflation target, the distortion generated by that higher inflation and the remaining imperfect competition distortion that monetary policy could not have offset will affect the steady-state levels of the derivatives discussed above, and thus the optimal cyclical policy will be affected. In that sense, log-linearizing structural equations around their first-best non-distorted solutions may lead to different prescriptions for the optimal cyclical policy.

The following section illustrates this discussion with a simple example.

3. A Model with Limited Participation Constraint

3.1. Model Description

The purpose of this section is to illustrate with a simple model the influence of different steady-state inflation levels on the optimal monetary policy reaction to shocks.

As Cooley and Quadrini (1999) documented, the Federal Open Market Committee seems to monitor and react to changes in commercial borrowers' credit financing conditions. With that reading of the events, when, following a positive productivity shock, firms want to expand and the loan market gets tight, the Federal Reserve would inject liquidity in the system. As Cooley and Quadrini pointed out, the positive correlation between monetary aggregates and employment may be a consequence of a policy, which after a positive productivity shock, when output is below its potential level due to a tighter loan market, would call for a liquidity injection to allow firms to take fully advantage of the shock.

The economy considered here is subject to productivity shocks that affect the nominal interest rate at which commercial firms borrow from households through financial intermediaries. Given households' limited participation constraint in the financial market, loan market conditions are influenced by productivity shocks, and this affects labor demand decisions. The central bank has to decide whether to accommodate these shocks, through a liquidity effect.

The general setup, where open-market operations can generate liquidity effects on nominal interest rates, is provided by Lucas (1990). Fuerst (1992) incorporates the production side and interprets the nominal interest rate as the one firms face when paying their workers in advance. Christiano, Eichenbaum and Evans (1997b) provide empirical estimates of the liquidity effect on nominal interest rates and production following a shock to the money supply, and calibrate a limited participation constraint model with adjustment costs that could replicate their empirical results.

The underlying model mechanism is as follows. Firms have to use savings from households to pay their inputs in advance of production, thus factors demand and output will depend on the nominal interest rate in the loan market. Households are subject to a limited participation constraint or adjustment costs that introduce some sluggishness to their savings decision. As a result, the monetary authority can influence the nominal interest rate, and thus production, by injecting money directly to the loan market through the financial intermediaries. In other terms, the central bank can directly affect the relative holding of cash between the different sectors of the economy. After a positive productivity shock, firms will want to expand their activity. However, as the supply of loans will not change, due to the limited participation constraint, there will be upward pressure on the nominal interest rate. A procyclical monetary policy, which in that case increases the money supply, would dampen the upward pressures on the nominal interest rate and thus allow firms to optimally expand their activity.

A strictly positive nominal interest rate acts like a tax by scaling down employment and production. The first-best outcome is thus obtained by keeping the nominal interest rate at zero, increasing the money supply when the economy is hit by a positive productivity shock. In case of a negative shock, the optimal monetary policy would be indeterminate, because of the non-negativity constraint on the nominal interest rate. The way previous studies[5] have dealt with this issue is to consider only policies where cash-in-advance constraints for both households and firms are binding,

[5] See e.g. Fuerst (1994).

which imply a procyclical policy even in the event of a negative productivity shock. When we depart from the first-best outcome, and allow for a positive average inflation rate, the indeterminacy disappears but we are left with second-best policies. Previous studies have compared different second-best policies for a given distortion. Carlstrom and Fuerst (1996) show that a nominal interest rate peg is superior, in terms of welfare, to a money growth peg with the same average distortion level. They relate that finding to the result that a constant tax is preferred to a variable one, as a strictly positive nominal interest rate acts as a tax on labor demand in this model. Moreover, Carlstrom and Fuerst (1995) provide numerical results in favor of a nominal interest rate peg over a constant money supply growth rate rule by comparing two economies with the same non-stochastic steady-state level of capital and nominal interest rate.

Here I use the setup proposed by Carlstrom and Fuerst, and examine the consequences of varying steady-state inflation levels for the optimal cyclical monetary policy. Instead of ranking the two pegs, i.e., money growth and interest rate pegs, I derive the optimal policy, which can be somewhere in between the two pegs, and examine how different steady-state distortions affect the optimal policy. If for low steady-state inflation levels it is optimal to allow firms to adjust their labor demand to productivity shocks, for higher inflation levels and thus larger distortions, consumption smoothing concerns become more important, and monetary policy will not necessarily dampen the offsetting effect of interest rate movements on output.

The section is organized as follows. In section 3.2, the model and its equilibrium are presented. Section 3.3 compares the first-best with equilibrium outcomes. In section 3.4, the second-best policy for a given average nominal interest rate is derived. Calibration is presented in section 3.5, and results from the calibrated model are displayed in section 3.6.

3.2. The Model

3.2.1. Household

An infinitely lived representative household maximizes its utility

$$E_{-1}\sum_{t=0}^{\infty}\beta^{t}U\left(C_t, L_t\right), \tag{9}$$

where C and L are consumption and labor, respectively, and β is the discount factor.

The household is subject to a cash-in-advance constraint; it keeps the amount of cash $(M_t - N_t)$ to purchase goods at period t, and sends N_t to the financial intermediary, at the beginning of the period, where M_t is the money balances carried over from $t-1$. At the end of the period, it gets the return R_tN_t from its deposit (R is the gross nominal interest rate), and the profits R_tX_t from the financial intermediary and D_t from the firm.

The constraints it faces are thus

$$P_tC_t \leq M_t - N_t + W_tL_t, \tag{10}$$

$$M_{t+1} = R_t\left(N_t + X_t\right) + D_t + \left(M_t - N_t + W_tL_t - P_tC_t\right), \tag{11}$$

and

$$0 \leq N_t \leq M_t, \tag{12}$$

where P_t, W_t and X_t are the price, wage, and monetary injection from the central bank to the financial market, respectively. The problem of the household is thus to choose the sequence $\{C_t, L_t, N_t, M_{t+1}\}_{t=0}^{\infty}$ to maximize its utility subject to the constraints above.

The household is subject to a limited participation constraint, i.e. N_t has to be function only of M_t and variables dated $t-1$ and earlier. Thus the household has to choose the amount it sends to the intermediary without knowing the realization of the current shock on production; neither does it know the size of the money injection, $X_t = M_{t+1}^S - M_t^S$, if the latter is a function of the shock of the current period.

The information sets are represented as follow:

Ω_t includes all the variables dated t and earlier;

Ω_t^N includes the variables known to the household when the latter chooses N_t, i.e. Ω_{t-1} and M_t.

To express the household's problem in a recursive form, we normalize the nominal variables by the beginning of period money supply M^S. Let us define

$$p = \frac{P}{M^S},\ w = \frac{W}{M^S},\ n = \frac{N}{M^S},\ m = \frac{M}{M^S},\ x = \frac{X}{M^S},\ 1 + x = \frac{M^{S\prime}}{M^S},\ d = \frac{D}{M^S}.$$

where a prime denotes next period variable.

The Bellman equation is then

$$V(m) = \max_{n \in [0,m]} E_{\Omega^N} \left\{ \max_{C,L,m'} \left[U(C, L) + \beta V(m') \right] \right\} \tag{13}$$

such that

$$pC \leq m - n + wL \tag{14}$$

and

$$(1+x)\, m' = R\,(n+x) + d + (m - n + wL - pC)\,. \tag{15}$$

We assume that productivity shocks are i.i.d. First order conditions are presented in appendix 1.

The utility function is given by

$$\frac{\left(C - \psi_0 \frac{L^{1+\psi}}{1+\psi}\right)^{1-\sigma}}{1-\sigma}, \qquad \text{for } 0 \; < \; \sigma, \sigma \neq 1, \tag{16}$$

$$\ln\left(C - \psi_0 \frac{L^{1+\psi}}{1+\psi}\right), \qquad \text{for } \sigma \; = \; 1. \tag{17}$$

The labor supply elasticity is $\frac{1}{\psi}$. Christiano, Eichenbaum and Evans (1997a) propose a way to reconcile this utility function with balanced growth. This form of utility function, which implies a zero income effect on leisure, is chosen so that first-best and equilibrium employment are both procyclical in the model, for any coefficient of risk aversion. This would not be the case with the functional form $U = \frac{C^{1-\sigma}-1}{1-\sigma} - V(L)$, where $V' \geq 0$ and $V'' \geq 0$, given the chosen form of the production function. With the latter utility function, equilibrium employment would not respond to productivity shocks when money growth is constant, and optimal employment would be countercyclical for $\sigma > 1$.

3.2.2. Firm

The production function has the form

$$y = \overline{K} + \theta L^{\alpha}. \tag{18}$$

The productivity shock θ has a mean of unity and variance σ_θ^2. This production function was suggested by Carlstrom and Fuerst (1996) to capture some stylized facts with a simple model. Physical capital is assumed constant and enters additively. With this specification, nominal interest rates move procyclically when money growth is constant. This would be the case with a Cobb-Douglas production function when physical capital accumulation is taken into account. With $\overline{K} = 0$, the nominal interest rate does not react to productivity shocks, and moves only with changes in the money growth rate, as explained in appendix 2. However, the results of this paper remain the same whether $\overline{K} = 0$ or $\overline{K} \neq 0$: the optimal cyclical monetary policy does differ for different steady-state inflation levels.

We assume that the firm chooses labor demand to maximize the discounted value of its dividend payments to the household (shareholder), and is subject to a cash-in-advance constraint: it has to borrow to pay its wage bill at the beginning of the period. Its problem is thus

$$\max_{\{L_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\beta^{t+1} \frac{U_{C,t+1}}{P_{t+1}} \right) D_t, \tag{19}$$

where

$$D_t = P_t \left(\overline{K} + \theta_t L_t^{\alpha} \right) - R_t W_t L_t. \tag{20}$$

As the household is also subject to a cash-in-advance constraint, the firm discounts the profit at time t with the marginal utility of consumption and price at time $t+1$.

The first order condition is then

$$\alpha P_t \theta_t L_t^{\alpha-1} = R_t W_t. \tag{21}$$

3.2.3. Financial Intermediary

At the beginning of the period, the representative competitive financial intermediary collects the money the household sends to it, N_t, and receives a money injection from the central bank, $X_t = M_{t+1}^S - M_t^S$. It then lends that entire amount to the firm, clearing the financial market:

$$W_t L_t = N_t + X_t. \tag{22}$$

At the end of the period, the financial intermediary collects the loan (capital plus interest) from the firm, $R_t (N_t + X_t)$, and pays back the deposit (capital plus interest) to the household, $R_t N_t$. The profit, $R_t X_t$, is then also distributed to the household.

3.2.4. Monetary Policy

The central bank is assumed to be able to react to the contemporaneous productivity shock. The monetary policy could be modeled either as

$$x = \overline{x} + \delta_x (\theta - 1), \tag{23}$$

or as

$$R = \overline{R} + \delta (\theta - 1). \tag{24}$$

If the central bank sets the nominal interest rate, the money supply will be determined endogenously, in the sense that it will have to be adjusted by the central bank to implement the interest rate rule.

3.2.5. Equilibrium

An equilibrium consists of allocation functions $C(\theta), L(\theta), n$, and price functions $R(\theta), p(\theta), w(\theta)$, such that the household maximizes utility and firms maximize profits given the price functions, and the goods, loan and money markets clear, for each realization of the productivity shock, given the monetary policy $x(\theta)$. The allocation and price functions must thus satisfy the Euler equations

$$\psi_0 L^{\psi} = \frac{w}{p}, \tag{25}$$

$$E_{\Omega^N}\left(\frac{U_C}{p}\right) = \beta E_{\Omega^N}\left(\frac{RU_{C'}}{(1+x)\,p'}\right), \tag{26}$$

$$\frac{wR}{p} = \alpha\theta L^{\alpha-1}, \tag{27}$$

the resource constraint

$$C = \overline{K} + \theta L^{\alpha}, \tag{28}$$

the loan market clearing

$$wL = n + x, \tag{29}$$

the household cash constraint, which combined with (29) leads to

$$pC = 1 + x, \tag{30}$$

given the monetary policy

$$x = \overline{x} + \delta_x(\theta - 1). \tag{31}$$

The labor and money market clearing conditions have been imposed. By Walras Law, the household's budget constraint is satisfied.

In the analysis below I will characterize the second-best policy in terms of a nominal interest rate rule of the form $R = \overline{R} + \delta\,(\theta - 1)$. This latter equation will thus replace equation (31), and money supply will be determined by equilibrium conditions.

3.3. Solving for First-Best and Equilibrium Employment

Using (25) and (27), we can obtain equilibrium employment

$$L_e = \left(\frac{\alpha}{\psi_0}\frac{\theta}{R}\right)^{\frac{1}{(1+\psi-\alpha)}}, \tag{32}$$

whereas first-best employment in this model is given by

$$L^* = \left(\frac{\alpha}{\psi_0}\theta\right)^{\frac{1}{(1+\psi-\alpha)}}. \tag{33}$$

Thus, from equation (33), optimal employment is procyclical. Let us set $\psi_0 = \alpha$, so that optimal employment is unity at steady-state.

The behavior of equilibrium employment over the cycle depends on the covariance between the nominal interest rate and productivity shocks. The ratio of optimal over equilibrium employment is given by

$$\frac{L^*}{L_e} = R^{\xi}, \tag{34}$$

where

$$\xi = \frac{1}{(1 + \psi - \alpha)}.$$

If monetary policy is non-activist, i.e. with a constant money growth rule, R will be positively correlated with the productivity shock, as explained in appendix 2, and this ratio will vary over the cycle, increasing in booms, and decreasing in recessions. Optimality is obtained by keeping R at unity, which implies injecting money after a positive productivity shock that would otherwise raise the nominal interest rate. A zero net nominal interest rate would however lead to a non-unique optimal monetary policy, as nominal interest rates cannot go below zero. For example, Fuerst (1994) deals with this multiplicity problem by assuming that both cash-in-advance constraints (household and firm) are always binding, thus monetary policy is procyclical even after a negative productivity shock.

Targeting a strictly positive inflation level will imply a steady-state gross nominal interest rate higher than unity. The fundamental issue addressed in this paper is to figure out what is the second-best ratio (34) as a function of productivity shocks, for a given sub-optimal steady-state gross interest rate higher than one, and what this does imply for monetary policy, or equivalently, what is the optimal behavior of the nominal interest rate when the central bank targets a strictly positive inflation level. The strictly positive inflation target is exogenously imposed, thus the maximization problem becomes a second-best problem. The motivations for that exogenously imposed sub-optimal inflation target were discussed in section 2.

Section 3.4 characterizes the optimal cyclical monetary policy, and section 3.6 provides numerical results. In appendix 2 an equilibrium analysis of the model is presented, where the implications of the assumption that $\overline{K} \neq 0$ are developed.

3.4. Solving for the second-best policy

In this section, the optimal cyclical behavior of the nominal interest rate is characterized by deriving an optimal nominal interest rate rule[6] for different pre-specified levels of steady-state nominal interest rate corresponding to different steady-state inflation levels. We will be looking at a monetary policy rule of the form $R = \overline{R} + \delta(\theta - 1)$.

Using equation (32) for equilibrium employment and the utility function (16), the utility of the representative household can be expressed as a function of the nominal interest rate and the productivity shock:

$$U = \frac{\left[\overline{K} + \theta\left(\frac{\theta}{R}\right)^{\alpha\xi} - \psi_0 \frac{\left(\frac{\theta}{R}\right)^{\xi(1+\psi)}}{1+\psi}\right]^{1-\sigma}}{1-\sigma} \tag{35}$$

As in section 2.1, we can take the expectation of the second-order Taylor approximation of this expression around the steady-state values $R = \overline{R}$ and $\theta = 1$, and obtain

$$E(U) = \overline{U} + \overline{U_{\theta R}}\sigma_{\theta R} + \frac{1}{2}\overline{U_{RR}}\sigma_R^2 + \frac{1}{2}\overline{U_{\theta\theta}}\sigma_\theta^2, \tag{36}$$

where the upper-bar means that these derivatives are evaluated at their steady-state values. As explained in section 2, the goal of the policy maker is to maximize $\left(\overline{U_{\theta R}}\sigma_{\theta R} + \overline{U_{RR}}\sigma_R^2\right)$, where $\overline{U_{RR}} < 0$, for a given steady-state $\overline{R}$ that has been exogenously imposed.

If $\overline{U_{\theta R}} = 0$, which would be the case for a utility function of the form $U = \ln(C) - L$ in this simple model, a policy that pegs the nominal interest rate is better

[6] The optimal behavior of the money supply can be recovered from the equilibrium conditions.

than any other policy, with same mean $\overline{R}$, that causes variations in the nominal interest rate. This is the result obtained by Carlstrom and Fuerst (1996) when seigniorage is kept constant across the two regimes[7]. However, as mentioned in section 3.2.1, with that utility function equilibrium employment does not react to productivity shocks when money growth is constant, and first-best employment becomes countercyclical for higher degrees of risk aversion. Adopting the utility function (16) allows us to get around both these features and will lead to interesting results for second-best monetary policy. Moreover, with more fully specified models, there is no reason to expect that $\overline{U_{\theta R}} = 0$.

In case $\overline{U_{\theta R}} > 0$, a monetary policy implying a positive covariance between interest rates and productivity shocks, like a constant money growth rate for example, could potentially dominate a nominal interest rate peg. We will see below that this can be the case. The most important aspect with regard to the issue addressed in this paper is that $\overline{U_{\theta R}}$ varies with different steady-state nominal interest rates and thus with different inflation targets.

As discussed in section 2.1, the second-best policy is defined as the optimal reaction of the nominal interest rate to the productivity shock, and takes the form

$$R = \overline{R} + \delta^* (\theta - 1) \tag{37}$$

where δ^* is chosen so as to maximize expected utility (36), for a pre-specified $\overline{R}$.

[7] Given that $\overline{U_{\theta R}} = 0$ in their model, optimal monetary policy will be determined by the sign of $\overline{U_{RR}}$. With their model specifications, fluctuations in interest rate per se increase labor and output, thus fluctuations in interest rates will increase welfare by raising average output, and decrease welfare by increasing average labor and by causing fluctuations in consumption. For $R > 2$, $\overline{U_{RR}}$ becomes positive in their model, thus the optimal monetary policy then would be to generate an infinite variance of the interest rate.

Given this policy function, as seen in section 2.1, we can obtain the optimal reaction coefficient

$$\delta^* = \frac{\overline{U_{\theta R}}}{-\overline{U_{RR}}}. \tag{38}$$

We will then compare the optimal response of the nominal interest rate to productivity shocks, i.e. δ^*, to the equilibrium response of the nominal interest rate when monetary policy is non-activist, i.e. when money supply growth is kept constant. We approximate the behavior of the nominal interest rate in the non-activist case by taking the first order Taylor expansion of the right-hand side of equation (49) around $\theta = 1$, thus getting an expression of the form

$$R = \overline{R} + \delta^e (\theta - 1). \tag{39}$$

δ^e will then be compared to δ^* in (37) for different values of $\overline{R}$, in order to see how much fluctuations in the nominal interest rate are smoothed (or amplified) with the optimal monetary policy compared to the non-activist policy, for different $\overline{R}$.

Here a non-activist policy is understood as a policy that keeps money growth constant. Thus a positive covariance of the nominal interest rate with the productivity shock does not necessarily mean that the monetary policy is activist, as the limited participation constraint causes the equilibrium interest rate to increase with a productivity shock when money growth is kept constant. A pro- (counter-)cyclical monetary policy will thus dampen (exacerbate) the interest rate response to a productivity shock, relative to its equilibrium response in the non-activist case, by injecting (withdrawing) money to (from) the financial sector.

3.5. Calibration

α is set to 0.7. In order to obtain a steady-state labor share of 0.64, $\overline{K}$ is set to 0.09375. The period unit is a quarter, thus β is set to $1.03^{-.25}$. $\psi = 1$, implying a unit labor supply elasticity. Results for different coefficients of risk aversion σ will be displayed.

3.6. Results

The following results illustrate the potential importance of varying the steady-state inflation level for the optimal cyclical monetary policy. Results for different average quarterly gross nominal interest rates, and the corresponding annual inflation rates, are presented in Table 1. $\overline{R}$ and δ^* are from the optimal rule (37).

$\overline{R}$	Inflation (%)	δ^* when $\sigma = 1$	δ^* when $\sigma = 3$	δ^* when $\sigma = 5$
1	-3	0	0	0
1.0075	0	-.003	.04	.08
1.0125	2	-.005	.06	.13
1.02	5	-.008	.10	.22
1.03	10	-.013	.16	.33

Table 1. Optimal Policy, Inflation Target and Risk Aversion

The equilibrium response[8] of the nominal interest rate to productivity shocks when the monetary policy is non-activist, i.e. with a constant money supply growth rule, δ^e in equation (39), is 0.28. The first-best monetary policy completely smooths the nominal interest rates at its zero level, i.e. $\delta^* = 0$, and allows employment to react to shocks in an optimal way.

Consider now the last column, i.e. when $\sigma = 5$. When the average inflation rate is

[8] δ^e varies only very slightly with the different steady-states.

0 percent, the optimal monetary policy is still procyclical as it dampens fluctuations in the nominal interest rate, i.e. $\delta^* < \delta^e$. However, as average inflation increases, monetary policy becomes less procyclical and allows the nominal interest rate to vary more. If the central bank chooses the inflation target between 5 and 10 percent for which δ^* equals .28, it should adopt a constant growth rate of the money supply, without reacting to economic disturbances, as in that case δ^*, the optimal reaction to shocks, is equal to δ^e, the equilibrium reaction to shocks when monetary policy is non-activist, i.e. does not react to shocks. For higher inflation rates, the optimal monetary policy becomes countercyclical, i.e. the central bank withdraws money when a positive productivity shock occurs, thus exacerbating the rise in interest rate.

Considering the two last columns, i.e. when $\sigma = 3$ and $\sigma = 5$, we see that the optimal cyclical policy becomes less accommodative when the inflation target increases, i.e. the optimal covariance between the policy instrument R and productivity shocks increases with inflation. Optimal cyclical policy switches from procyclical to countercyclical as the inflation target increases, reflecting the fact that in a more distorted economy, fluctuations in consumption become more painful to households, thus the central bank will dampen output fluctuations following productivity shocks, as discussed in section 2.2. Note also that the higher the degree of risk aversion, the more monetary policy will dampen output fluctuations, as the more these fluctuations are painful for risk-averse households.

Thus, when the average distortion level is small, it is worth it to smooth nominal interest rates. This will allow equilibrium employment to react as first-best employment does and thus achieve a higher average consumption level. However, from a certain level of risk aversion, as we depart significantly from the first-best outcome it becomes less attractive to have employment reacting to shocks and utility smoothing concerns become more important.

Considering now the case where $\sigma = 1$, we see that monetary policy becomes more procyclical, i.e. goes beyond nominal interest rate smoothing, when the level of distortion increases. This reflects the fact that for less risk-averse households, the gains in obtaining a higher output level on average, by letting employment covary positively with productivity shocks, more than offset the loss of higher output fluctuations, as they suffer less from them. This comes from the fact that the larger the distortion, the larger the marginal utility of that extra average consumption will be.

4. Conclusions

The goal of this paper was to try understanding how different steady-state inflation levels could affect optimal cyclical monetary policy. Alternatively, the paper examines how accounting for steady-state distortions affect the derivation of optimal cyclical policy. Due to the presence of short-term rigidities, monetary policy can affect the equilibrium response of macroeconomic variables to shocks. This paper shows that the optimal policy response to shocks is dependent of the degree of steady-state distortion, thus the derived optimal monetary policy rule is dependant of assumptions about the steady-state inflation level and other steady-state distortions. The main idea behind this result is that introducing a second distortion can improve welfare, as by varying the short-term interest rate in response to shocks, the central bank can allow households to attain the allocation they would have chosen in the absence of short-term rigidities for a given steady-state distortion.

The implications for monetary policy and for research on monetary policy rules are that, first, the issue of optimal cyclical monetary policy should be addressed simultaneously with the choice of optimal inflation target. And second, whether or not we account for various steady-state distortions will affect the results regarding optimal cyclical monetary policy.

While this paper has examined different mechanisms and provided with a simple numerical example showing the potential importance of different inflation targets on optimal cyclical monetary policy, a more fully specified and calibrated model, with different types of short-term rigidities, shocks, and distortions justifying a higher inflation target, are needed to obtain quantitative assessment of the importance of the issue addressed. This is left for future work.

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Appendix 1: First order conditions of the Household's Problem

Let ν and μ be the multipliers associated with the cash-in-advance constraint and the law of motion of money demand, respectively. The first order and envelope conditions are then as follows:

FOC w/r/to C:

$$U_C = (\nu + \mu)\, p \tag{40}$$

FOC w/r/to L:

$$U_L + (\nu + \mu)\, w = 0 \tag{41}$$

FOC w/r/to m':

$$\beta V_{m'} = \mu\, (1 + x) \tag{42}$$

FOC w/r/to n:

$$E_{\Omega^N}\left[\nu + \mu\, (1 - R)\right] = 0 \tag{43}$$

Env. w/r/to m:

$$V_m = E_{\Omega^N}\left[\nu + \mu\right] \tag{44}$$

Combining the FOC w/r/to C and L, we get

$$U_L + U_C \frac{w}{p} = 0. \tag{45}$$

Combining the FOC and Envelope condition w/r/to m' and m, we get

$$\mu = \frac{\beta}{1 + x} E_{\Omega^N}\left[\nu' + \mu'\right]. \tag{46}$$

Appendix 2: Equilibrium Analysis

In this appendix, the special case where $\alpha = 1$ is examined, which leads to closed form solutions for employment and the nominal interest rate. Results will be derived alternatively for $\overline{K} \neq 0$ and $\overline{K} = 0$.

Combining (29) and (30), we obtain

$$\frac{wL}{pC} = s = \frac{n+x}{1+x}, \tag{47}$$

where s is defined as the share of the money stock, after the current period injection, held by intermediaries. As n must be chosen before the productivity shock occurs, s depends on the shocks only through the response of monetary policy to shocks, i.e. via $x(\theta)$ in (31).

Let $\psi = 1$. Substituting (25) and (28) in (47), we obtain employment as a function function of the productivity shock and the money supply (through s)

$$L = \frac{s\theta + \sqrt{s^2\theta^2 + 4s\overline{K}}}{2}, \tag{48}$$

thus

$$0 < \frac{dL}{d\theta} \leq s.$$

When $\overline{K} = 0$, we have

$$L = \frac{\theta}{R} = s\theta,$$

thus

$$\frac{dL}{d\theta} = s = \frac{1}{R}.$$

With $\overline{K} > 0$, the increase in the nominal interest rate dampens the equilibrium response of employment to productivity shocks, whereas, as we will see below, in the case $\overline{K} = 0$ nominal interest rate does not react to shocks when money growth x is constant.

Equalizing (48) with (32) leads to an expression determining the behavior of the equilibrium gross nominal interest rate

$$R = \frac{2\theta}{s\theta + \sqrt{s^2\theta^2 + 4s\overline{K}}}. \tag{49}$$

Thus the nominal interest rate is procyclical when money growth is constant.

If $\overline{K} = 0$, we have

$$R = \frac{1}{s},$$

thus R would be independent of the productivity shock when money growth is kept constant, as s depends on monetary policy only.

The reason why we need a strictly positive $\overline{K}$ for the nominal interest rate to react to productivity shocks in this model can be understood as follows. Multiplying the marginal productivity of labor by equilibrium employment and further by the price level in (30), we obtain the wage bill, which has to be financed by loans, at the nominal interest rate R. We thus obtain

$$(1+x)\,\frac{\alpha\theta L^{\alpha}}{\overline{K} + \theta L^{\alpha}} = RwL = R\,(n+x)\,.$$

However, when the monetary policy is non-activist, wL is independent of the productivity shock, as it is equal to the loans supply which is determined before the shock is observed. Thus if $\overline{K} = 0$, R would be unresponsive to productivity shocks, as movements in the price level would exactly offset changes in productivity.