STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Timing Tax Evasion

Dirk Niepelt

Working Paper 04.07

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Dirk Niepelt†

Institute for International Economic Studies, Stockholm University
and
Study Center Gerzensee

October 2004

Abstract

Standard models of tax evasion implicitly assume that evasion is either fully detected, or not detected at all. Empirically, this is not the case, casting into doubt the traditional rationales for interior evasion choices. I propose two alternative, dynamic explanations for interior tax evasion rates: Fines depending on the duration of an evasion spell, and different vintages of income sources subject to aggregate risk and fixed costs when switched between evasion states. The dynamic approach yields a transparent representation of revenue losses and social costs due to tax evasion, novel findings on the effect of policy on tax evasion, and a tractable framework for the analysis of tax evasion dynamics.

KEYWORDS: Dynamic tax evasion; detection risk; duration dependent fine; inaction range; optimal taxation.
JEL CODE: E6, H2.

1 Introduction

Illegal, intentional underreporting of income reduces the income tax revenue for the U.S. Department of the Treasury by about 15 to 20 percent. In other developed economies, the revenue loss due to tax fraud appears to be of the same order of magnitude, if not larger.[1] Not only is the volume of tax evasion and semi-legal tax avoidance quantitatively important, the elasticity of evasion and avoidance is also high. In fact, evasion and avoidance may be at least as relevant for tax policy as the labor supply or savings responses on which traditional public finance has its focus.[2]

Existing theories of tax evasion emphasize risk aversion or endogenous detection probability in order to rationalize the observation that households evade some, but typically not all, taxes.[3]

*I have benefited from comments by Emmanuel Saez, three anonymous referees, Per Krusell, Claes Trygger, and participants at seminars and the 2002 SED conference. Editorial assistance by Christina Lönnblad is gratefully acknowledged.

†106 91 Stockholm, Sweden. E-mail: dirk.niepelt@iies.su.se; ph: +46 8 162527; fax: +46 8 161443.

[1] For estimates of the tax gap in the U.S. and other countries, see Andreoni, Erard and Feinstein (1998) or Slemrod and Yitzhaki (2002) and the sources cited therein. These estimates abstract from hypothetical tax revenues from illegal sources of income; see also Cowell (1990).

[2] See, for example, MaCurdy (1992), Slemrod (1992), Feldstein (1995), Agell, Englund and Södersten (1996), and Auerbach and Slemrod (1997).

[3] The seminal papers are Allingham and Sandmo (1972) and Yitzhaki (1974), building on Becker's (1968) work. Andreoni et al. (1998) and Slemrod and Yitzhaki (2002) review the literature.

None of these factors is sufficient, however. In fact, the finding of an interior evasion rate in the standard model crucially depends on the implicit assumption that evasion is either fully detected, triggering fines proportional to the total amount of taxes evaded, or not detected at all. While central for existing theories of tax evasion, this "all-or-nothing" assumption is often implausible (for example in the context of internationally diversified financial investments) and at odds with the data (see below). But relaxing the assumption and replacing it with the opposite extreme of uncorrelated detection risk, results in a corner solution because risk aversion or endogenous detection probability no longer give rise to convex costs of evasion.

This paper argues that there are other forces at work that push towards an interior evasion rate, and that these forces arise from *dynamic* considerations. I explore two mechanisms: The first relies on fines upon detection of evasion that depend on the duration of an evasion spell. Such fines, for example as a function of the cumulative evaded tax, imply that privately optimal evasion choices are characterized by a stopping time: Income is first evaded, and later reported, in order to maximize the expected return net of taxes, fines and other costs. The second mechanism relies on a cross-section of vintages of otherwise identical income sources that are subject to aggregate return risk and fixed costs when switched between evasion states (i.e., between being declared or not declared to the tax authority). These fixed costs imply that old sources of income are only sluggishly switched between evasion states, while the status of new sources immediately responds to shocks. In equilibrium, the evasion rate is typically interior, displays hysteresis, and strongly responds to changes in various institutional parameters.

The potential importance of dynamic considerations for a household's tax evasion strategy has been noted before. Allingham and Sandmo (1972, section 5) discuss an extension of their static argument, with detection of evasion triggering investigations on prior reporting by the tax authority. Engel and Hines (1999) document the empirical relevance of such a link between detection and investigations on prior reporting. They propose a model where this link operates over one period. The settings of both Allingham and Sandmo (1972) and Engel and Hines (1999) combine dynamic and static sources of convex evasion costs. This makes it difficult to identify the exact role played by the different assumptions, and it precludes closed form solutions. The model considered here carefully distinguishes between the various aspects rendering the cost of tax evasion convex. The focus on dynamic considerations generates a transparent representation of revenue losses and social costs due to tax evasion; it yields closed form solutions; and it allows us to analyze an extension with aggregate risk, and with households rationally accounting for that risk.

The remainder of the paper is structured as follows: Section 2 clarifies the central role played by the "all-or-nothing" assumption in the standard model, motivates the dynamic approach adopted in the paper, and relates it to the traditional setup. Section 3 analyzes the effect of duration dependent fines. Section 4 turns to the environment with aggregate risk, and Section 5 concludes.

2 Detection Risk

Consider the tax evasion program of a household at a given point in time. The household owns many sources of income, indexed by $i = 1, \ldots, I$, that pay a constant return, normalized to r/I. One way of considering the sources of income is in terms of dollars deposited to (potentially many different) savings accounts paying a uniform pre-tax yield, another is in terms of hours allocated to (potentially different) jobs paying a uniform pre-tax wage.

For each i, the household chooses whether to declare the source of income to the tax authority. Choice of the former option is denoted by $e_i = 1$, choice of the latter by $e_i = 0$; e denotes the vector of evasion decisions. Declaring i to the tax authority implies that a tax, amounting to the fraction τ of the income generated by i, must be paid. Not declaring i to the tax authority implies that a fine, amounting to the fraction π of the income generated by i, must be paid to the tax authority if evasion is detected. To make the problem interesting, I assume that $\pi > \tau$. Detection occurs randomly. Denote by $\delta_i = 1$ the event that the tax authority scrutinizes income source i (triggering payment of a fine if i was evaded) and by $\delta_i = 0$ the event of no scrutiny. Further, let δ denote the vector of detection events. The realization of δ does not only depend on exogenous sources of uncertainty but also, potentially, on the household's evasion decision, e; $f(\delta|e)$ denotes the probability of a particular realization δ, conditional on the evasion choice e.

The household's objective is to maximize expected utility of consumption, where consumption equals income after taxes and fines. Letting $u(\cdot)$ denote the utility function, this program can be stated as

$$\max_e \sum_\delta u\left(\sum_{i=1}^{I}[1-\tau+e_i(\tau-\delta_i\pi)]r/I\right) f(\delta|e).$$

In the models of Allingham and Sandmo (1972) and Yitzhaki (1974), tax evasion is either not detected at all, or fully detected. Under this "all-or-nothing" assumption (which imposes a restriction on the conditional probabilities $f(\cdot|e)$[4]), expected utility of the household equals

$$(1-p(e))u\left(\sum_{i=1}^{I}[1-\tau+e_i\tau]r/I\right) + p(e)u\left(\sum_{i=1}^{I}[1-\tau+e_i(\tau-\pi)]r/I\right),$$

with $p(e)$ denoting the probability of full detection. The cost of tax evasion is convex, and the evasion rate is thus interior ($e_i \neq e_j$ for some i, j), if either the household is risk averse or p increases in $e_i, i = 1, \ldots, I$.[5] If the household is risk averse, the increased volatility of consumption due to a higher evasion rate renders evasion increasingly costly. If p increases in e, a higher evasion rate raises the marginal expected fine, once more rendering evasion increasingly costly.[6] If neither the first nor the second condition is satisfied, the tax evasion program yields a corner solution, since the expected net benefit of an increase in the evasion rate is independent of that rate.

Both traditional rationales for an interior evasion rate crucially depend on the "all-or-nothing" assumption. To see this more clearly, consider the opposite case with i.i.d. detection risk across all sources of income. Formally, the restriction on $f(\cdot|e)$ which is implicit in the models of Allingham and Sandmo (1972) and Yitzhaki (1974) is replaced by the assumption that each $\delta_i, i = 1, \ldots, I$, is independent of e, as well as i.i.d. according to $g(\cdot)$ say, the marginal distribution of every individual detection event. Letting z denote the fraction of non-reported sources of income, $z \equiv \sum_i e_i/I$, expected consumption equals

$$r(1-\pi z g(1) - \tau(1-z)),$$

[4]The restriction is the following: For any i, j, if $e_i = e_j = 1$ then $f(\delta|e) = 0$ for all δ featuring $\delta_i \neq \delta_j$. This condition is satisfied, for example, if the tax authority either scrutinizes all sources of income, or none.

[5]The second condition is satisfied, for example, if the tax authority adopts a two-step procedure to investigate tax evasion. In the first step, it randomly picks one source of income and checks whether income from that particular source was declared. If this was not the case, then the authority scrutinizes all other sources of income in a second step.

[6]A small increase in the evasion rate only triggers a small increase in the detection risk, but detection triggers increasingly large fines since these are proportional to total evaded income.

and the variance of consumption converges to zero for $I \to \infty$, due to statistical independence. Uncorrelated detection risk across many sources of income thus implies that the household is perfectly insured—higher evasion does not increase the volatility of consumption. It also implies that expected consumption is linear in z—higher evasion does not increase the marginal expected fine. As a consequence, the optimal evasion rate in the static model is *not* interior, even if the household is risk averse, and although the probability of evasion being detected is increasing in the evasion rate.

The crucial role of the "all-or-nothing" assumption in the static model is disturbing. For once, the assumption appears inconsistent with empirical evidence, according to which tax evasion is typically not fully detected, even conditional on a taxpayer being audited (Andreoni et al., 1998).[7] Moreover, it often is not very plausible. After all, many situations are characterized by weakly rather than fully correlated detection risk. This is particularly true in situations where households diversify their sources of income, as for example financial investments. Arguably, tax authorities push for more wide-ranging international information-sharing, exactly because uncorrelated detection risk removes the threat of consumption volatility, and thereby fosters tax evasion.[8]

This paper offers two dynamic explanations that are able to reconcile uncorrelated detection risk with interior evasion choices (for individual types of income). First, fines that are increasing in the duration of a tax evasion spell because detection triggers fines proportional to the cumulative evaded tax; in the above equation, this amounts to π increasing over time. Second, a cross-section of vintages of otherwise identical income sources that are subject to aggregate risk and fixed costs when switched between evasion states. These dynamic factors result in convex evasion costs when detection risk is uncorrelated, but they can also generate interior evasion rates under much more general conditions, and in combination with other sources of convexity. To keep the analysis transparent, however, I focus on the benchmark case where the results are exclusively driven by these dynamic aspects. In other words, I exclude all sources of convexity present in the static model. As discussed earlier, this is consistent with three alternative sets of assumptions:

i. Risk neutrality, detection "all-or-nothing" and independent of e; or

ii. risk neutrality and uncorrelated detection risk across many sources of income; or

iii. risk aversion and uncorrelated detection risk across many sources of income.

In the setting without aggregate risk considered in Section 3, the results hold under any of these three sets of assumptions. In the setting with aggregate risk considered in Section 4, however, the assumption of risk aversion would introduce substantial complications. Therefore, I impose risk neutrality in that section, consistent with either i. or ii.

[7] Andreoni et al. (1998, p. 850) report that conditional on an audit by the U.S. Internal Revenue Service, approximately one half of the concealed income typically remains undetected.

[8] See, for example, recent proposals by the European Commission (2001, Proposal 400, `http://europa.eu.int/comm/ taxation_customs/ proposals/ taxation/ tax_prop.htm#COM2001400`) that EU member states should provide each other with information on interest income accrued to their residents (instead of just taxing at the source), or the OECD (1998) proposal on information measures to counteract "harmful tax competition".

3 Duration Dependent Fines

3.1 The Model

I analyze the household's dynamic tax evasion program in continuous time. Households discount the future at rate ρ. They face a time invariant tax system, a constant pre-tax yield r ($0 < r < \rho$) on their sources of income, or "capital", and a constant detection rate λ on any unit of capital not declared to the tax authorities. Uncorrelated detection risk reduces the tax evasion strategy to one of maximizing the expected return after taxes and fines; as discussed earlier, risk neutrality (i. or ii.) has the same implication. The fundamental unit of analysis is therefore the unit of capital, not the individual household.[9]

A unit of capital is characterized by two properties: First, whether income from the unit is reported to the tax authority, for brevity referred to as "in state w", or not, referred to as "in state v"; second, by the time t that has passed since the unit was last switched between states v and w. There is a fixed cost k per unit of capital for voluntarily switching between state w and state v, capturing the cost of disguising the sudden appearance or disappearance of income sources in the tax declaration.[10] Such a cost might arise, for example, because an advisor has to be hired who knows how to convincingly make a case vis-à-vis the tax authority. Or it may arise because hiding capital and letting it reappear involves some transactions that temporarily reduce the return. The statutory tax rate is $\tau > 0$. The dividend yield thus equals r for units of capital neither declared nor detected, and $r(1-\tau)$ for truthfully reported ones.

A unit of capital switched to state w remains in that state for a minimum duration of T. This assumption cuts short on a micro founded argument according to which paying taxes has some private benefit of reducing potential future fines, for example by creating a reputation of "honesty" discouraging investigations by the tax authority in case of detection during a successive evasion spell (see Appendix A.1 for an exposition).

Units of capital in state v are detected at the rate λ. Upon detection, two actions are triggered. First, the tax authority starts an audit and investigates for how long income from that particular source has been evaded. The fine $\pi(t)$, assumed to be a smooth function, accounts for (some of the) prior evaded tax payments, such that $\pi'(t) > 0$. Second, the unit must be switched from state v to state w.

Denote the value of one unit of capital in state (v, t) by $V(t)$ and the value of one unit of capital in state (w, t) by $W(t)$. Upon detection, the continuation value of one unit of capital in state (v, t) is given by $-\pi(t) + W(0)$. Moreover,

$$\begin{aligned} W(t) &= \int_t^T \mathrm{e}^{-\rho(x-t)} r(1-\tau)\, dx + \mathrm{e}^{-\rho(T-t)} W(T) = \\ &= \mathrm{e}^{-\rho(T-t)} \left(W(T) - \frac{r(1-\tau)}{\rho} \right) + \frac{r(1-\tau)}{\rho}, \qquad (1) \\ W(T) &= \max_{y \geq 0} \int_0^y \mathrm{e}^{-\rho x} r(1-\tau)\, dx + \mathrm{e}^{-\rho y}[V(0) - k]. \qquad (2) \end{aligned}$$

The first condition defines the value of a unit of capital in state (w, t) as the present discounted value of payoffs from the unit. The second condition defines the value of a unit of capital in state (w, T): If tax evasion is profitable, households will switch from state w to state v as soon as they can, implying $y = 0$ and $W(T) = V(0) - k$. If households choose not to immediately

[9] This excludes, for example, progressive taxation or household-specific investigations by the tax authority.

[10] The assumption that the switching cost is the same in both directions can easily be relaxed.

switch from state w to state v after duration T, they will never switch, $y = \infty$. In this case, $W(T) = W(0) = W(t) = r(1-\tau)/\rho$.

To derive $V(t)$, consider the value of a unit of capital in state (v, t) that may *not* be switched to state w before the infinitesimally small time span dt has passed. Denote this value by $\tilde{V}(t)$, and denote by s the time span after which switching from state v to state w is optimal. Clearly, $\tilde{V}(t) = V(t)$ for $t < s$. As long as $t < s$, the value of a non-reported unit of capital with spell duration t therefore equals the flow payoff, $r\,dt$, plus the probability weighted discounted continuation values in case of detection and no detection, respectively. We thus have

$$V(t) = \lim_{dt \to 0} r\,dt + \mathrm{e}^{-\rho\,dt}[(1-\lambda\,dt)V(t+dt) + \lambda\,dt(W(0) - \pi(t+dt))],\ t < s,$$

implying that a unit in state (v, t) must satisfy the following standard no-arbitrage relationship:

$$(\rho+\lambda)V(t) = r + V'(t) + \lambda(-\pi(t) + W(0)),\ t < s. \tag{3}$$

Equation (3) is derived in Appendix A.2. It states that, for $t < s$, a unit of capital in state (v, t) must pay the risk adjusted required return, $\rho + \lambda$, in the form of either flow payoffs or expected capital gains. If the optimal stopping time s is finite, the household must be indifferent between keeping a unit of capital in state (v, s) and switching it to state $(w, 0)$:

$$V(s) = W(0) - k \text{ if } s < \infty. \tag{4}$$

The fundamental, bubble free solution of (3) is therefore given by

$$V(t) = \int_t^s \mathrm{e}^{-(\rho+\lambda)(x-t)}(r + \lambda(-\pi(x) + W(0)))\,dx + \mathrm{e}^{-(\rho+\lambda)(s-t)}(W(0) - k),\ t < s. \tag{5}$$

Either T or k must be strictly positive for the dynamic tax evasion problem to be well defined. If both parameters were equal to zero, households could switch capital from state v to state w after an infinitesimally short duration (to "reset" the fine to $\pi(0)$) and then immediately back to state v. A strictly positive value for k induces households to keep the capital in state v for some time before switching it to state w. However, it does not induce them to keep it in state w for some time.[11] In contrast, a strictly positive value for T enforces a minimum duration in state w. It also induces households to keep capital in state v for some time before switching it to state w, even if k is zero. Throughout this section, I assume $T > 0, k \geq 0$, implying that the optimal duration s is bounded away from zero, provided that tax evasion is profitable at all. An interior solution for s results if taxes are evaded and if (from (5)) $\int_t^\infty \mathrm{e}^{-(\rho+\lambda)(x-t)}(r + \lambda(-\pi(x) + W(0)))\,dx < W(0) - k$ for some $t > 0$, such that $s < \infty$.

The optimal stopping time s can be obtained by combining conditions (1), (2), and (5) with the smooth pasting condition $V'(s) = 0$ (see Dixit and Pindyck, 1994).[12] Alternatively, one may maximize the function $\mathcal{V}_0(s)$, defined as $V(0)$ subject to conditions (1), (2), and (5). This function is given by[13]

$$\begin{aligned}\mathcal{V}_0(s) \;=\; & \left\{-\lambda \int_0^s \mathrm{e}^{-(\rho+\lambda)x}\pi(x)\ dx + q(s)\left(r + \lambda\frac{r(1-\tau)}{\rho}(1-\mathrm{e}^{-\rho T})\right) + \right.\\ & \left.\left(\frac{r(1-\tau)}{\rho}(1-\mathrm{e}^{-\rho T}) - k\right)\mathrm{e}^{-(\rho+\lambda)s} - k(\lambda \mathrm{e}^{-\rho T} q(s) + \mathrm{e}^{-\rho T - (\rho+\lambda)s})\right\} / \\ & \left(1 - \lambda \mathrm{e}^{-\rho T} q(s) - \mathrm{e}^{-\rho T - (\rho+\lambda)s}\right),\end{aligned}$$

[11] If it *is* optimal to switch capital to state v at some point, then it is optimal to do so as soon as possible.
[12] Combined with (4), the smooth pasting condition implies $(\rho+\lambda)(W(0) - k) = r + \lambda(-\pi(s) + W(0))$.
[13] See Appendix A.3.

with $q(s) \equiv \int_0^s \mathrm{e}^{-(\rho+\lambda)x}dx = \frac{1-\mathrm{e}^{-(\rho+\lambda)s}}{\rho+\lambda}$. The optimal duration of a tax evasion spell is then characterized by the condition $\mathcal{V}_0'(s) = 0$.[14] In the remainder of this section, I assume that $\lim_{t\to\infty} \pi'(t)/\pi(t) < \rho + \lambda$, implying the integral term in $\mathcal{V}_0(s)$ to be bounded. Since all other terms in $\mathcal{V}_0(s)$ are also bounded and the denominator is strictly positive, $\mathcal{V}_0(s)$ is bounded.[15]

The difference between r/ρ, the social value of one unit of capital in the absence of tax evasion, and $\mathcal{V}_0(s)$ reflects the present discounted value of taxes and fines as well as switching costs. These two present discounted values are recursively defined by, respectively,

$$\begin{aligned}
\mathrm{PDV}^{\tau,\pi}(s) &= \int_0^s \mathrm{e}^{-\rho x} f(x) \left[\pi(x) + \int_0^T \mathrm{e}^{-\rho y} r\tau dy + \mathrm{e}^{-\rho T}\mathrm{PDV}^{\tau,\pi}(s)\right] dx \\
&\quad + (1-F(s))\mathrm{e}^{-\rho s}\left[\int_0^T \mathrm{e}^{-\rho y} r\tau dy + \mathrm{e}^{-\rho T}\mathrm{PDV}^{\tau,\pi}(s)\right], \\
\mathrm{PDV}^{k}(s) &= \int_0^s \mathrm{e}^{-\rho x} f(x) \left[\mathrm{e}^{-\rho T}(k + \mathrm{PDV}^{k}(s))\right] dx + (1-F(s))\mathrm{e}^{-\rho s}\left[k + \mathrm{e}^{-\rho T}(k + \mathrm{PDV}^{k}(s))\right].
\end{aligned}$$

Here, $f(x)$ and $F(x)$ denote the p.d.f. and c.d.f. of an exponential(λ) distribution, respectively. Straightforward manipulations yield

$$\mathrm{PDV}^{\tau,\pi}(s) = \frac{\lambda \int_0^s \mathrm{e}^{-(\rho+\lambda)x}\pi(x)dx + \left(\lambda q(s) + \mathrm{e}^{-(\rho+\lambda)s}\right)\frac{r\tau}{\rho}(1-\mathrm{e}^{-\rho T})}{1-\lambda \mathrm{e}^{-\rho T}q(s) - \mathrm{e}^{-\rho T-(\rho+\lambda)s}}, \tag{6}$$

$$\mathrm{PDV}^{k}(s) = k\frac{\lambda \mathrm{e}^{-\rho T}q(s) + \mathrm{e}^{-(\rho+\lambda)s}(1+\mathrm{e}^{-\rho T})}{1-\lambda \mathrm{e}^{-\rho T}q(s) - \mathrm{e}^{-\rho T-(\rho+\lambda)s}}. \tag{7}$$

Based on (6) and (7), the effective tax-plus-fine rate, $\theta(s) \equiv \mathrm{PDV}^{\tau,\pi}(s)\rho/r$, and the switching-cost rate, $\kappa(s) \equiv \mathrm{PDV}^{k}(s)\rho/r$, can be defined.[16] We then have

$$\mathcal{V}_0(s) = \frac{r(1-\theta(s)-\kappa(s))}{\rho},$$

which yields yet another representation of the tax evasion program, conditional on tax evasion being more profitable than full compliance: $\min_s \theta(s) + \kappa(s)$.

Proposition 1. Let $\pi(0) = 0$, $\pi'(t) \geq 0$, $\pi''(t) \geq 0$ $\forall t \geq 0$; moreover $\pi'(0) > 0$ or $\pi''(0) > 0$ (strict convexity implies these conditions). Under the maintained assumptions

i. there exists a unique, finite $s^\star > 0$ that minimizes $\theta(s) + \kappa(s)$;

ii. there exists a unique, finite $s^\diamond > 0$ that minimizes $\theta(s)$;

iii. if $k > 0$, then $s^\diamond < s^\star$ and $\theta(s^\star) > \theta(s^\diamond)$;

iv. households evade taxes if $\theta(s^\star) + \kappa(s^\star) < \tau$.

Proof. See Appendix A.4. □

[14] Combined with (4), this latter condition reduces to the smooth pasting condition.

[15] Note that $0 \leq q(s) < 1/(\rho+\lambda)$ and the denominator of $\mathcal{V}_0(s)$ equals $1 - \mathrm{e}^{-\rho T}(\rho \mathrm{e}^{-(\rho+\lambda)s} + \lambda)/(\rho+\lambda)$.

[16] Equivalently, $\kappa(s)$ can be derived as $\frac{-k\ \partial\mathcal{V}_0(s)/\partial k}{r/\rho}$.

If there exist stopping times s such that $\theta(s) + \kappa(s) < \tau$, then households evade taxes. Once the marginal benefit from continued evasion in the form of tax savings and lower average switching costs is outweighed by the expected fine, households switch capital back to state w. Under the assumptions of Proposition 1, this is generally the case after a finite duration, $s^\star$.[17] Since households minimize the expected sum of taxes, fines, and switching costs, the privately optimal tax evasion strategy does *not* minimize the government's tax and fine collections, as long as k is strictly positive.

Figures 1 and 2 display a numerical example under the assumption that the fine is given by $\pi(t) = \alpha(e^{rt} - e^{r(1-\tau)t}), \alpha > 1$.[18] Such a fine is natural to consider; it requires, upon detection, repayment of α times the exact accumulated amount gained by tax evasion. It also satisfies the assumptions of the Proposition. In this example, $s^\star$ turns out to be approximately 8.58. Since $\mathcal{V}_0(s^\star) \approx 0.64$ exceeds $r(1-\tau)/\rho = 0.56$ (or τ exceeds $\theta(s^\star)+\kappa(s^\star)$), households optimally evade taxes until duration $s^\star$, or until the evasion is detected.

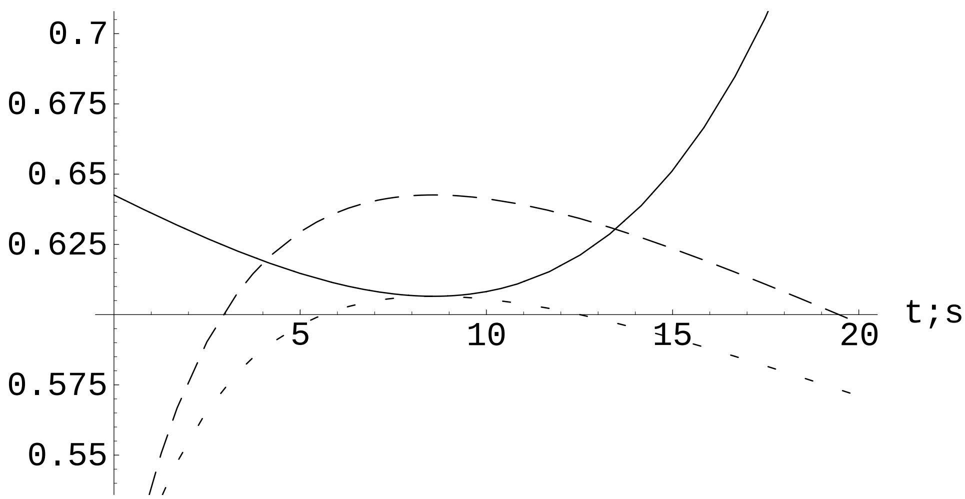

Figure 1: $V(t)[s^\star]$ (downward sloping section of line only), $\mathcal{V}_0(s)$, $W(0)[s] - k$ (in order of decreasing length of line segments).

3.2 Comparative Statics and Optimal Policy

An increase in the statutory tax rate τ increases $s^\star$ and thus, the average duration of tax evasion because

$$\frac{\partial \theta'(s)}{\partial \tau} < 0, \quad \frac{\partial \kappa'(s)}{\partial \tau} = 0.$$

[17] While real world tax laws often imply an initially increasing fine, they also frequently feature a statute of limitation that applies to offences committed more than $\bar{t}$ periods in the past. With such a statute of limitation, the fine $\tilde{\pi}(t)$ becomes $\min[\pi(t), \pi(\bar{t})]$, thereby contradicting the assumptions of the Proposition. As a consequence, the optimal duration $s^\star$ need no longer be finite.

[18] The parameter values in the example are: $\lambda = 0.05; \rho = 0.05; r = 0.04; \tau = 0.3; T = 5; k = 0.01; \alpha = 1.475$. The choice of α is inspired by empirical evidence discussed in the following section.

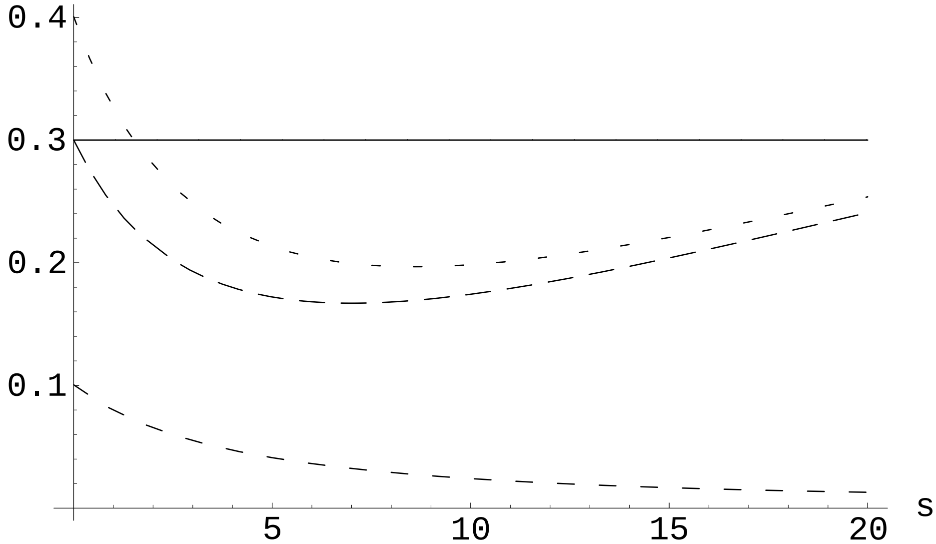

Figure 2: τ, $\theta(s)$, $\kappa(s)$, $\theta(s)+\kappa(s)$ (in order of decreasing length of line segments).

This result contrasts with the finding in static models, where a higher statutory tax rate might have ambiguous effects (Allingham and Sandmo, 1972; Yitzhaki, 1974). While these models stress the income and substitution effects of changes in τ on the demand for state contingent consumption, the dynamic perspective proposed here stresses the effect on expected returns: A higher statutory tax rate induces households to wait longer, and face higher expected fines before switching to reporting accrued income.

Other comparative statics results can easily be derived. The derivations become particularly simple under the convenient assumption that $T \to \infty$ (i.e., once a unit of capital is in state w, income from that unit is never again evaded), which implies that $W(t)$ is independent of $s^\star$, such that the circular effect of $V(0)$ via $W(0)$ on $V(0)$ disappears. We then have

$$\begin{aligned}
\lim_{T\to\infty} \mathcal{V}_0(s) &= -\lambda \int_0^s \mathrm{e}^{-(\rho+\lambda)x}\pi(x)\ dx + q(s)\left(r + \lambda\frac{r(1-\tau)}{\rho}\right) + \left(\frac{r(1-\tau)}{\rho} - k\right)\mathrm{e}^{-(\rho+\lambda)s}, \\
\lim_{T\to\infty} \theta(s) &= \frac{\rho}{r}\lambda \int_0^s \mathrm{e}^{-(\rho+\lambda)x}\pi(x)dx + \left(\lambda q(s) + \mathrm{e}^{-(\rho+\lambda)s}\right)\tau, \\
\lim_{T\to\infty} \kappa(s) &= \frac{\rho}{r}k\mathrm{e}^{-(\rho+\lambda)s}
\end{aligned}$$

and the optimal tax evasion strategy simplifies to

$$\lim_{T\to\infty} s^\star = \pi^{-1}\left(\frac{r\tau + (\rho+\lambda)k}{\lambda}\right).$$

Increases in r, τ, ρ, or k raise $s^\star$, because they increase the benefit of not paying taxes (given by $r\tau$) and the cost of switching to state w. A decrease in λ raises $s^\star$, because it reduces the expected cost of evasion, by rendering detection less likely.

Switching costs borne by households and detection efforts by the government are socially wasteful. From an optimal taxation perspective, the former play a similar role as the deadweight burden associated with tax induced substitution effects. In parallel to the optimal taxation literature initiated by Ramsey (1927), the government's problem of efficiently raising revenue subject to the household's optimal evasion choice may thus be considered. In the current setup, this problem assumes a very transparent form. Following the approach pioneered by Diamond and Mirrlees (1971), it may be analyzed in terms of the household's indirect utility function, which is to be maximized subject to the government's budget constraint. Let $\alpha \equiv (\lambda, k, T, \tau, \pi(\cdot))$[19], let $s^\star(\alpha)$ denote the $\arg\min_s \theta(s, \alpha) + \kappa(s, \alpha)$, and let $\mathbf{1}_{[s^\star(\alpha)>0]}$ denote the indicator function that equals 1 if $s^\star(\alpha) > 0$. Normalizing by the household's stock of capital and r/ρ, the government program reads

$$\begin{array}{ll} \max\limits_{\alpha} & 1 - \theta(s^\star(\alpha), \alpha) - \mathbf{1}_{[s^\star(\alpha)>0]}\kappa(s^\star(\alpha), \alpha) \\ \text{s.t.} & \theta(s^\star(\alpha), \alpha) = C(\alpha) + \text{PDV(normalized government spending)}. \end{array}$$

The first line represents the household's indirect utility function: Normalized utility equals $1 - \theta(0, \alpha) = 1 - \tau$ in case of no evasion and $1 - \theta(s^\star(\alpha), \alpha) - \kappa(s^\star(\alpha), \alpha)$ otherwise. Due to the separability of θ and κ, the social losses of tax evasion appear in much more transparent form than the social losses of tax avoidance in standard models. The second line represents the normalized budget constraint where the cost function $C(\cdot)$ depends on the parameters of the tax system. Letting μ denote the shadow value of government funds and confining ourselves to an interior equilibrium with tax evasion, the first-order condition for this problem is given by

$$\begin{aligned} \mu C_\alpha &= \theta_\alpha(\mu - 1) - \kappa_\alpha + s^\star_\alpha(\theta_s(\mu - 1) - \kappa_s) = \\ &= \theta_\alpha(\mu - 1) - \kappa_\alpha + s^\star_\alpha \theta_s \mu, \end{aligned}$$

where all derivatives are evaluated at $(s^\star, \alpha)$. The left-hand side of this equation represents the cost for the government of a marginal increase in any of the tax parameters. The right-hand side represents the net gain from the same adjustment. This gain consists of higher revenue, evaluated at the shadow value of government funds, minus the income loss for the household due to taxes, fines, and switching costs; these changes in government revenue and private income occur both directly and indirectly, i.e., through an induced change in $s^\star$.[20]

Should the government employ a strictly positive switching cost k, if it can influence this parameter?[21] The government's optimality condition suggests a negative answer to that question, subject to qualifications. Starting from an initial value $k = 0$, a marginal increase in k is detrimental. It neither delivers a direct revenue gain for the government ($\theta_k = 0$ from (6)), nor an indirect one (for $k = 0$, $\kappa_s(s^\star, \alpha) = -\theta_s(s^\star, \alpha) = 0$ by the household's optimality condition) but induces first-order switching costs and administrative costs ($\kappa_k > 0$, $C_k \geq 0$). For further increases in k, however, this argument need no longer hold as θ_s is then positive.

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[19] For notational simplicity, I assume that $\pi(\cdot)$ can be represented as a vector.

[20] If policy changes involved no direct resource cost ($C_\alpha = 0$, as generally assumed in the optimal taxation literature) and neither directly nor indirectly affected switching costs ($\kappa_\alpha = 0$, and $\kappa_s = 0$ such that $\theta_s = 0$ by the household's optimality condition), then μ would equal unity: the shadow values of public and private funds would coincide and the government could costlessly transfer resources from the private to the public sector.

[21] One way for the government to affect k could be to demand more background information on units of capital that newly appear in or disappear from the tax declaration.

4 Aggregate Risk

4.1 Inaction and Hysteresis

To analyze the dynamic properties of tax evasion rates, it is necessary to go beyond the stationary environment discussed above. This section extends the previous model to a setting with aggregate risk under the maintained assumptions that households optimally evade taxes and form rational expectations. As explained earlier, I assume risk neutrality to keep the model tractable.

Aggregate risk arises in the form of dividend yield (or productivity) risk. I assume the return on capital to fluctuate randomly around an average value, $\bar{r}$. The relative deviation of the return from $\bar{r}$, denoted by r, follows the mean reverting Ito process

$$dr = -\eta r \, dt + \sigma \, dZ, \;\; \eta, \sigma > 0,$$

with dZ denoting the increment of a standard Brownian motion, and t denoting calendar time.

I assume switching costs to be strictly positive. When deciding whether to declare income to the tax authority, households trade off these costs against the benefit of switching capital from state v to state w, or vice versa. Since these benefits depend on expected future dividend yields (and thus on the current yield), they vary stochastically over time. Even in the absence of duration dependent fines, the tax evasion program is therefore non-trivial. For simplicity, I thus completely abstract from duration dependent fines and let $T = 0$; the duration of an evasion spell is no longer a relevant state variable.

As in models of entry and exit under uncertainty, switching costs give rise to an inaction range. Households do not immediately switch capital from state w to state v when the net flow benefit from a unit in state v exceeds the flow from a unit in state w, i.e., when the dividend yield is high, such that the tax savings due to evasion are high. Nor do they immediately switch capital from state v to state w when the dividend yield is low, such that the net flow benefit from a unit in state w exceeds the flow from a unit in state v. They would rather wait until the difference between the two flows has become sufficiently large to compensate for two cost components: First, "annualized" switching costs, and second, the cost of foregoing the possibility of costlessly returning to the current (pre-switching) state. This second cost component reflects the risk that the difference between the flow benefits quickly reverts, such that incurring the switching cost becomes unprofitable ex post.[22] Each cost component drives a wedge between the upper boundary of the inaction range, r^h say (associated with the dividend yield $\bar{r}(1 + r^h)$), at which it is optimal to switch from state w to state v, and the lower boundary, r^l, at which it is optimal to switch from state v to state w. The presence of an inaction range, in turn, gives rise to hysteresis: The effect of a change in r on tax evasion is not immediately reversed if r returns to its initial value. Even if r is such that all new units of capital are reported to the tax authority, say, old units may still not be reported if the flow benefit differential is not sufficiently large. The fraction of taxes evaded therefore typically remains interior.

As in the previous section, optimal household behavior can be characterized by the values of a unit of capital in state v and state w. Also as in the previous section, these values are interdependent. The two value functions (which now depend on the new state variable, r) must thus be solved simultaneously. Within the inaction range, these functions are characterized by

$$\begin{aligned} (\rho + \lambda)V(r) &= \bar{r}(1 + r) - V'(r)\eta r + 1/2V''(r)\sigma^2 + \lambda(-\pi(r) + W(r)) - \psi, & (8) \\ \rho W(r) &= (1 - \tau(r))\bar{r}(1 + r) - W'(r)\eta r + 1/2W''(r)\sigma^2. & (9) \end{aligned}$$

[22] See Dixit (1989) for a detailed exposition.

Equations (8) and (9) represent no-arbitrage conditions, similar to (3). The required return on one unit of capital equals $\rho + \lambda$ if the unit is subject to detection risk, and ρ otherwise. The return consists of flow dividends, pre or after tax, expected capital losses due to detection risk in the case of V, and expected capital gains or losses due to changes in dividend yield (see Appendix A.5 for the derivation). Equation (8) features an additional flow cost parameter, ψ, which serves calibration purposes and is discussed below.

As noted earlier, switching costs and dividend risk imply that entry and exit into/from tax evasion are characterized by an inaction range, $[r^l, r^h]$: Switching capital from state v to state w (from state w to state v) is optimal once r reaches the lower (upper) boundary of that range. To characterize r^l and r^h, the value matching and smooth pasting conditions need to be solved. The former state that switching occurs when the values of both alternatives, net of switching costs, are equal: $V(r^h) - k^h = W(r^h)$ and $W(r^l) - k^l = V(r^l)$.[23] The latter specify that these equalities extend to small variations around the optimal trigger points and thus, that there is no gain from delaying the switching decision for an infinitesimally short duration: $V'(r^h) = W'(r^h)$, $V'(r^l) = W'(r^l)$.[24]

Since we are not interested in the functions $V(r)$ and $W(r)$ per se, but only in the optimal decision rules, we can simplify this problem. Let $X(r) \equiv V(r) - W(r)$. The no-arbitrage conditions and the value matching and smooth pasting conditions can then be rewritten as

$$(\rho + \lambda)X(r) = \tau(r)\bar{r}(1 + r) - \lambda\pi(r) - X'(r)\eta r + 1/2X''(r)\sigma^2 - \psi, \tag{10}$$
$$X(r^h) = k^h, \quad X(r^l) = -k^l, \quad X'(r^h) = 0, \quad X'(r^l) = 0.$$

Consider a linear tax rate function $\tau(r) = \tau_0 + \tau_1 r$ and let fines be some multiple ζ of currently evaded taxes, $\pi(r) = \zeta\tau(r)\bar{r}(1 + r)$. As argued below, this is a realistic assumption; note that $\lambda\zeta$ needs to be smaller than unity for there to be an incentive to evade taxes. Appendix A.6 shows that in this case, the solution to equation (10) is of the form

$$X(r) = A\ {}_1F_1\left(\frac{\rho + \lambda}{2\eta}, \frac{1}{2}, \frac{r^2\eta}{\sigma^2}\right) + Br\sqrt{\frac{2\eta}{\sigma^2}}\ {}_1F_1\left(\frac{\rho + \lambda + \eta}{2\eta}, \frac{3}{2}, \frac{r^2\eta}{\sigma^2}\right) + \phi_0 + \phi_1 r\sqrt{\frac{2\eta}{\sigma^2}} + \phi_2 r^2 \frac{2\eta}{\sigma^2}. \tag{11}$$

A and B denote arbitrary constants; ϕ_0, ϕ_1, and ϕ_2 denote functions of the parameters, specified in Appendix A.6; and ${}_1F_1(\cdot)$ denotes the confluent hyper-geometric function or Pochhammer's function. The four value matching and smooth pasting conditions together with (11) can be solved for the four unknowns A, B, r^h, r^l.

Figure 3 displays an example. The boundaries of the inaction range are $r^l \approx -22.5$ percent and $r^h \approx 6.6$ percent. If *new* units of capital are assigned to states v or w with equal cost, then new units are assigned to state v whenever r exceeds the value at which $X(r)$ equals zero; in the figure, this value is ≈ 0.5 percent. Call that value r^c ($r^l < r^c < r^h$). Old units of capital are not immediately switched between states v and w, once r reaches r^c, however. Households rather wait with switches of that sort until r reaches the upper or lower boundary of the inaction range.

Figure 3 and the benchmark simulation reported below are based on the parameter values summarized in Table 1. I assume a yearly discount rate of 5 percent. This approximately translates into $\rho = 0.0125$ (in the model, time is measured in quarters). Slemrod and Yitzhaki

[23] In contrast to the previous section, I allow the switching costs on the upper and lower boundary of the inaction range to differ.

[24] See Dixit (1993) or Dixit and Pindyck (1994).

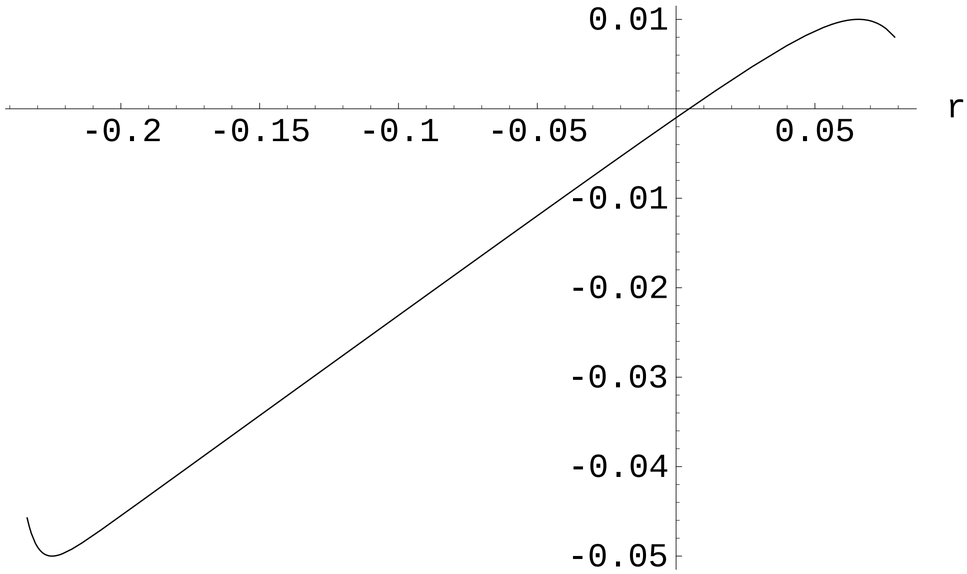

Figure 3: $X(r)$, benchmark calibration.

(2002) and Andreoni et al. (1998, p. 820) report for the U.S. that 1.5 to 1.7 percent of the tax returns are audited per year, which translates into a flow detection rate of $\lambda = 0.00375$. They also report that fines are levied at rates between 20 and 75 percent of the evaded income tax. The mean of these values translates into $\zeta = 1.475$ (which satisfies the condition $\lambda\zeta < 1$). I take the statutory tax rate, τ_0, to be 30 percent. With regard to switching costs, I assume that it is easier to let a unit of capital "disappear" than "reappear" vis-à-vis the tax authorities; I set $k^l = 0.05$ and $k^h = 0.01$. I assume the average annual yield on capital, broadly defined, to be 20 percent, and the annual growth rate of the capital stock (introduced later in the model) to be 3 percent. This implies $\bar{r} = 0.05$ and $\gamma = 0.0075$. To calibrate η and σ, I use standard assumptions from the Real Business Cycle literature about the persistence and volatility of the Solow residual. An AR(1)-regression on quarterly U.S. data of the Solow residual yields an auto-regressive coefficient of about 0.95, and a standard deviation of the innovation term of about 1 percent (see, for example, Hansen, 1985). These estimates imply a mean reversion and variance rate of the diffusion process of $\eta = 1 - 0.95$ and $\sigma^2 = 0.01^2 2\eta/(1 - \mathrm{e}^{-2\eta}) = 0.010251^2$, respectively (Dixit and Pindyck, 1994, p. 77). Finally, I set $\psi = \bar{r}\tau_0(1-\lambda\zeta) = 0.014917$, implying that under normal conditions, the difference between the flow benefit of a unit of capital in state v and a unit in state w is zero. This is to account for the fact that the model obviously does not comprise all relevant factors determining the amount of tax evasion.[25] Fixing ψ at a lower value twists the inaction range towards lower (negative) values of r, reflecting the fact that the small risk of detection in the benchmark calibration renders tax evasion very lucrative.

Variations in the parameter values shift the inaction range in Figure 3 or change its width. An increase in σ widens the inaction range. A more volatile dividend yield increases the option value of waiting and thus, induces households to wait "longer" before incurring the switching costs. A rise in η also increases the width of the inaction range (almost exclusively by reducing r^l). Stronger mean reversion reduces the likelihood of persistent deviations of the fundamental

[25] Beyond the factors analyzed in static models or in the previous section of this paper, tax evasion might depend on mental costs and many other aspects, see Cowell (1990).

Table 1: Benchmark Calibration

Model Parameter	Benchmark Value
ρ	0.0125
λ	0.00375
ζ	1.475
τ_0	0.3
τ_1	0.00
k^l	0.05
k^h	0.01
$\bar{r}$	0.05
γ	0.0075
η	0.05
σ	0.010251
ψ	0.014917

from its mean, which makes it less worthwhile to incur the switching costs and induces more cautious behavior by households. The cyclicality of tax rates also influences the width of the inaction range. Pro-cyclical tax rates render the flow benefit of tax evasion (the difference between the flow benefits of units in states v and w) larger (smaller) for positive (negative) values of r. A positive value for τ_1 thus pushes the boundaries of the inaction range inwards, while a negative value pushes them outwards. Since the inaction range is not initially symmetric around the origin, however, the changes in r^h are less pronounced than the changes in r^l (see Figure 4). Variations in k^l and k^h practically only affect the corresponding boundary of the inaction range. A reduction in k^l, for example, shifts r^l inwards without a sizeable effect on r^h. Finally, changes in ζ and τ_0 (and ψ) shift the inaction range more than they affect its width. A higher fine or a lower average tax rate shifts r^l, r^c, and r^h to the right.

4.2 Aggregate Implications

I embed the household's problem in a stylized macroeconomic framework. Households inhabit an economy à la Lucas (1978) with a stock of capital growing at rate γ and yielding stochastic dividends. The law of motion for dividends, tax rates and fines are as specified above. Dividends after taxes and fines are either consumed or saved in the form of government debt. The instantaneous interest rate on government debt is ρ, because households are risk neutral. At any point in time, new capital is either assigned state v or state w, depending on whether r exceeds r^c. Moreover, old capital is shifted from state w to state v (from state v to state w) if r exceeds r^h (r^l exceeds r).

In the discrete (Δt = one quarter) approximation to the continuous time economy, the

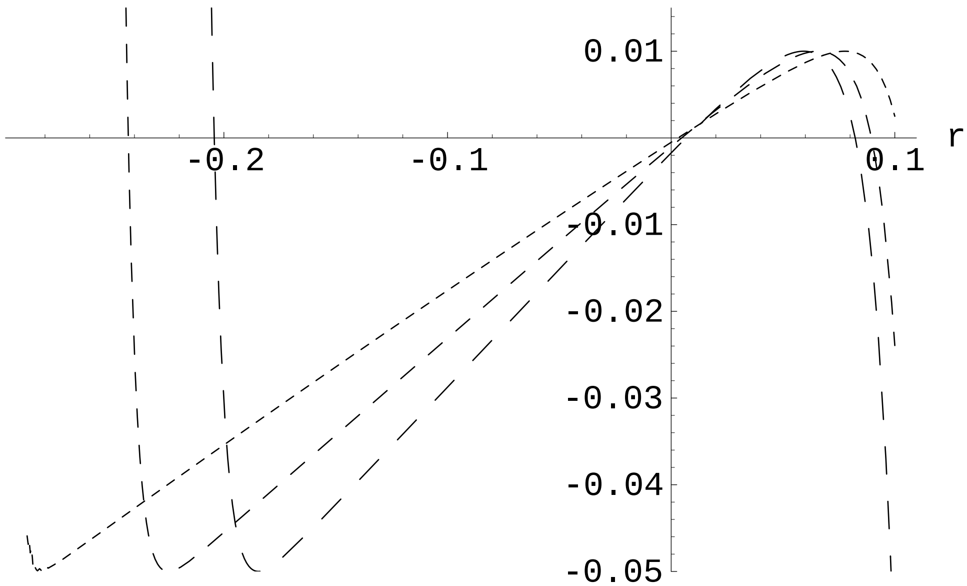

Figure 4: Effect of cyclicality of tax rates on $X(r)$: $\tau_1 = -0.75, 0, 0.75$ (in order of increasing length of line segments).

following laws of motion hold:

$$
\begin{aligned}
r_t &= \mathrm{e}^{-\eta} r_{t-1} + h_t, \quad h_t = \pm\sigma \text{ with equal probability}, \\
\tau_t &= \tau_0 + \tau_1 r_t, \\
T_t &= \bar{r}(1+r_t)\tau_t w_t, \\
\Pi_t &= \bar{r}(1+r_t)\tau_t \zeta \lambda v_t, \\
g_t &= g_T(T_t + \Pi_t) + g_d d_t, \\
\delta_t &= \rho d_t - T_t - \Pi_t + g_t, \\
d_{t+1} &= d_t + \delta_t, \\
c_t &= \bar{r}(1+r_t)(w_t + v_t) + \rho d_t - T_t - \Pi_t - \delta_t = \bar{r}(1+r_t)(w_t+v_t) - g_t, \\
w_{t+1} - w_t &= +\lambda v_t + \mathbf{1}_{[r<r^l]} v_t (1-\lambda)(1-k^l) - \mathbf{1}_{[r>r^h]} w_t \qquad + (1 - \mathbf{1}_{[r>r^c]})\Gamma_0 (1+\gamma)^t, \\
v_{t+1} - v_t &= -\lambda v_t - \mathbf{1}_{[r<r^l]} v_t (1-\lambda) \qquad + \mathbf{1}_{[r>r^h]} w_t (1-k^h) + \quad \mathbf{1}_{[r>r^c]} \Gamma_0 (1+\gamma)^t.
\end{aligned}
$$

Here, v_t and w_t denote the time t stock of non-reported and reported capital, respectively; T_t, Π_t, g_t, δ_t, d_t, and c_t denote tax collections, fines, government spending, deficit, debt, and private consumption, respectively; and $\mathbf{1}_{[q]}$ represents the indicator function for event q. The first equation of the dynamic system discretely approximates the diffusion process for r (see Dixit and Pindyck, 1994, pp. 69, 76). The following three equations define the statutory tax rate, tax collections, and fines, respectively. Government spending is assumed to linearly depend on revenue and the stock of debt. In the simulations, I set $g_T = 1.1$ and $g_d = -0.03$.[26] The

[26] On a balanced growth path, this spending rule and the definitions of d_t and δ_t imply

$$(\gamma - \rho - g_d) d_t = (g_T - 1)(T_t + \Pi_t).$$

The values for g_T and g_d satisfy this equation for quarterly debt and tax-and-fine quotas of 160 percent and 40 percent, respectively.

next three equations define the government's deficit, debt accumulation, and household consumption.[27] The last two equations link the household's optimality conditions (the boundaries of the inaction range as well as r^c) to the accumulation of reported and non-reported capital, respectively. With a total stock of capital in period $t = 0$ equal to 1 and $\Gamma_0 \equiv \gamma/(1+\gamma)$, the inflow of new capital in period t equals $\Gamma_0(1+\gamma)^t$. I also set $d_0 = 0$.

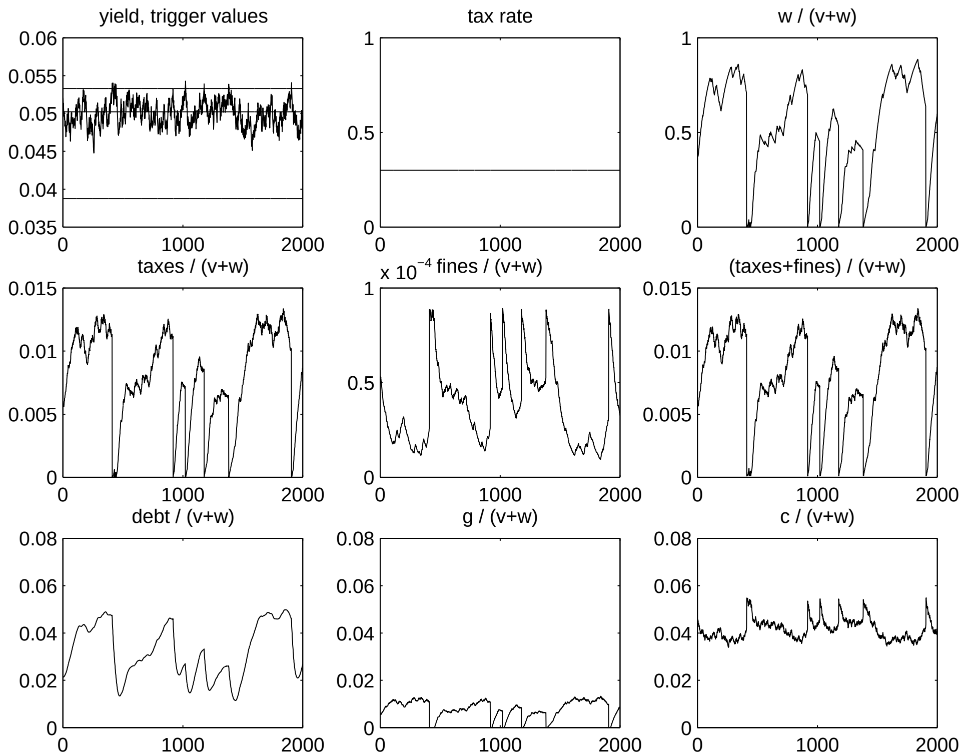

Figure 5: Tax evasion dynamics.

Figure 5 illustrates the aggregate consequences of a sequence of 2000 realizations of the dividend yield. The upper left panel displays the dividend yield realizations, together with the household's optimal trigger points (r^l, r^c, r^h), subject to the stochastic properties of the process r and the tax system. If r exceeds r^c, households allocate new capital to state v, which is reflected in a decrease in $w_t/(v_t + w_t)$. If the dividend yield also exceeds $\bar{r}(1 + r^h)$, all capital is shifted to state v and w_t drops to zero. The corresponding fall in tax revenue strongly outweighs the increase in fines and leads to a sharp drop in government spending, and a significant reduction in the debt quota. On the other hand, the increase in households' disposable income together with the fall in the government's borrowing requirement increases consumption. Note that we never observe sharp *increases* in the fraction of reported wealth in this sample because r^l is sufficiently low to never induce households to fully report their wealth.

When evaluating the simulation, in particular the high volatility of key macroeconomic variables, it should be kept in mind that the model abstracts from various elements that would reduce this volatility. One such element is risk aversion, another relates to the fact that large fractions of income cannot be evaded in practice, because the tax authorities are directly no-

[27] Interest income from government bonds is assumed to be untaxed.

tified about them (for instance, by the employer).[28] Nevertheless, several conclusions can be drawn from this benchmark simulation. First, the macroeconomic responses to changes in r are highly non-linear, due to different behavioral responses within the inaction range and at its boundaries. Second, the macroeconomic time series display asymmetry, due to the fact that only the upper boundary of the inaction range is "tested". Finally, relatively small variations in r can have a large impact. This finding sharply contrasts with the prediction of the standard tax evasion model, where tax evasion is governed by a succession of static evasion decisions. In that framework, small changes in the conditional expectation of future income flows will only indirectly and mildly affect the evasion decision.

Table 2 and Figures 6–9 summarize the effects of changes in tax policy on macroeconomic performance. Throughout these scenarios, the sequence of dividend yields driving the aggregate dynamics is the same as in the benchmark scenario.

Table 2: Effects of Different Tax Policies

Variable	Effects of Change in ... [a]							
	τ_1 to ...		k^l to ...		ζ to ...		τ_0 to ...	
	-0.75	0.75	0.3	0.1	5.475	9.475	0.295	0.305
mean$(v_t + w_t)$	1.0003	0.9999	1.0000	0.9973	1.0026	1.0026	1.0026	0.9990
mean$(w_t/(v_t + w_t))$	1.0699	0.9182	1.0000	1.0519	1.8176	1.8405	1.8273	0.2551
mean(T_t)	0.9998	1.0204	1.0000	1.2923	1.8091	1.8265	1.7873	0.3313
mean(Π_t)	1.0061	0.9697	1.0000	0.6385	0.0788	0.0000	0.0106	1.8482
mean(g_t)	0.9996	1.0201	1.0000	1.3071	1.8172	1.8348	1.7953	0.3451
mean(c_t)	1.0005	0.9960	1.0000	0.9381	0.8467	0.8434	0.8509	1.1240
d_{2000}	0.9933	1.0189	1.0000	1.7014	2.1716	2.2014	2.1498	0.4986

[a] Relative to the outcome in the benchmark scenario. See the explanations in the text.

Consider first the effect of changes in the cyclicality of the tax rate (Figure 6). We saw earlier (cf. Figure 4) that a counter-cyclical tax rate pushes the boundaries of the inaction range outwards. Households thus less often switch their capital from state w to state v, which increases the capital stock due to lower total switching costs. In addition to reducing the frequency of switches, a negative τ_1 also reduces the trigger value r^c. This implies that households become more aggressive in terms of allocating new funds to state v. In "normal times" (with r around r^c), the inflow into the reported capital stock is thus lower. The combination of (a) less frequent switching into state v but (b) larger direct flows into state v implies that the effect of the cyclicality of τ on both tax revenue and fines is ambiguous. Channel (a) increases the capital stock and the fraction of reported income, and thus tends to raise tax revenue and reduce fines. Channel (b) reduces the fraction of reported income and thus tends to reduce tax revenue and raise fines. Which of the two effects is predominant is history dependent and varies with the circumstances. The growth rate of the economy, for example, is of importance because it affects the strength of channel (b). In the particular simulation example considered here, effect (b) outweighs effect (a) on average, but not in all sub-periods. Effect (b) becomes decisive towards

[28] Andreoni et al. (1998, p. 821) report for the U.S. that the Internal Revenue Service receives direct notification about approximately 75 percent of all income.

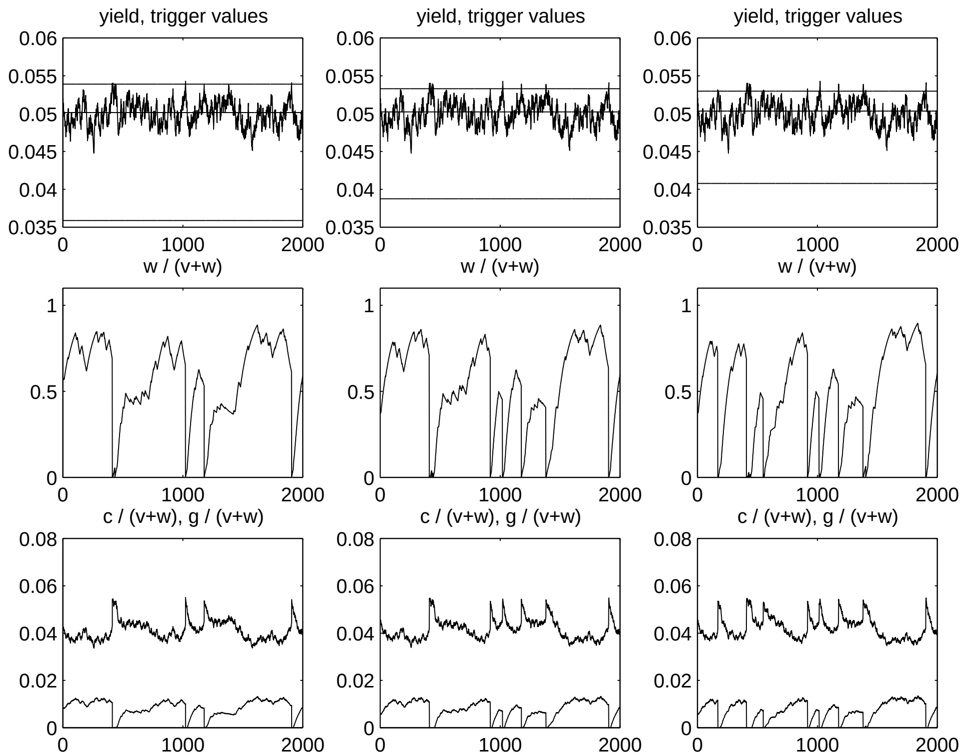

Figure 6: Aggregate implications of variations in τ_1: $\tau_1 = -0.75, 0, 0.75$ (left to right).

the end of the simulation period, where switches occur equally often under a negative and a positive τ_1, such that effect (a) does not differ across policies.

Figure 7 compares the outcomes under different assumptions about the switching costs. Reducing k^l to 0.03 has no effect on the equilibrium outcome at all: Both r^h and r^c remain practically unchanged, and the lower value of r^l is irrelevant in so far as the dividend yield never drops to as low a level as to induce households to switch from state v to state w. This changes with a further decrease in k^l to 0.01. Now, $\bar{r}(1+r^l)$ lies in the range of realized dividend yields, and we do not only observe downward jumps of $w_t/(v_t+w_t)$ to zero (whenever r exceeds r^h) but also upward jumps of the same variable to one (whenever r falls below r^l). The ratio of reported to total capital becomes more erratic but increases on average, thereby reducing the evasion rate and thus increasing the tax base. The additional switching from state v to state w raises the total switching costs which, in turn, reduces the capital stock, and thus the tax base. These two opposing effects imply ambiguous consequences for revenue, spending, and consumption; tax revenue increases over the whole time period, but not in all sub-periods.[29]

Figure 8 compares the effect of different fines. An increases in ζ raises the capital stock by increasing the threshold at which shifting funds from state w to state v is optimal (higher r^h). An even further increase of ζ also raises $\bar{r}(1+r^c)$ beyond the range of realized dividend yields, and altogether eliminates tax evasion. Further increases in ζ would also shift the lower trigger point into the relevant range. This would have no effect, however, since there would be

[29]The increase in final period government debt is due to the fact that towards the end of the simulation period, the ratio of reported to total capital and thus also tax revenue and government spending increase more sharply when r^l is lower.

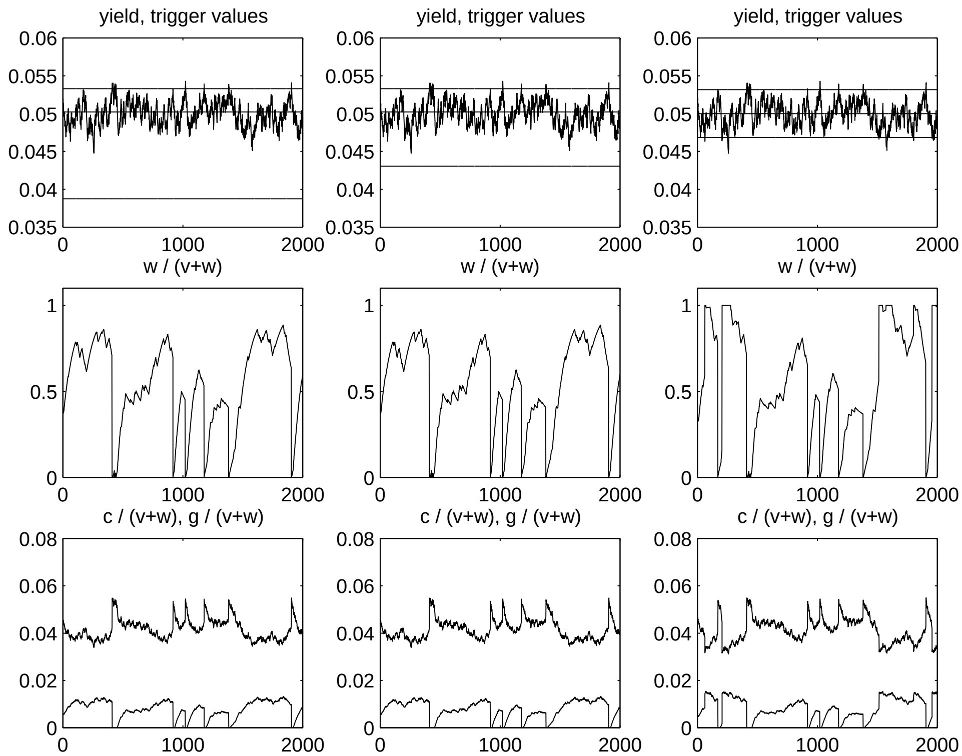

Figure 7: Aggregate implications of variations in k^l: $k^l = 0.05, 0.03, 0.01$ (left to right).

no non-reported capital to be shifted from state v to state w to start with. By discouraging switches from state w to state v, the first increase in ζ raises the capital stock. Moreover, the reduction, and finally the elimination of tax evasion, strongly amplifies the positive effect on tax revenue, raises government spending and debt, and reduces private consumption. It also significantly reduces the volatility of these variables.

Finally, Figure 9 shows that the economy is located on the declining portion of the Laffer curve. A small increase in the average tax rate from 29.5 percent to 30.5 percent shifts the inaction range downward by about fifty basis points. This, in turn, induces a near collapse of government activity. Tax revenue falls by more than eighty percent. Payments of fines skyrocket but total government revenue collapses and government spending is contracted.

In conclusion, the simulations suggest that tax evasion dynamics severely complicate the government's task. The high sensitivity of tax evasion and its history dependence require sure instinct by the government, and some luck. Once tax evasion starts to occur, tax policy becomes difficult, and once it starts to spread, the sustainability of government activity is quickly at risk.

5 Conclusions

While risk aversion and endogenous detection probability do not guarantee interior solutions to a household's tax evasion program, dynamic aspects tend to induce interior tax evasion rates. Two dynamic lines of argument have been explored in the paper: The first is based on duration dependent fines, the second on a cross-section of vintages subject to dividend risk and switching costs. Both arguments imply behavioral responses different from those predicted by standard

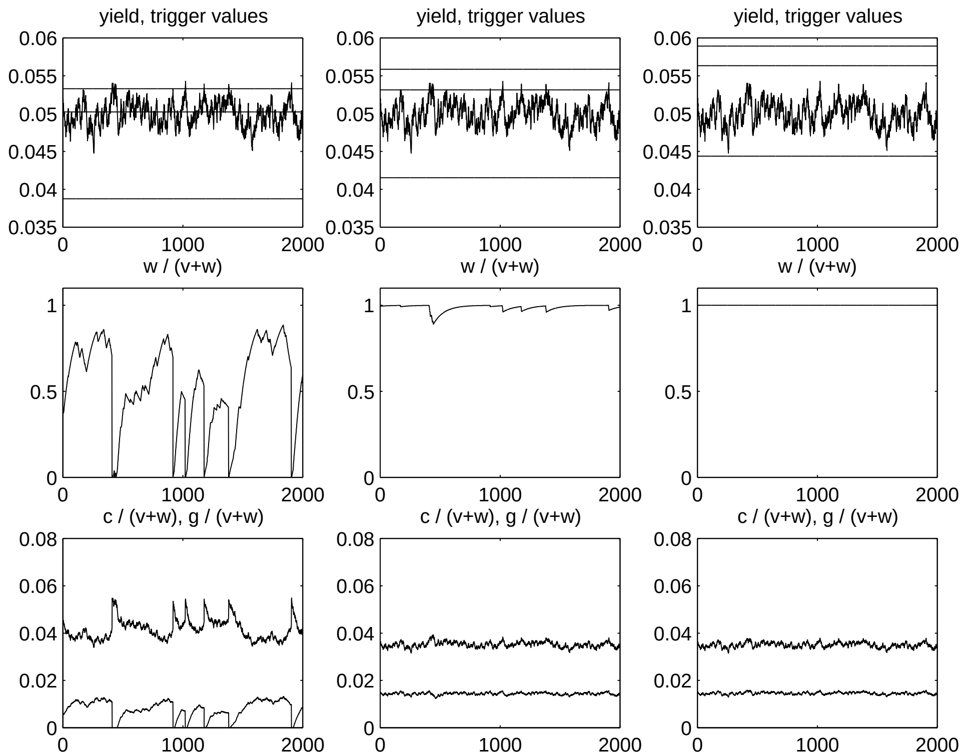

Figure 8: Aggregate implications of variations in ζ: $\zeta = 1.475, 5.475, 9.475$ (left to right).

theory. Static models stress the income and substitution effects of changes in the statutory tax rate on the demand for state-contingent consumption. The dynamic perspective proposed in this paper stresses the maximization of expected returns—a higher statutory tax rate encourages households to wait longer before they report their accrued income. This prediction is testable. While beyond the scope of this paper, a careful empirical analysis controlling for the various other aspects affecting households' evasion decisions thus seems a promising avenue for future research.

Throughout the analysis, I have abstracted from risk aversion. This was unimportant in the model without dividend risk where households are fully insured. In the model with dividend risk, however, the assumption was substantive. Introducing risk aversion here would complicate the household's portfolio choice problem, and change the riskfree rate on government debt.[30] Other potential extensions include heterogeneous costs or benefits of tax evasion across the population, or multiple sources of aggregate risk.[31] Introducing such considerations is likely to dampen the volatility observed in the simulations without changing the flavor of the arguments. Finally, an interesting extension would generalize the government's spending rule, allowing for the possibility of default. Interest on government debt would then include a risk premium, which varies with the level of debt and the anticipated extent of tax evasion. Such a model could, I suspect, enhance our understanding of fiscally driven crises.

[30]For related work in the finance literature, see Constantinides (1986), Grossman and Laroque (1990), and Vayanos (1998).

[31]An example for the latter extension that immediately comes to mind is the combination of nominal interest rate risk and inflation rate risk.

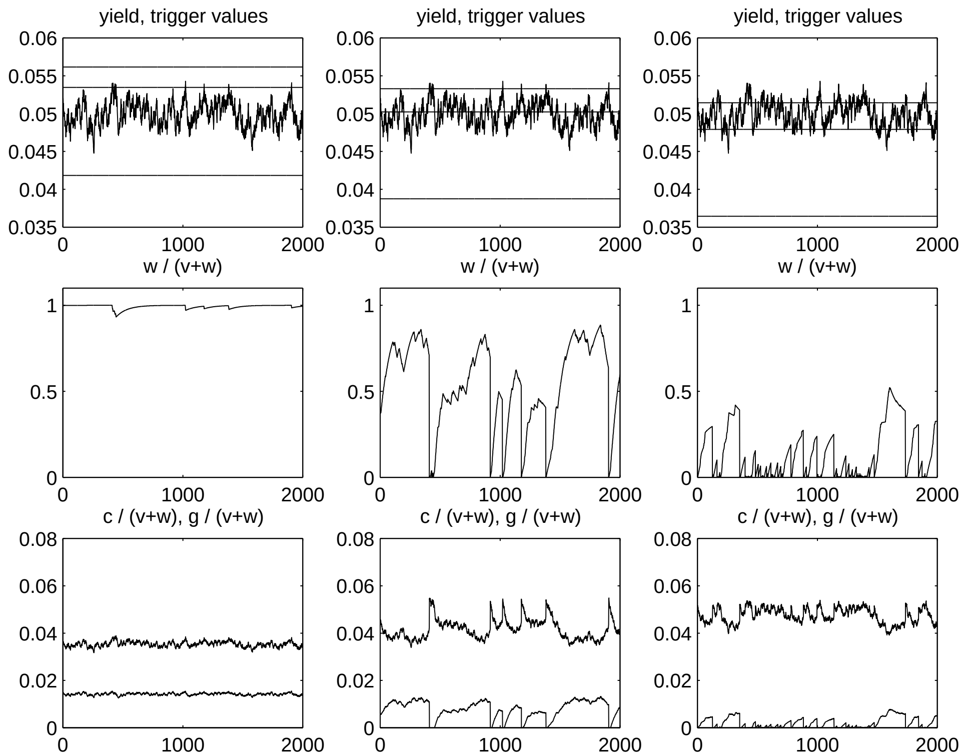

Figure 9: Aggregate implications of variations in τ_0: $\tau_0 = 0.295, 0.3, 0.305$ (left to right).

A Appendix

A.1 Micro Foundations for $T > 0$

To rationalize the assumption that $T > 0$, a duration dependent benefit of paying taxes must be introduced. One way is the following. Assume as in the main model that tax evasion pays a flow return of r and exposes the unit of capital to a risk of detection, which triggers duration dependent fines and a switch to w. In addition, assume that the duration at the start of a new evasion spell is not necessarily zero as in the main model; it rather depends on the duration of the previous spell of tax payments. Paying taxes thus constitutes an investment in good-will or reputation, reducing expected fines in the successive tax evasion spell.

To avoid the introduction of an additional state variable, posit that the duration of a tax paying spell reduces the "effective duration" in the successive tax evasion spell by one-to-one. The accumulation of reputation thus reduces t, while evasion increases t; both $V(t)$ and $W(t)$ are decreasing in their argument. It is then optimal to keep a unit of capital in v until the effective duration has increased to some value, $\bar{t}$ say, and keep it in w until the effective duration has decreased to some other value, $\underline{t}$ say. Between these trigger points, the value of a unit of capital in v and w, respectively, is characterized by

$$\begin{aligned}(\rho+\lambda)V(t) &= r + V'(t) + \lambda(-\pi(t) + W(t_0)), \quad \underline{t} \leq t \leq \bar{t}, \\ \rho W(t) &= r(1-\tau) - W'(t), \quad \underline{t} \leq t \leq \bar{t},\end{aligned}$$

as well as the value matching and smooth pasting conditions $V(\bar{t}) = W(\bar{t}) - k$, $V'(\bar{t}) = W'(\bar{t})$, $V(\underline{t}) = W(\underline{t}) + k$, $V'(\underline{t}) = W'(\underline{t})$.

For $\pi(\underline{t}) = 0$, and $T = \bar{t} - \underline{t}$, the setup of the main model is replicated.

A.2 Derivation of Equation (3)

Let $U(t+dt) \equiv (1-\lambda\, dt)V(t+dt) + \lambda\, dt(W(0) - \pi(t+dt))$. Using the differentiability of $V(t)$, we then have

$$
\begin{aligned}
V(t) &= \lim_{dt\to 0} r\, dt + \mathrm{e}^{-\rho\, dt} U(t+dt),\ t<s, \\
V(t) &= \lim_{dt\to 0} r\, dt + (1+\rho\, dt)^{-1} U(t+dt),\ t<s,\ (\text{using } \rho\, dt \approx \ln(1+\rho\, dt)) \\
\lim_{dt\to 0}(1+\rho\, dt)V(t) &= \lim_{dt\to 0} r\, dt(1+\rho\, dt) + U(t+dt),\ t<s, \\
\lim_{dt\to 0}(\lambda+\rho)\, dt V(t) &= \lim_{dt\to 0} r\, dt(1+\rho\, dt) + [U(t+dt) - (1-\lambda\, dt)V(t)],\ t<s, \\
\lim_{dt\to 0}(\lambda+\rho)\, dt V(t) &= \lim_{dt\to 0} r\, dt(1+\rho\, dt) + [(1-\lambda\, dt)V'(t)\, dt + \lambda\, dt(W(0)-\pi(t+dt))],\ t<s, \\
(\lambda+\rho)V(t) &= \lim_{dt\to 0} r(1+\rho\, dt) + [(1-\lambda\, dt)V'(t) + \lambda(W(0)-\pi(t+dt))],\ t<s, \\
(\rho+\lambda)V(t) &= r + V'(t) + \lambda(-\pi(t) + W(0)),\ t<s.
\end{aligned}
$$

A.3 Derivation of $\mathcal{V}_0(s)$

Under the assumption $s > 0$: From (5), we have

$$
V(0) = \int_0^s \mathrm{e}^{-(\rho+\lambda)x}(r + \lambda(-\pi(x) + W(0)))\, dx + \mathrm{e}^{-(\rho+\lambda)s}(W(0) - k).
$$

From (1), (2) this implies

$$
\begin{aligned}
\mathcal{V}_0(s) &= \int_0^s \mathrm{e}^{-(\rho+\lambda)x}\left[r + \lambda\left(-\pi(x) + \left[\mathrm{e}^{-\rho T}\left(W(T) - \frac{r(1-\tau)}{\rho}\right) + \frac{r(1-\tau)}{\rho}\right]\right)\right] dx + \\
&\quad \mathrm{e}^{-(\rho+\lambda)s}(W(0) - k) = \\
&= \int_0^s \mathrm{e}^{-(\rho+\lambda)x}\left[r + \lambda\left(-\pi(x) + \left[\mathrm{e}^{-\rho T}\left(\mathcal{V}_0(s) - k - \frac{r(1-\tau)}{\rho}\right) + \frac{r(1-\tau)}{\rho}\right]\right)\right] dx + \\
&\quad \mathrm{e}^{-(\rho+\lambda)s}\left[\mathrm{e}^{-\rho T}\left(\mathcal{V}_0(s) - k - \frac{r(1-\tau)}{\rho}\right) + \frac{r(1-\tau)}{\rho} - k\right] = \\
&= \int_0^s \mathrm{e}^{-(\rho+\lambda)x}(r - \lambda\pi(x))\ dx + \lambda\frac{r(1-\tau)}{\rho}(1-\mathrm{e}^{-\rho T})q(s) + \\
&\quad \lambda(\mathcal{V}_0(s) - k)\mathrm{e}^{-\rho T}q(s) + \left(\frac{r(1-\tau)}{\rho}(1-\mathrm{e}^{-\rho T}) - k\right)\mathrm{e}^{-(\rho+\lambda)s} + (\mathcal{V}_0(s) - k)\mathrm{e}^{-\rho T-(\rho+\lambda)s}, \\
\mathcal{V}_0(s) &= \left\{-\lambda\int_0^s \mathrm{e}^{-(\rho+\lambda)x}\pi(x)\ dx + q(s)\left(r + \lambda\frac{r(1-\tau)}{\rho}(1-\mathrm{e}^{-\rho T})\right) + \right. \\
&\quad \left.\left(\frac{r(1-\tau)}{\rho}(1-\mathrm{e}^{-\rho T}) - k\right)\mathrm{e}^{-(\rho+\lambda)s} - k(\lambda\mathrm{e}^{-\rho T}q(s) + \mathrm{e}^{-\rho T-(\rho+\lambda)s})\right\} / \\
&\quad \left(1 - \lambda\mathrm{e}^{-\rho T}q(s) - \mathrm{e}^{-\rho T-(\rho+\lambda)s}\right).
\end{aligned}
$$

A.4 Proof of Proposition 1

I assume throughout that $\rho, \lambda, T > 0$ and $k, s \geq 0$. Denote the denominator of (6) or (7) by $D(s)$. We have that $D(s) > 0, D'(s) > 0$. The derivatives of $\theta(s)$ and $\kappa(s)$ are given by

$$\begin{aligned}\theta'(s) &= \frac{\mathrm{e}^{-(\rho+\lambda)s}}{D(s)^2}\frac{\rho}{r}\left[D(s)\lambda\pi(s) - r\tau(1-\mathrm{e}^{-\rho T}) - \lambda\rho\mathrm{e}^{-\rho T}\int_0^s \mathrm{e}^{-(\rho+\lambda)x}\pi(x)dx\right],\\ \kappa'(s) &= \frac{\mathrm{e}^{-(\rho+\lambda)s}}{D(s)^2}\frac{k\rho}{r}\left[\lambda(\mathrm{e}^{-\rho T}-1) - \rho(1+\mathrm{e}^{-\rho T})\right] < 0,\end{aligned}$$

such that

$$\theta'(s) + \kappa'(s) = \frac{\mathrm{e}^{-(\rho+\lambda)s}}{D(s)^2}\frac{\rho}{r}\left[D(s)\lambda\pi(s) - \lambda\rho\mathrm{e}^{-\rho T}\int_0^s \mathrm{e}^{-(\rho+\lambda)x}\pi(x)dx - \Psi(k)\right],$$

with $\Psi(k) > 0, \Psi'(k) > 0$, and $\Psi(k)$ independent of s. Denote the term in square brackets in the last expression by $S(s)$.

i. $S(0) < 0$. $S'(s) = \lambda(D'(s)\pi(s) + D(s)\pi'(s)) - \lambda\rho\mathrm{e}^{-\rho T}\mathrm{e}^{-(\rho+\lambda)s}\pi(s) = \lambda D(s)\pi'(s) \geq 0$, ($>$ 0 for $s > 0$). Moreover, $\lim_{s\to\infty} S'(s)$ is bounded away from zero because $\lim_{s\to\infty} D(s) > D(0) > 0$. By continuity, there exists a unique, finite $s^\star > 0$ such that $S(s^\star) = 0$. Since $\frac{\mathrm{e}^{-(\rho+\lambda)s}}{D(s)^2}\frac{\rho}{r} > 0$, $s^\star$ minimizes $\theta(s) + \kappa(s)$.

ii. The above argument holds for any $k \geq 0$. For $k = 0$, $\kappa(s) = 0\ \forall s$.

iii. By the above argument, $s^\star$ increases in $\Psi(k)$ and thus, in k. Since $s^\star = s^\diamond$ for $k = 0$, $s^\star > s^\diamond$ and $\theta'(s^\star) > \theta'(s^\diamond)$ for $k > 0$.

iv. Households evade taxes if the minimal tax-plus-fine and switching-cost rates sum to less than the statutory tax rate.

A.5 Derivation of Equations (8), (9)

Derivation for a value function $M(r)$ with a flow payoff $\phi(r)$ and a flow probability ν of switching to the other state with the associated value function $N(r)$. Within the inaction range, we have

$$M(r) = \lim_{dt\to 0} \phi(r)\,dt + (1+\rho\,dt)^{-1}\mathrm{E}[(1-\nu\,dt)M(r+dr) + \nu\,dtN(r+dr)]$$

or, equivalently,

$$\lim_{dt\to 0}(\rho+\nu)\,dtM(r) = \lim_{dt\to 0}\phi(r)(1+\rho dt)\,dt + \mathrm{E}[(1-\nu\,dt)dM(r) + \nu\,dt(N(r)+dN(r))].$$

By Ito's lemma and $dr = -\eta r\,dt + \sigma\,dZ$,

$$\mathrm{E}dM(r) = -M'(r)\eta r\,dt + 1/2M''(r)\sigma^2\,dt,$$

and parallel for $\mathrm{E}dN(r)$. Dividing by dt and taking the limit, we find that

$$(\rho+\nu)M(r) = \phi(r) - M'(r)\eta r + 1/2M''(r)\sigma^2 + \nu N(r).$$

Equations (8) and (9) follow by direct substitution for $M(r)$, $N(r)$, $\phi(r)$, and ν.

A.6 Solution of the ODE (10)

Let $a \equiv 2\eta/\sigma^2 > 0$; $b \equiv 2(\rho+\lambda)/\sigma^2 > 0$; and $c \equiv b/a$. To solve the homogeneous part of (10), we must find a function $x(r)$ solving

$$x''(r) - arx'(r) - bx(r) = 0.$$

Let $z \equiv r\sqrt{a}$ and $y(z) \equiv x(r)$. An equivalent representation of the homogeneous equation is

$$y''(z) - zy'(z) - cy(z) = 0;$$

see Kamke (1956, 2.54). The solution to this equation is

$$y^h(z) = A\ {}_1F_1\left(\frac{c}{2}, \frac{1}{2}, \frac{z^2}{2}\right) + Bz\ {}_1F_1\left(\frac{1+c}{2}, \frac{3}{2}, \frac{z^2}{2}\right),$$

for A, B arbitrary constants and ${}_1F_1(\cdot)$ the confluent hyper-geometric function or Pochhammer's function; see Kamke (1956, 2.44).

A particular solution to the original ODE in the modified representation,

$$y''(z)\eta - \eta zy'(z) - (\rho+\lambda)y(z) = -(1-\lambda\zeta)\bar{r}\left(\tau_0 + \frac{\tau_0+\tau_1}{\sqrt{a}}z + \frac{\tau_1}{a}z^2\right) + \psi,$$

is given by $y^p(z) = \phi_0 + \phi_1 z + \phi_2 z^2$ with

$$\begin{aligned}
\phi_0 &= \frac{(1-\lambda\zeta)\bar{r}\tau_0 - \psi + \frac{\sigma^2\bar{r}\tau_1(1-\lambda\zeta)}{2\eta+\lambda+\rho}}{\lambda+\rho}, \\
\phi_1 &= \frac{(1-\lambda\zeta)\bar{r}(\tau_0+\tau_1)}{\sqrt{a}(\eta+\lambda+\rho)}, \\
\phi_2 &= \frac{(1-\lambda\zeta)\bar{r}\sigma^2\tau_1}{2\eta(2\eta+\lambda+\rho)}.
\end{aligned}$$

The solution to the original ODE in the modified representation is thus $y(z) = y^h(z) + y^p(z)$, subject to the value matching and smooth pasting conditions $y(z^h) = k^h$, $y'(z^h) = 0$, $y(z^l) = -k^l$, $y'(z^l) = 0$. Solutions for the four unknowns A, B, z^h, z^l can be numerically obtained. Solutions for r^h, r^l follow directly. Alternatively, one derives $X(r)$ from $y(z)$ and solves $X(r)$ and the value matching and smooth pasting conditions numerically for A, B, r^h, r^l.

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