The Home Bias in Equities and Distribution Costs

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The Home Bias in Equities and Distribution Costs

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Abstract

We show that including distribution costs into a general equilibrium model of international portfolio choice contributes to explaining the “home bias” in international equity investment. Our model is able to replicate observed investment positions for a wide range of parameter values, even if agents have an incentive to hedge labor income risk by purchasing foreign equity. This is because the existence of a retail sector affects both the correlation of domestic returns with the domestic price level and the correlation between financial and nonfinancial income.

JEL Classification: F41, G11, G15

Keywords: International Financial Market Integration, International Risk Sharing, Home Bias

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1 Introduction

Since Obstfeld and Rogoff (2000) established their claim that trade costs contribute to explaining the "six major puzzles in open economy macroeconomics", a growing literature has scrutinized the relationship between frictions in goods trade and the observed home bias in equity holdings. In the presence of such frictions, technology shocks lead to stronger fluctuations in the national price level, and the home bias may reflect agents' desire to insure against "real exchange rate risk": if domestic firms' profits are high whenever the domestic price level is high, i.e. the real exchange rate appreciates, this is a compelling reason to increase the share of domestic equities relative to the frictionless benchmark. Variations in the real exchange rate may be due to a relatively large weight of domestic goods in agents' consumption baskets – i.e. a home bias in consumption (Kollmann 2006) –, the presence of goods that are non-tradable (Stockman and Dellas 1989, Collard et al. 2007), or the existence of trading costs which drive a wedge between the domestic and the foreign price of a good. In Obstfeld and Rogoff (2000), Obstfeld (2007) and Coeurdacier (2009), these costs take the form of “iceberg costs”, i.e. a leak in the bucket which transports goods from one country to another.

Both Coeurdacier and Obstfeld point towards two important challenges which this literature faces: Firstly, they illustrate that trade costs in goods markets cannot generate a home bias in equities under standard preference assumptions. Secondly, the equilibrium equity portfolios they derive are extremely sensitive to variations in the model parameters. In this paper we propose “distribution costs” as an alternative source of international price differences and real exchange rate fluctuations. We argue that including these costs into an otherwise standard model of international portfolio choice goes a long way in explaining observed equity investment patterns – even if consumers have no particular preference for domestic traded goods, and even if there is an incentive to hedge labor-income risk by purchasing foreign equities (Baxter and Jermann 1997, Baxter and King 1998). Hence, although we put ourselves into an unfavorable starting position relative to endowment-economy models, our framework is able to generate a diversification pattern that is close to real-world portfolio shares. Moreover, we demonstrate that, in such a setup, agents' portfolio choices do not exhibit the dramatic sensitivity to small parameter variations that characterizes many other models of international risk sharing.

Our results are derived within a stochastic two-country general equilibrium model in which agents purchase shares of domestic and foreign firms in order to maximize their expected util-

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1Surveys of both the empirical and theoretical literature on the “home bias puzzle” are provided by Lewis (1997) and Obstfeld (2007). Sercu and Vanpee (2007) show that the portfolio share of domestic equities is on average 70% in 20 industrialised countries, ranging from 49% in Belgium to around 90% in Greece and Japan.
ity. Agents consume domestic and foreign traded goods as well as nontraded goods. We follow Burstein et al. (2003) in assuming that every traded good has to be augmented by a certain amount of (nontraded) retail services. While it has long been recognized that such a constellation results in international price differences at the retail and wholesale levels (Corsetti et al. 2008a) and that the existence of distribution services raises the effective share of nontraded goods in GDP (Collard et al. 2007), we argue that accounting for distribution costs adds several features that are important for agents’ portfolio choice: first, the existence of retail services increases the demand for nontraded goods at given prices and reduces the effective elasticity of the demand for traded goods. Both aspects enhance the importance of profits in financing agents’ consumption and thus magnify any shock that causes a variation in profits. Moreover, the existence of distribution costs limits consumers’ scope for expenditure switching and thus reinforces the positive relationship between nontraded goods firms’ prices and their profits. Conversely, it cushions the impact of terms of trade fluctuations on traded goods firms’ profits. Finally, distribution costs reinforce the influence of nontraded goods prices on the real exchange rate.

These features affect agents’ demand for domestic and foreign assets by influencing the second moments that are crucial for agents’ portfolio choices: domestic equities are a good hedge against “real exchange rate risk” if the covariance between domestic profits and the real exchange rate is negative.2 If domestic firms’ profits are high whenever the domestic price level is high, agents find it advantageous to invest a large share of their wealth in domestic equity, since the loss in purchasing power is compensated by an increase in income. Conversely, agents have an incentive to hedge their non-diversifiable labor income by purchasing foreign equities if domestic profits are positively correlated with labor income. The existence of distribution costs influences these covariances: higher distribution costs raise the (negative) covariance between domestic profits and the real exchange rate and reduce the (positive) covariance between profit and labor income. Thus, distribution costs affect agents’ portfolio choice by reducing the incentive to hedge labor income risk and by enhancing the incentive to hedge real exchange rate risk – with the latter motive becoming less important as distribution costs grow large.

As we will show, our model generates realistic portfolio choices for a wide range of parameter values. Hence, even if we depart from our benchmark parameterization, the resulting portfolio shares do not assume implausibly high or low values. This surprising stability – which is in contrast to the findings of Obstfeld (2007) and Coeurdacier (2009) – is due to the fact that distribution costs cushion the influence of parameter variations on prices and incomes. Thus, in contrast to Coeurdacier and Gourinchas (2009), who demonstrate that an asset structure consisting not only of equities but also of internationally traded bonds can help to overcome

\footnote{We are using the price notation. An \textit{increasing} real exchange rate thus corresponds to a \textit{real depreciation}.}
the problems raised above, we show that even if we stick to the traditional assumption that agents only trade equity, distribution costs go a long way in explaining the “home bias” in international equity investment and help to resolve the high sensibility of equity portfolios to preference parameters.

The rest of the paper is structured as follows: section 2 presents and motivates the building blocks of our model. In section 3, we explain how to extract optimal portfolio shares from the log-linearized version of the model and how they are affected by the presence of distribution services. Here we also offer numerical results, focusing, in particular, on the parameter that represents the size of distribution costs. Section 4 concludes.

2 The Model

2.1 Basic structure

We use a stochastic two-economy general equilibrium model whose structure is based on Obstfeld and Rogoﬁ (2000). Yet it differs crucially on two points. First, output is determined by (endogenous) employment. Second, and more importantly, we choose a different source of price segmentation: while Obstfeld and Rogoﬁ (2000) analyse the impact of iceberg-type trade costs on the covariance between ﬂuctuations in the real exchange rate and ﬁnancial income, we focus on the consequences of service costs arising from distributing tradable goods to consumers.

The two economies are identical in terms of preferences, technologies, market structure as well as size. We will use an asterisk (*) to denote foreign magnitudes. Home and foreign agents are indexed over the interval $[0, 1]$. In both countries, varieties of a tradable and a non-tradable good are produced. The number of differentiated goods in each of the four sectors is defined by a continuum of unit mass. Varieties of the domestic (foreign) tradable goods are indexed by $h \in [0, 1]$ ($f \in [0, 1]$), while varieties of the nontraded goods in home and foreign are indexed by $n \in [0, 1]$ and $n^* \in [0, 1]$. Sectoral aggregates such as total proﬁts or overall demand for a good are denoted by $H$, $F$, $N$, and $N^*$, respectively. Each of the ﬁrms is a monopolistic supplier of a single variety and sets prices ﬂexibly. The productivity of ﬁrms in the four industries may each be hit by a technology shock, the realization of which is uncertain ex ante. Apart from directly serving consumers, nontraded goods ﬁrms also offer their output to a perfectly competitive retail sector. This sector combines traded goods (purchased at the wholesale level) and nontraded goods when meeting ﬁnal consumers’ demand for tradables.\footnote{Goldberg and Campa (2010) provide empirical evidence. By contrast, typical nontraded goods like housing, health and education do not require wholesaling and retailing. They are delivered directly to ﬁnal consumers.}
2.2 Timing and preferences

The economy we consider lasts for two periods: in period 0, agents choose the share of their wealth that they invest in domestic and foreign equities. In period 1, they decide on their labor supply, collect wage incomes as well as the returns on their investments, and consume. Expected utility of a representative domestic household as of period 0 is increasing in the aggregate consumption index $C$ and decreasing in work effort $L$ in period 1:

$$U = E_0 \left[ \frac{C^{1-\rho} - 1}{1-\rho} - \kappa L \right], \quad (1)$$

In equation (1), $E_0$ is the expectation operator across states of natures, $\rho$ is the degree of relative risk aversion, and $\kappa$ is a strictly positive parameter. The assumption of a constant marginal disutility of labor implies an infinite wage elasticity of labor supply, which simplifies the analysis. The household supplies labor to all firms in the nontraded and traded goods sectors,

$$L = L_N + L_H, \quad L_H = \int_0^1 L_H(h) dh, \quad L_N = \int_0^1 L_N(n) dn. \quad (2)$$

The goods basket consumed by the representative domestic household is given by

$$C = \left( \gamma^\frac{\theta}{\theta+1} C_T^\theta + (1 - \gamma)^\frac{\theta}{\theta+1} C_N^\theta \right)^\frac{1}{\theta} \quad \text{with} \quad P = \left( \gamma P_T^{1-\theta} + (1 - \gamma) P_N^{1-\theta} \right)^\frac{1}{1-\theta}. \quad (3)$$

$C_T$ is a bundle of tradables, $C_N$ is (final) consumption of non-tradables and $0 < \gamma \leq 1$ is the share of traded goods in the overall consumption basket. The consumer price index is $P$. The elasticity of substitution between traded and nontraded goods is $\theta > 0$. The traded goods aggregate depends on the consumption of domestic tradables ($C_H$) and foreign tradables ($C_F$), with $\eta > 0$ denoting the elasticity of substitution between the two types of traded goods:

$$C_T = \left( a^\frac{\theta}{\theta+1} C_H^{\frac{\theta}{1-\sigma}} + (1 - a)^\frac{\theta}{\theta+1} C_F^{\frac{\theta}{1-\sigma}} \right)^\frac{1}{\theta} \quad \text{with} \quad P_T = \left( a P_H^{1-\eta} + (1 - a) P_F^{1-\eta} \right)^\frac{1}{1-\eta}. \quad (4)$$

The parameter $a$ measures the overall share of home goods in domestic traded goods consumption. If $a = \frac{1}{2}$, there is no home bias in traded goods consumption. Conversely, if $\frac{1}{2} < a \leq 1$, agents allocate a larger share of their tradables consumption to domestic goods. Note that the aggregate traded goods price index $P_T$ consists of retail prices for home and foreign tradables.

Both traded and nontraded goods are, again, bundles of differentiated products,

$$C_H = \left( \int_0^1 C_H(h)^{\sigma-1} dh \right)^{1-\sigma}, \quad C_F = \left( \int_0^1 C_F(f)^{\sigma-1} df \right)^{1-\sigma}, \quad C_N = \left( \int_0^1 C_N(n)^{\sigma-1} dn \right)^{1-\sigma},$$

with $\sigma$ denoting the elasticity of substitution, which we assume to be the same for traded and nontraded varieties, respectively. The resulting price indices are

$$P_H = \left( \int_0^1 P_H(h)^{1-\sigma} dh \right)^{1-\sigma}, \quad P_F = \left( \int_0^1 P_F(f)^{1-\sigma} df \right)^{1-\sigma}, \quad P_N = \left( \int_0^1 P_N(n)^{1-\sigma} dn \right)^{1-\sigma}.$$

Agents in the foreign country have identical preferences.
2.3 International financial markets and budget constraints

In period 0, international trade in equity shares takes place. We follow Coeurdacier (2009) in assuming that households in both countries can choose to hold positions in two equity assets: claims on a share of the sum of future profits generated by domestic (tradables and non-tradables) firms and claims on a share of the sum of future profits generated by foreign (tradables and non-tradables) firms. Hence, it is not possible to invest in the traded and nontraded goods industries separately. At the beginning of the first period, households fully own their local firms. The supplies of domestic and foreign shares are normalized to one, respectively, and the representative domestic and foreign consumers face the following budget constraints:

\[ \phi p + \varphi Sp^* = p \quad \text{and} \quad \phi^* p + \varphi^* Sp^* = Sp^*, \]

where \( \phi \) (\( \varphi \)) is the amount of domestic (foreign) shares purchased by domestic consumers and \( p \) (\( p^* \)) is the price for an equity share denoted in the domestic (foreign) currency. The nominal exchange rate \( S \) (in price notation) transforms foreign prices into domestic-currency units. Note that \( \phi \) also represents the share of an agent’s wealth allocated to domestic equities. Since we normalize the total stock of domestic and foreign shares to one, the equilibrium is characterized by \( \phi + \phi^* = 1 \) and \( \varphi + \varphi^* = 1 \). Moreover, it follows from our symmetry assumptions that \( \phi = \varphi^* \) and \( \varphi = \phi^* = 1 - \phi \). The domestic and foreign first-period budget constraints can thus be rewritten as \( \phi p + (1 - \phi)Sp^* = p \) and \( (1 - \phi)p + \phi Sp^* = Sp^* \). Domestic agents thus purchase claims on \( \phi \cdot 100 \) percent of domestic firms’ future profits, and a claim on \( (1 - \phi) \cdot 100 \) percent of foreign firms’ future profits. Our goal is to derive the optimal value of \( \phi \).

After the realization of shocks in period 1, households at home and abroad decide about optimal consumption and labor supply subject to their respective budget constraint

\[ PC = \phi \Pi + (1 - \phi) (\Pi N^*) + WL \quad \text{and} \quad SP^*C^* = (1 - \phi) \Pi + \phi \Pi N^* + SW^*L^*, \]

where \( W \) (\( W^* \)) is the nominal wage, while \( \Pi = \Pi_H + \Pi_N \) and \( \Pi^* = \Pi_{P^*} + \Pi_{N^*} \) are the sum of nominal profits in the traded and nontraded goods industries at home and abroad, respectively. The optimality conditions for consumption and labor supply for a representative domestic household are derived from the objective function (1) and the budget constraint (6):

\[ \mu = \frac{C^*}{P^*}, \mu = \frac{\kappa}{W} \quad \text{and, hence,} \quad \frac{W}{P} = \frac{\kappa}{C^*}, \]

where the Lagrange multiplier for the second-period budget constraint is denoted by \( \mu \). The optimality conditions yield the relationship between the real wage and the marginal utility of consumption. The foreign agent’s choices are described by identical first order conditions.
The Euler equation that characterizes domestic agents’ optimal portfolio choice equalizes the relative price of domestic equity to the relative gains in terms of expected marginal utility:

$$\mu_0(p - Sp^*) = E_0[\mu(\Pi - S\Pi^*)],$$

(8)

where $\mu_0$ is the Lagrange multiplier referring to the budget constraint in period 0. Due to our symmetry assumption, equity prices do not differ across countries, i.e. $p = Sp^*$. Using this result and (8) as well as the analogous condition for the foreign country yields

$$E_0[C^{-\rho}P^{-\Pi}] = E_0[C^{-\rho}S\Pi^*]$$

(9)

The portfolios at home and abroad are chosen optimally if the covariances between equity payoffs (future profits) and marginal utility from consumption are the same for both assets. Thus, in equilibrium households trade equity shares across borders until none of the two assets provides a better hedge against (relative) consumption risk:

$$E_0[(\Pi - S\Pi^*) (C^{-\rho}P - C^{-\rho}S\Pi^*)] = 0.$$ 

(10)

### 2.4 Goods markets and distribution costs

#### 2.4.1 Demand for goods

Domestic consumption demand for variety $h$ of the home traded good depends on the retail price for this variety relative to the aggregate retail price index in this sector. It also depends on the aggregate demand for domestic tradables. The demand of consumers for a nontraded good variety $n$ has an analogous form, so that

$$C_H(h) = \left(\frac{P_H(h)}{P_H}\right)^{-\sigma} C_H \quad \text{and} \quad C_N(n) = \left(\frac{P_N(n)}{P_N}\right)^{-\sigma} C_N.$$ 

(11)

The relative demand for domestic and foreign tradables as well as nontraded goods equals

$$\frac{C_H}{C} = \gamma a \left(\frac{P_H}{P_F}\right)^{-\eta} \left(\frac{P_T}{P_T}\right)^{-\theta} \quad C_F = \gamma (1 - a) \left(\frac{P_F}{P_T}\right)^{-\eta} \left(\frac{P_T}{P_T}\right)^{-\theta} \quad C_N = (1 - \gamma) \left(\frac{P_N}{P_T}\right)^{-\theta}.$$ 

In modelling distribution costs, we follow the approach of Burstein et al. (2003) as well as Corsetti and Dedola (2005): a perfectly competitive retail sector combines any unit of traded goods (home or foreign) that it sells to domestic consumers with $\delta$ units of a composite nontraded good. Aggregated individual nontraded goods varieties used by retail firms are

$$\delta = \left(\int_0^1 \delta(n) \frac{P_T}{P_T} \, dn\right)^{\frac{1}{\theta}},$$ 

(12)

where $\delta(n)$ is the amount of nontraded variety $n$ used by retailers. Due to perfect competition, the domestic retail prices of domestic and foreign traded varieties – $P_H(h)$ and $P_F(f)$ – equal
retail firms’ marginal costs. These, in turn, are the sum of wholesale prices – \( \hat{P}_H(h) \) and \( \hat{P}_F(f) \) – and the costs of nontraded services:

\[
P_H(h) = \hat{P}_H(h) + \int_0^1 \delta(n)P_N(n)dn \quad \text{and} \quad P_F(f) = \hat{P}_F(f) + \int_0^1 \delta(n)P_N(n)dn.
\]

(13)

Domestic retail firms minimize their costs, given (13) subject to (12). Their demand for non-traded variety \( n \) is given by

\[
\delta(n) = \left( \frac{P_N(n)}{P_N} \right)^{-\sigma} \delta C_T.
\]

(14)

Combining this with the demand by final consumers (11) yields

\[
C_N(n) + \delta(n) = \left( \frac{P_N(n)}{P_N} \right)^{-\sigma} (C_N + \delta C_T),
\]

(15)

which is total demand for the non-tradable variety \( n \). In the foreign country, analogous functions apply, with retail prices depending on wholesale prices and the price of foreign retail services.

2.4.2 Price setting and profits

Total profits in the home traded goods sector equal

\[
\Pi_H = \int_0^1 \hat{P}_H(h)C_H(h)dh + S\int_0^1 \hat{P}_H^*(h)C_H^*(h)dh] - WL_H.
\]

(16)

where \( C_H(h) \) (\( C_H^*(h) \)) is domestic (foreign) demand for the domestic traded variety \( h \). \( \hat{P}_H(h) \) is the domestic wholesale price for a differentiated domestic traded good \( h \), while \( \hat{P}_H^*(h) \) is the respective price charged to the foreign retail sector and denoted in the foreign currency. The output of firm \( h \) is given by labor employed in its production and the productivity level in this sector \( A_H \): \( Y_H(h) = A_H L_H(h) \). In the nontraded goods sector total profits are

\[
\Pi_N = \int_0^1 P_N(n)(C_N(n) + \delta(n))dn - WL_N.
\]

(17)

Similarly to the traded sector production technology, the output of firm \( n \) in the nontraded sector is given by labor employed in its production and the productivity level in this sector \( A_N \): \( Y_N(n) = A_N L_N(n) \). The productivity parameters \( A_H \) and \( A_N \) can be seen as random shifts in sectoral productivity with a mean value of \( E_0 (\ln A_H) = E_0 (\ln A_N) = 0 \), finite variances \( \sigma^2_A_H = E_0 (\ln A_H^2) - E_0 (\ln A_H)^2 \), \( \sigma^2_A_N = E_0 (\ln A_N^2) - E_0 (\ln A_N)^2 \) and the covariance \( \sigma_{A_H A_N} = E_0 (\ln A_H \ln A_N) \).

The overall resource constraints equal \( Y_H = C_H + C_H^* \) and \( Y_N = C_N + \delta C_T \), where \( Y_H \) and \( Y_N \) reflect total production in the two sectors. Similar conditions hold for the foreign country.

Firms in the domestic nontraded sector are monopolistic suppliers and maximize profits given the demand function (15) and their marginal costs \( MC_N(n) \). As a result, their price is set as

\[
P_N(n) = \frac{\sigma}{\sigma - 1} MC_N(n),
\]

(18)
with the marginal costs given by $MC_N = W/A_N$. Similar conditions hold for the foreign country. Due to symmetry, all firms in the nontraded sector at home and abroad set the same price in equilibrium, i.e. $P_N(n) = P_N$ and $P_N^*(n^*) = P_N^*$. Combined with (13) this implies that domestic retail prices of domestic and foreign tradables are given by

$$P_H(h) = \hat{P}_H(h) + \delta P_N \quad \text{and} \quad P_F(f) = \hat{P}_F(f) + \delta P_N. \quad (19)$$

Producers of traded goods varieties maximize (16), taking into account their marginal costs $MC_H$, the domestic demand function (11) as well as its foreign counterpart. Note that, while firms set their prices at the wholesale level – $\hat{P}_H(h)$ and $\hat{P}_H^*(h)$ – consumers’ demand depends on retail prices $P_H(h)$ and $P_H^*(h)$. Substituting (19) into (11) yields

$$C_H(h) = \left( \frac{\hat{P}_H(h) + \delta P_N}{P_H + \delta P_N} \right)^\sigma C_H \quad \text{and} \quad C_H^*(h) = \left( \frac{\hat{P}_H^*(h) + \delta^* P_N^*}{P_H^* + \delta^* P_N^*} \right)^\sigma C_H^*. \quad (20)$$

Taking these demand functions into account, firms in the domestic traded goods industry set their prices for the home and foreign market equal to

$$\hat{P}_H = \frac{\sigma}{\sigma - 1} \left( MC_H + \frac{\delta}{\sigma} P_N \right) \quad \text{and} \quad \hat{P}_H^* = \frac{\sigma}{\sigma - 1} \left( MC_H + \frac{\delta^*}{\sigma} SP_N \right). \quad (21)$$

The marginal costs in the traded goods sector equal $MC_H = W/A_H$. Foreign traded goods firms set their prices similarly. Note that wholesale prices set by traded goods firms depend both on their marginal costs and on the price of non-tradables. The reason is that the necessity to combine traded and nontraded goods implicitly lowers the demand elasticity for traded goods and increases the markup. Combining equation (21) with (19) and assuming that $\delta$ equals $\delta^*$ yields the retail prices of the domestically produced traded goods at home and abroad:

$$P_H = \frac{\sigma}{\sigma - 1} (MC_H + \delta P_N) \quad \text{and} \quad SP_H^* = \frac{\sigma}{\sigma - 1} (MC_H + \delta SP_N^*). \quad (22)$$

Note that if $\delta > 0$, retail and wholesale prices for the same good may differ across countries. Hence, the law of one price does not hold at the wholesale and retail levels. Moreover, (22) indicate that markups are responsive to shocks in the nontraded sector.

The terms of trade ($\tau$) are defined on the basis of relative marginal costs,

$$\tau \equiv \frac{MC_H}{SMC_F^*} \quad (23)$$

Since both wholesale and retail prices of traded goods are influenced by nontraded goods prices, this definition allows us to isolate price variations that originate in the tradables sector. Finally, the real exchange rate is defined to be $Q \equiv \frac{SP_F^*}{P_H} \quad$ i.e. an increase of $Q$ reflects a real depreciation.
2.5 Equilibrium and steady state conditions

The optimality and market clearing conditions are used to determine the endogenous variables in equilibrium – in particular, the portfolio share \( \phi \). The rational expectations equilibrium is a set of values for relative consumption, output, labor, real wages, prices and the optimal portfolio share, given the distribution of shocks \( \{A_H, A_F, A_N, A_N^*\} \). The model is solved by log-linearizing around the symmetric steady state where \( \overline{A}_H = \overline{A}_F = \overline{A}_N = \overline{A}_N^* = 1 \).

The steady state ratio of the nontraded goods price over the (retail) price of traded goods is given by

\[
\frac{P_N}{P_T} = \frac{1}{1+\theta} \frac{\sigma-1}{\sigma+1} \leq 1.
\]

If \( \delta = 0 \) – i.e. if there are no distribution costs – it follows that \( \Omega = 1 \) and \( \hat{P}_N = \hat{P}_T \). Once \( \delta > 0 \), \( \Omega \) is smaller than one. This is because – as shown in (22) – the retail price of traded goods exceeds its “markup price” by \( \delta \sigma / (\sigma - 1) \).

The steady-state ratio of nontraded goods firms’ to traded goods firms’ profits is given by

\[
\frac{\Pi_N}{\Pi_H} = \omega + \delta \Omega \quad \text{with} \quad \omega \equiv \frac{1-\gamma}{\gamma} \Omega^{1-\theta} > 0.
\]

If \( \delta = 0 \), this ratio only depends on the weights of nontraded and traded goods in agents’ consumption aggregator. Whether the existence of distribution costs raises or reduces this expression depends on \( \theta \), the elasticity of substitution between traded and nontraded goods: if \( \theta > 1 \), a higher value of \( \delta \) raises \( \Pi_N / \Pi_H \) both because the need to combine traded goods with retail services raises the demand for nontraded goods at given prices, and because the higher relative price of traded goods shifts demand towards nontradables. By contrast, if \( \theta < 1 \) the effect is ambiguous since consumers’ demand reaction to increasing tradables prices is less pronounced.

The (retail) value of domestic traded goods production relative to domestic nominal spending is given by \( \Lambda_T = \overline{P}_H \overline{Y}_H / \overline{PC} = 1 / (1 + \omega) \) while the value of domestic nontradables production relative to domestic spending is \( \Lambda_N = \overline{P}_N \overline{Y}_N / \overline{PC} = (\omega + \delta \Omega) / (1 + \omega) \). Without distribution costs, these ratios are \( \gamma \) and \( (1 - \gamma) \), respectively. With positive distribution costs, \( \Lambda_T < \gamma \) if \( \theta > 1 \) while \( \Lambda_T > \gamma \) if \( \theta < 1 \). By contrast, the effect of \( \delta \) on \( \Lambda_N = (1 - \Lambda_T) + \delta \Omega \Lambda_T \) is ambiguous: steady state revenues of nontraded goods firms are affected by the fact that these firms charge a lower price (the first part of the equation). At the same time, however, they meet higher demand at given prices since every traded good that is sold is associated with \( \delta \) units of the (composite) nontraded good (the second part of the equation).

Finally, the steady-state share of total profits in agents’ income equals \( \overline{\Pi} / \overline{PC} = (1 + \delta \Omega \Lambda_T) / \sigma \). The share of labor income is \( \overline{WL} / \overline{PC} = 1 - (1 + \delta \Omega \Lambda_T) / \sigma \). While the former expression is strictly increasing in \( \delta \), the latter is strictly falling. This is an important result: by lowering the effective elasticity of demand for traded goods and by increasing the demand for nontraded goods at given prices, the necessity to use retail services raises the share of agents’ spending that is financed out of profit income while it reduces the importance of labor income.
2.6 Calibration

We will later compute the equilibrium portfolio share $\phi$ for varying sets of parameter values. The parameters that enter our benchmark calibration are presented in Table 1.

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<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<td>$\theta$</td>
<td>Elasticity of substitution between traded and nontraded goods</td>
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<tr>
<td>$\eta$</td>
<td>Elasticity of substitution between home and foreign traded goods</td>
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<td>$\sigma$</td>
<td>Elasticity of substitution between varieties</td>
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<tr>
<td>$\delta$</td>
<td>Units of non-tradables required for delivering tradables</td>
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<td>$\rho$</td>
<td>Degree of relative risk aversion</td>
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<td>$\gamma$</td>
<td>Preference for tradables</td>
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<tr>
<td>$\alpha$</td>
<td>Preference for domestic tradables</td>
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</tbody>
</table>

Standard estimates for $\theta$, the elasticity of substitution between traded and nontraded goods, vary between 0.44 (Tesar and Stockman, 1995) and 1.2 (Ostry and Reinhart, 1992). We choose the midway and set $\theta$ equal to 0.9 which is, in fact, only slightly higher than the estimate provided in Mendoza (1991). The elasticity of substitution between home and foreign tradables $\eta$ equals 1.5 and is based on the estimate by Backus et al. (1994), a widely used value in open economy macro studies. In the sensitivity analysis carried out later on, we will also account for the recent contribution by Imbs and Mejean (2008). They reconcile estimates for $\eta$ in macro studies with those based on micro-level data and suggest considerably higher values for $\eta$ than 1.5.

The elasticity of substitution between varieties of a good ($\sigma$) is set to 7 (Corsetti et al., 2008b), which implies a steady state markup of 16 percent in the nontraded goods sectors. The calibration is thus in line with Oliveira Martins et al. (1996), who find markups generally ranging between 10 and 35 percent. In the presence of distribution costs, the assumption of identical elasticities of substitution across sectors implies that the steady state markup for the traded goods sector is above the markup in the nontraded goods sector. Its size depends on the value chosen for $\delta$, the amount of non-tradables services required for the delivery of a unit of a tradable good to consumers. According to evidence provided in Burstein et al. (2003), the distribution margin in retail prices for US tradables ranges between 40 and 50 percent. These figures are broadly confirmed by the empirical analysis in Goldberg and Campa (2010) who document distribution margins between 30 and 50 percent of purchaser prices for a sample of twenty-one OECD countries. We follow these findings and set $\delta$ to 0.8 which corresponds to a distribution margin ($\delta \Omega \times 100$) of around 40 percent of retail prices. Accordingly, the markup in the traded goods sector amounts to 32 percent over marginal costs.

The coefficient of relative risk aversion $\rho$ equals 2 (see Backus et al., 1994). The preference

...
of the domestic representative household for tradables, $\gamma$, is picked such that it corresponds to a steady-state share of non-tradables in GDP of 60 per cent. This ratio, in turn, is based on the worldwide service to GDP ratio which was, according to the World Development Indicators (WDI), around 65 per cent on average between 1990 and 2006. By subtracting 5 percent we account for the fact that some services are indeed tradable. Note, that in the presence of a distribution sector, the weight of non-tradables in the consumption basket $(1 - \gamma)$ is smaller than the (adjusted) service share because a part of the demand for non-tradables stems from the household’s demand for tradable goods. The more units of retail services are required for the delivery of one unit of tradables to consumers, i.e. the higher the distribution margin, the smaller is the implied weight of non-tradables in consumption.

The preference for domestic tradables $a$ is chosen to match an import-to-GDP ratio of 25 per cent, which is in line with WDI data for high income OECD countries between 1990 and 2006 and includes imports of services. Note that in the presence of a distribution sector consumers purchase tradables at retail prices while imports are recorded at wholesale prices, which differ from consumer prices by the distribution margin. Moreover, the ratio between prices for traded and nontraded goods, $\Omega$, is not equal to one in the steady state. Hence, unlike in many other models, $(1 - a)$, the household’s preference for imported tradables, does not simply equal $\text{Import Share}$. With a given import share, $a$ increases in the weight of tradables in the aggregate consumption basket – which is, among others, dependent on the size of distribution costs. Interestingly, this procedure yields a value of 0.53, i.e. our benchmark calibration implies almost no home bias in traded goods consumption.

3 The optimal portfolio share of domestic equities

3.1 Financial and nonfinancial income and the real exchange rate

In this section we derive the optimal share of domestic equities in domestic agents’ portfolio $(\phi)$. We denote $\tilde{X}$ as a variable’s percentage deviation from its steady-state level $\bar{X}$: $\tilde{X} = \ln X - \ln \bar{X} \equiv (X - \bar{X})/\bar{X}$, approximated up to first-order. We take a second-order approximation of (9), which relates the covariance between domestic returns and domestic nominal spending to the covariance between excess returns and the real exchange rate:

$$E_0[(\tilde{\Pi} - \tilde{\Pi}^*)(\Delta \tilde{PC})] = \frac{1 - \rho}{2\rho} E_0[(\tilde{\Pi} - \tilde{\Pi}^*)(\tilde{Q})].$$

---

4 For more details, also for the preference parameter $a$, see part 2 of the appendix.
5 We relate the portfolio share to differences between domestic and foreign variables. Thus, the magnitudes of domestic spending, profits and labor incomes, are always understood relative to their foreign counterparts.
The difference between domestic and foreign nominal expenditures on consumption, i.e. “excess” domestic nominal spending, is denoted by \( \Delta \hat{P}C \equiv \hat{P}C - \hat{P}^*C^* \) and depends on financial income generated by total profits in the domestic economy and nonfinancial (labor) income, respectively. To see this more clearly consider the budget constraint (6) whose log-linear approximation is

\[
\Delta \hat{P}C = (2\phi - 1)\Delta \hat{Y}_{Fin} + \Delta \hat{Y}^{Non}_{Fin}, \quad \text{with} \quad (25)
\]

\[
\Delta \hat{Y}_{Fin} = \frac{1 + \delta \Delta T}{\sigma} (\hat{P} - \hat{P}^*) \quad \text{and} \quad (26)
\]

\[
\Delta \hat{Y}^{Non}_{Fin} = (1 - \frac{1 + \delta \Delta T}{\sigma})(W_L - S\hat{W}^*L^*). \quad (27)
\]

Equation (25) shows that nonfinancial income directly affects agents’ consumption, while the portfolio share \( \phi \) determines to what extent consumption is influenced by domestic financial income. Using these relationships, we can rewrite (24):

\[
\phi = \frac{1}{2} \left\{ \frac{1 + \frac{\rho}{\rho} E_0[\Delta \hat{Y}_{Fin} \hat{Q}]}{E_0[(\Delta \hat{Y}_{Fin})^2]} - \frac{E_0[\Delta \hat{Y}_{Fin} \Delta \hat{Y}^{Non}_{Fin}]}{E_0[(\Delta \hat{Y}_{Fin})^2]} \right\}. \quad (28)
\]

This equation shows that the optimal share of domestic equities (\( \phi \)) in agents’ portfolio depends on three components: The covariance between domestic financial income \( \Delta \hat{Y}_{Fin} \) and the real exchange rate \( \hat{Q} \), the covariance between nonfinancial (labor) income \( \Delta \hat{Y}^{Non}_{Fin} \) and financial (profit) income as well as the variance of domestic financial income.

If the real exchange rate fluctuates – be it because there is a home bias in traded goods consumption or because some goods are nontraded – the expression \( E_0[\Delta \hat{Y}_{Fin} \hat{Q}] \) may differ from zero. Risk-averse investors (\( \rho > 1 \)) have an incentive to insure against "real exchange rate risk". Domestic equities are a good hedge against this risk if domestic firms’ profits are high whenever the domestic price level is high, i.e. the real exchange rate appreciates. In this case the covariance \( E_0[\Delta \hat{Y}_{Fin} \hat{Q}] \) is negative, and the optimal share of domestic equities increases.

This effect is offset by the incentive of households to hedge their non-diversifiable labor income, \( \Delta \hat{Y}^{Non}_{Fin} \). When domestic financial income is positively correlated with the household’s nonfinancial income, i.e. \( E_0[\Delta \hat{Y}_{Fin} \Delta \hat{Y}^{Non}_{Fin}] > 0 \), that is, if the return on domestic assets is high when labor incomes are high as well, households have an incentive to hedge this risk by purchasing foreign equities such that \( \phi \) decreases. For our model and benchmark parameterization, this will indeed be the case. Thus, the second expression puts us into an unfavorable starting position to explain equity home bias since the non-diversifiable labor income theoretically raises the incentive to purchase foreign equity (see Baxter and Jermann 1997).\(^6\) A lower variance of financial income,\(^6\) Note, that we consider an economy with flexible prices and wages. Engel and Matsumoto (2009) show that in the presence of nominal rigidities financial and labor incomes may be negatively correlated. In this case, labor income risk reinforces the bias towards domestic equities arising from the desire to hedge against real exchange risk.
$E_0[(\Delta \hat{Y}_{Fin})^2]$, magnifies both effects. In what follows, we will explore how distribution costs affect the covariance and variance terms in (28).

3.2 Relative profits and prices

Fluctuations of $\Delta \hat{Y}_{Fin}$ and $\Delta \hat{Y}_{Fin}^{Non}$ are driven by technology shocks in the traded and nontraded goods sectors. These shocks affect the different types of income through their impact on relative profits and prices. Consider first financial income, which can also be written as

$$\Delta \hat{Y}_{Fin} = \frac{[\Lambda_T(\hat{\Pi}_H - S\hat{\Pi}_F) + \Lambda_N(\hat{\Pi}_N - S\hat{\Pi}_N)]}{\sigma},$$

i.e. as a function of profits in the traded and the nontraded goods sectors. A larger value of the elasticity of substitution between different varieties of the composite traded/nontraded good ($\sigma$) reduces firms’ markup and therefore lowers the importance of profits in agents’ total income. The elasticities of financial income with respect to traded and nontraded goods firms’ profits are given by $\Lambda_T$ and $\Lambda_N$, respectively. We can rewrite nonfinancial income as

$$\Delta \hat{Y}_{Fin}^{Non} = (\sigma - 1)\{\Delta \hat{Y}_{Fin} + \frac{\Lambda_T (\Omega - 1)}{\sigma} [\hat{\Pi}_H - S\hat{\Pi}_F - \Omega(\lambda \Delta \hat{P}_N + \hat{\tau})]\},$$

with $\lambda \equiv 1 - 2a = 0$ if there is neither a home bias nor a foreign bias in consumption ($a = 0.5$), and $\lambda < 0$ if domestic traded goods have a greater weight in agents’ utility function ($a > 0.5$). Moreover, $\hat{\tau}$ represents variations in the terms of trade (defined on the basis of relative marginal costs) while $\Delta \hat{P}_N \equiv \hat{P}_N - S\hat{P}_N$ reflects variations in relative nontraded goods’ prices.

Equation (30) shows that financial and nonfinancial income are perfectly correlated if there are no distribution costs (i.e. if $\Omega = 1$). In this case, $E_0[\Delta \hat{Y}_{Fin}, \Delta \hat{Y}_{Fin}^{Non}] / E_0[\Delta \hat{Y}_{Fin}^2] = \sigma - 1$. Letting $\delta$ rise above zero reduces $\Omega$ and activates the second term in curled brackets. This neatly illustrates that accounting for distribution costs also affects agents’ portfolio choice through its impact on the covariance between financial and nonfinancial income.

Given (29) and (30) we relate profits in the traded and the nontraded goods sector in more detail to relative prices and the value of nominal spending:

$$\hat{\Pi}_H - S\hat{\Pi}_F = \Omega \{[(1 - \lambda^2)(1 - \eta) + (1 - \theta)\lambda^2(1 - \Lambda_T)] \hat{\tau} + (1 - \theta)\lambda(1 - \Lambda_T)\Delta \hat{P}_N\} - \lambda \Delta \hat{P}C,$$

$$\hat{\Pi}_N - S\hat{\Pi}_N = \frac{\Lambda_T}{\Lambda_N} \Omega [\delta \Omega + (1 - \theta)(1 - \delta \Omega)(1 - \Lambda_T)][\lambda \hat{\tau} + \Delta \hat{P}_N] + \Delta \hat{P}C.$$

The terms of trade, $\hat{\tau}$, and the relative nontraded goods price, $\Delta \hat{P}_N$, can be related to the underlying shocks by invoking the production function (7) as well as (23):

$$\hat{\tau} = \rho \Delta \hat{P}C + (\rho - 1)\hat{Q} - (\hat{A}_H - \hat{A}_F)$$

and

$$\Delta \hat{P}_N = \rho \Delta \hat{P}C + (\rho - 1)\hat{Q} - (\hat{A}_N - \hat{A}_N^*).$$

$^7$More details are given in part 3 and 4 of the Technical Appendix.
Note that an increase in \( \tilde{\tau} \) reflects an “improvement” of the terms of trade. An increase in domestic consumption raises the domestic real wage and thus the terms of trade by reducing labor supply. For \( \rho > 1 \), a real depreciation also raises the terms of trade through its effect on the nominal wage. Finally, a favorable technology shock reduces costs and prices and thus "worsens" the terms of trade. The relative price of nontraded goods follows the same logic.

The real exchange rate is related to the underlying tradables and non-tradables prices by

\[
\hat{Q} = \lambda \Lambda_T \Omega \tilde{\tau} - (1 - \Lambda_T \Omega) \Delta \hat{P}_N. \tag{33}
\]

The terms of trade do not affect the real exchange rate if there is no home bias in traded consumption (\( \lambda = 0 \)). If \( \lambda < 0 \) a rise of the terms of trade results in a real appreciation. If there are distribution costs, \( \Omega < 1 \), the impact of fluctuations in nontraded goods prices on the real exchange rate is reinforced while the impact of the terms of trade is dampened.

### 3.3 Understanding the effect of distribution costs

Equations (29)–(33) in combination with (28) hold the key for understanding the influence of distribution costs on agents’ portfolio choice. We start by analyzing a very simple special case: suppose that there are no distribution costs (\( \delta = 0 \) and \( \Omega = 1 \)), that all goods are tradable (\( \gamma = 1 \)) and that there is no home bias in consumption (\( \alpha = 0.5 \) and \( \lambda = 0 \)). In this scenario, the real exchange rate is constant and there is no need to use equity to hedge real exchange rate risk (i.e. \( E_0[\Delta \hat{Y}_{F_{in}} Q] = 0 \)). Combining this with the fact that \( E_0[\Delta \hat{Y}_{F_{in}} \Delta \hat{Y}_{F_{in}^{N_{on}}}]/E_0[(\Delta \hat{Y}_{F_{in}})^2] = (\sigma - 1) \) if \( \delta = 0 \) and substituting these results into (28) yields

\[
\phi = 1 - \frac{\sigma}{2}. \tag{34}
\]

This expression has a straightforward interpretation: the higher the elasticity of substitution between individual variants (\( \sigma \)), the smaller the markup of monopolistic firms, the higher the share of wages in total income, and the higher the portfolio position households need to hold in order to insure themselves against labor income risk.

Once we introduce nontraded goods, but abstract from a home bias in traded goods consumption – i.e. \( \gamma < 1 \) but \( \lambda = 0 \) – real exchange rate risk emerges as a possible reason to purchase home equities. In this case, we have

\[
\hat{\Pi}_H - \hat{\Pi}_F \vert_{\lambda = 0} = \Omega (1 - \eta) \hat{\tau}, \tag{35}
\]

\[
\hat{\Pi}_N - \hat{\Pi}_N^* \vert_{\lambda = 0} = \frac{\Lambda_T}{\Lambda_N} \Omega \left[ \delta \Omega + (1 - \theta) (1 - \delta \Omega) (1 - \Lambda_T) \right] \Delta \hat{P}_N + \Delta \hat{P}_C, \tag{36}
\]

\[
\Delta \hat{Y}_{F_{in}}^{N_{on}} \vert_{\lambda = 0} = (\sigma - 1) [\Delta \hat{Y}_{F_{in}} + \frac{\eta}{1 - \eta} \frac{\Lambda_T (1 - \Omega)}{\sigma} (\hat{\Pi}_H - \hat{\Pi}_F^*)], \tag{37}
\]

\[
\hat{Q} \vert_{\lambda = 0} = -(1 - \Omega \Lambda_T) \Delta \hat{P}_N. \tag{38}
\]
Equation (35) illustrates that, with $\lambda = 0$, profits in the tradables sector only depend on the terms of trade – with an “improvement” of the terms of trade raising (reducing) profits if $\eta$, the elasticity of substitution between domestic and foreign traded goods, is lower (greater) than one. Since $\Omega < 1$ for $\delta > 0$, positive distribution costs dampen the influence of terms-of-trade variations. This is because retail prices of tradables not only depend on marginal costs, but also on the price of retail services.

The influence of positive distribution costs on the elasticity of nontraded goods profits with respect to changes in $\Delta \widehat{P}_N$ is harder to determine and depends on the size of the elasticity of substitution between traded and nontraded goods, $\theta$: Equation (36) shows that if $\delta = 0$ and $\theta = 1$, profits are unaffected by movements in relative nontraded goods prices. Consequently, real exchange rate movements, induced by $\Delta \widehat{P}_N$, cannot be insured by purchasing equities. If $\delta = 0$ and $\theta < 1$, the reaction in spending on non-tradables that follows an increase in $\Delta \widehat{P}_N$ is positive, so that $\Delta \widehat{Y}_{Fin}$ increases. In this case purchasing domestic equities provides a hedge against real exchange rate risk. Once $\delta > 0$, the positive effect of an increase in $\Delta \widehat{P}_N$ on $\Delta \widehat{Y}_{Fin}$ is amplified. By contrast, if $\delta = 0$ and $\theta > 1$ the reaction of nontradable profits that follows an increase in $\Delta \widehat{P}_N$ is negative. Once $\delta > 0$, however, the negative effect of an increase in $\Delta \widehat{P}_N$ on nontradable profits is mitigated. Thus, the need to utilise distribution services reinforces the correlation between domestic profits and nontraded goods prices. This is because the presence of a retail sector implicitly reduces the elasticity of demand for nontraded goods.

The observation that distribution costs limit consumers’ scope for “escaping” increases in nontraded goods prices holds the key for understanding our results: the existence of distribution costs not only increases the de-facto share of nontraded goods in GDP, but also affects the pricing behavior of traded goods firms and thus enhances the correlation between nontraded goods prices and profits.

Equation (37) shows, once more, that financial and nonfinancial income are perfectly correlated if $\delta = 0$ (and thus $\Omega = 1$). Once there are positive distribution costs, the second term in (37) is no longer zero. Whether it raises or reduces the covariance between $\Delta \widehat{Y}_{Fin}$ and $\Delta \widehat{Y}_{NonFin}$ depends on the variance of profits in the traded goods sector and the covariance between profits in the traded and the nontraded goods sectors, respectively. Since the elasticity of demand for nontraded goods declines as distribution costs grow larger, the latter term in (37) might become positive, so that the covariance between $\Delta \widehat{Y}_{Fin}$ and $\Delta \widehat{Y}_{NonFin}$ would decline.

Finally, it follows from equation (38) that terms of trade movements do not influence the real exchange rate while fluctuations in nontraded goods prices have an even stronger impact on

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8This holds as long as the distribution margin $\delta \Omega > \frac{(\delta - 1)\Delta Y_{\omega}}{(1 + (\delta - 1)\Delta Y_{\omega})}$. 

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the real exchange rate if $\delta > 0$. This is not surprising: an increase in nontradables prices has a direct and an indirect effect on the domestic price level, with the indirect effect running via the retail price of nontraded goods.

Several conclusions emerge: first, the presence of distribution costs alters the correlation between financial and nonfinancial income. A higher value of $\delta$ potentially reduces agents’ incentive to hedge labor income risk by investing abroad. Conversely, the importance to diversify real exchange rate risk increases. As equations (35) and (36) indicate, distribution costs raise the importance of nontraded goods prices for agents’ financial income. Since these prices also dominate variations in the real exchange rate – and since this influence is magnified if $\delta > 0$ – the correlation between financial income and the real exchange rate increases. This provides an incentive to purchase home equities and shifts $\phi$ above the value given by (34).

To see this more clearly, consider the case where productivity shocks in the traded and nontraded goods sectors are identical. Thus, we allow for only two different shocks (at home and abroad). Since households have access to two equity-type assets, the spanning condition is satisfied and international financial markets are (locally) complete. Under internationally complete financial markets it follows that the Backus-Smith (1993) condition is satisfied (see Obstfeld, 2007):

$$\Delta PC = \frac{1 - \rho}{\rho} \hat{Q}. \quad (39)$$

From (39) and (35)–(38) households incentive to insure against "real exchange rate risk" equals

$$\frac{E_0[\Delta \hat{Y}_{Fin} \hat{Q}]}{E_0[(\Delta \hat{Y}_{Fin})^2]} \bigg|_{\lambda = 0} = \frac{\Omega \Lambda_T - 1}{\Theta \Delta \hat{Y}_{Fin}}, \quad (40)$$

while the incentive to hedge non-diversifiable labor income is driven by

$$\frac{E_0[\Delta \hat{Y}_{Fin} \Delta \hat{Y}_{Non}]}{E_0[(\Delta \hat{Y}_{Fin})^2]} \bigg|_{\lambda = 0} = (\sigma - 1) + \frac{(\sigma - 1) \Lambda_T (1 - \Omega)}{\sigma} \frac{\Theta \Delta \hat{Y}_{Fin}}, \text{ with } (41)$$

$$\Theta \Delta \hat{Y}_{Fin} \bigg|_{\lambda = 0} = \frac{\Lambda_T}{\sigma} \Omega [(1 - \eta) + (1 - \theta) (1 - \Lambda_T) + \delta \Omega (1 - (1 - \theta) (1 - \Lambda_T))] - \frac{1 - \rho}{\rho} \frac{\Lambda_N}{\sigma} (1 - \Omega \Lambda_T).$$

For the benchmark parameter specification in table (1) the expressions in (40) and (41) are both greater than zero if there are no distribution costs ($\delta = 0$), as shown in figure 1. Consequently, households would like to allocate a large portion of their wealth to foreign equities. This is a result one would expect. Consider for example a negative domestic productivity disturbance, which raises domestic prices. This causes domestic output and, hence, profits to decline. As domestic output is low, the real exchange rate appreciates due to the relative lower supply of domestic goods in the world economy. As a consequence, domestic profits and the real exchange rate are positively correlated, as shown in the left graph of Figure 1. Since labour income is positively correlated with domestic profits, as shown in the right graph of Figure 1, households
have an incentive to diversify their labour income. Thus, we should expect a portfolio choice towards foreign equities.

However, once distribution costs become strictly positive, both the covariance between financial income and the real exchange rate and the covariance between financial and nonfinancial income change. Positive distribution costs limit households’ possibility to "escape" the increase in domestic prices since distribution costs reduce the demand elasticity. This has positive effects on domestic profits. Consequently, the correlation between domestic profits and the real exchange rate turns negative and $E_0[(\Delta Y_{Fin})/(\Delta Y_{Fin})^2] < 0$, as illustrated by the left graph of Figure 1.

\[ E_0[\Delta Y_{Fin}Q]/E_0[(\Delta Y_{Fin})^2] < 0, \]

Figure 1: Covariance Variance Ratios

The right graph of Figure 1 shows that the expression in (41), though still positive, declines as $\delta$ becomes larger. Distribution costs amplify the correlation between profits in the traded and the nontraded goods sectors. This mitigates the incentive to hedge labour income risk. Hence, distribution costs affect agents’ portfolio choice by reducing the incentive to hedge labor income risk and by enhancing the incentive to hedge real exchange rate risk – with the latter motive becoming less important as $\delta$ grows large. This is an interesting finding in the light of the discussion raised by van Wincoop and Warnock (2010).

In the preceding paragraphs we have focused on the special case where consumers do not have a preference for domestic traded goods ($a = 0.5$). However, in section 2.5 we have argued that the data suggest a – albeit weak – home bias in traded goods consumption. Once $\lambda < 0$, the covariances between financial income and real exchange rate as well as nonfinancial income, respectively, are more complicated terms, but the qualitative properties do not change by much.\footnote{See part 5 of the Technical Appendix for the case of $\lambda < 0$.}

Figure 2 depicts $\phi$ as a function of $\delta$ for the benchmark parameters from table (1). For realistic distribution margins of around 40 percent, i.e. $\delta = 0.8$, the model yields a very weak home
bias in equity holdings. Domestic households allocate around 53 percent (\(\phi \cdot 100\)) of their initial wealth to domestic equity.

![Graph](image)

Figure 2: Distribution Costs and the Share of Domestic Equities in Domestic Portfolios (Complete Markets)

### 3.4 The incomplete markets case

#### 3.4.1 Solving for the optimal portfolio share

Real-world asset markets do not appear to be complete and the Backus-Smith condition in (39) is often found to be violated by the data (see Obstfeld, 2007). We therefore proceed by relaxing the assumption that productivity shocks in the traded and nontraded goods sectors are identical. Thus, since there are four different shocks while households have access to only two equity-type assets, the spanning condition is not satisfied, and international financial markets are incomplete. We can write nominal spending, financial and nonfinancial income as a function of the underlying technology shocks. To do so we treat the "portfolio based" financial income as exogenous and define

\[
\begin{align*}
\Delta \hat{Y}_{Fin}^{Ex} &= (2\phi - 1) \Delta \hat{Y}_{Fin}^{Ex}, \text{ with} \\
\Delta \hat{Y}_{Fin}^{Fin} &= \Theta_{\Pi} \mathbf{A} + \theta_{\Pi} \left( \Delta \hat{Y}_{Fin}^{Ex} + \Delta \hat{Y}_{Fin}^{Non} \right) \text{ and} \\
\Delta \hat{Y}_{Fin}^{Non} &= \Theta_{L} \mathbf{A} + \theta_{L} \Delta \hat{Y}_{Fin}^{Ex},
\end{align*}
\]

where \(\Theta_{\Pi}\) and \(\Theta_{L}\) are (1x2) vectors whose elements are functions of the model’s parameters, \(\theta_{\Pi}\) and \(\theta_{L}\) are scalars, and \(\mathbf{A'}=(\mathbf{A}_{H} - \mathbf{A}_{F}^{*}, (\mathbf{A}_{N} - \mathbf{A}_{N}^{*})\) is a (1x2) vector of sector-specific
technology shocks.\footnote{Details are given in part 6 of the Technical Appendix.} Combining (43) and (44) yields

\[ \Delta \tilde{Y}_{\text{Fin}} = R_1 \Delta \tilde{Y}_{\text{Ex}} + R_2 A, \]  

(45)

where \( R_1 \) is a scalar and \( R_2 \) is a 1x2 vector. Using (25), (29), (30), (42)–(44), we can derive

\[ \Delta \tilde{P}C - \frac{1}{\rho} \tilde{Q} = D_1 \Delta \tilde{Y}_{\text{Ex}} + D_2 A, \]  

(46)

which contrasts the Backus-Smith condition in (39). \( D_1 \) is a scalar and \( D_2 \) is a 1x2 vector. Now we recognize that \( \Delta \tilde{Y}_{\text{Ex}} \) rather than being exogenous is determined by (42), and (2\( \phi - 1 \)) is left to be solved for. Given (45) and (46), equation (24) can be written as \( R \Sigma D' = 0 \), where \( R = R_1 H + R_2 \), \( H = (2\phi - 1)(1 - (2\phi - 1)R_1)^{-1} R_2 \), \( D = D_1 H + D_2 \) are 1x2 vectors, and \( \Sigma \) is the 2x2 covariance matrix of the exogenous productivity disturbances in the traded and nontraded goods sector. Solving for \( \phi \) yields

\[ \phi = \frac{1}{2} \left\{ 1 + [R_2 \Sigma D_2' R_1' - D_1 R_2 \Sigma D_2' R_1' R_2 \Sigma D_2' R_1']^{-1} R_2 \Sigma D_2' R_1 \right\}, \]  

(47)

the solution to the share of domestic equities in domestic agents’ portfolio \( \text{(see Devereux and Sutherland, 2006)} \). With (47) in hand we can discuss the implications of distribution costs for the home bias in equities when international asset markets are incomplete.

3.4.2 Numerical results

\textbf{The variance and covariance of shocks } Once we drop the complete markets assumption we have to make an assumption on the variance and covariance of the productivity shocks. Working with several alternatives we found that the results were qualitatively and quantitatively very similar. We chose to follow Coeurdacier (2009) and set the correlations of shocks within a country to 0.3. Moreover we set all variances equal to one, thus assuming that the volatility of productivity shocks is equal across sectors and countries.

\textbf{The role of distribution costs for the equity home bias } Figure 3 illustrates how varying the size of distribution costs affects the share of domestic equities in the portfolio of domestic households. If \( \delta = 0 \), i.e. if there are no distribution costs, domestic households hold a large short position in domestic equities. As \( \delta \) increases, the share of domestic equities in the domestic portfolio initially rises quite rapidly, before it eventually starts to decrease again. However, even with very large values of \( \delta \), \( \phi \) is above zero and considerably higher than the position households would choose to hold if there were no distribution costs. When the distribution margin is 40\% \( (\delta = 0.8) \) the percentage share of domestic equities in the overall portfolio \( (\phi \cdot 100) \) equals around 75\%, as shown in Figure 3.

\[ \text{Details are given in part 6 of the Technical Appendix.} \]
To understand the effect of $\delta$ on $\phi$ we go back to the analysis in Section 3.3. There we have shown that, in a model without nontraded goods, without distribution costs and without a home bias in consumption, agents’ portfolio choice is driven by their goal to hedge labor income risk and only hinges on $\sigma$. Substituting our benchmark choice of $\sigma = 7$ into equation (34) yields $\phi = -2.5$. In our benchmark parameterization, nontraded goods enter the utility function with the same weight as traded goods ($\gamma = 0.5$), and consumers’ preferences are slightly biased towards domestic goods ($a = 0.53$). Both aspects give rise to real exchange rate fluctuations and thus possibly provide an incentive to hedge the associated risk. Note, however, that since our choice of the parameter $a$ is close to 0.5, there is almost no home bias in consumption, and variations in $\hat{Q}$ are predominantly driven by nontraded goods prices. In interpreting our numerical results we will therefore focus on the special case ($\lambda = 0$) as characterized by equations (35) to (38).

As shown in equation (28), the optimal value of $\phi$ increases if $E_0[\Delta \hat{Y}_{F,in} \hat{Q}] < 0$. The left column of Figure 4 indicates that for the incomplete markets case this expression is negative for all values of $\delta$. In this case, an increase of nontraded goods prices raises profits in the domestic nontraded goods sector and reduces $\hat{Q}$, i.e. it results in a real appreciation. Hence, even without distribution costs, $\Delta \hat{Y}_{F,in}$ and $\hat{Q}$ systematically move in opposite directions, and there is some scope for real exchange rate risk hedging via domestic equities. However, the motive of labor income hedging still plays a predominant role for optimal portfolio choice: The short position shown for $\delta = 0$ in Figure 3 is less pronounced than in the simplest reference case described by
(34), but the result for \( \phi \) is still far away from a home bias in equities.

Once distribution costs become strictly positive, the covariance between the real exchange rate and domestic financial income remains negative. Since \( a \) is close to 0.5 our interpretation is, again, based on equations (35) to (38).\(^\text{11}\) First, as demonstrated in Subsection 2.4, raising \( \delta \) increases the importance of profits in financing domestic consumption and therefore reinforces the impact of any shock that causes a variation in profits. Second, while positive distribution costs dampen the effect of terms of trade fluctuations on traded goods profits, they enhance the influence of \( \Delta \hat{P}_N \) on nontraded-goods profits. This is due to the fact that \( \delta > 0 \) increases the demand for nontraded goods at given prices. Moreover – as shown by (36) – higher nontraded goods prices unambiguously increase the respective firms’ profits if the elasticity of substitution between traded and nontraded goods (\( \theta \)) is smaller than one. As a result, both \( \Delta \hat{Y}_{Fin} \) and \( \hat{Q} \) are dominated by fluctuations in nontraded goods prices if \( \delta > 0 \), and this results in a negative value of \( E_0[\Delta \hat{Y}_{Fin} \hat{Q}] \).

In addition, raising \( \delta \) affects the covariance between \( \Delta \hat{Y}_{Fin} \) and \( \Delta \hat{Y}_{Non}^{Fin} \) (see equation 37). Whether this covariance increases or decreases relative to the benchmark value of \( (\sigma - 1) \) depends on how the variance of profits in the traded goods sector and the covariance between traded and nontraded goods profits are influenced by distribution costs. If the latter covariance is negative, \( E_0[\Delta \hat{Y}_{Fin} \Delta \hat{Y}_{Non}^{Fin}] \) may, in principle, increase in \( \delta \). However, for our benchmark calibration, the covariance between financial and non-financial income is a non-monotonic function of distribution costs. The right column of Figure 4 shows that this function decreases for small values of \( \delta \), then increases, and eventually decreases again in the “relevant range” – i.e. for values of \( \delta \) in the neighborhood of 0.8. This illustrates that, especially for realistic values of \( \delta \), the home bias in equities is not just driven by raising the correlation between domestic returns and the real

\(^{11}\)In fact, as we will show below, choosing higher values of \( a \) has almost no effect on \( \phi \) if \( \delta > 0 \).
exchange rate - a mechanism that has recently been criticized by van Wincoop and Warnock (2010) - but also by reducing the covariance between domestic financial and nonfinancial income.

**Stability Analysis**  It is the transmission of shocks to relative profits, labor income and the real exchange rate that is crucial for the understanding of agents’ portfolio choice. The response to shock-induced changes in relative prices depends on the response of consumers to these price changes and on the relative importance of the goods within aggregate consumption baskets. Hence, the value of \( \phi \) changes if we vary the parameters characterizing households’ preferences. However, as we will show, the reaction of \( \phi \) to such parameter changes is much less dramatic if \( \delta > 0 \) than in a scenario without distribution costs.

We first examine the sensitivity of agents’ portfolio choice to varying weights in the consumption baskets. Figure 5 illustrates the impact of \( \gamma \) on \( \phi \), keeping most parameters as defined in Table 1, but allowing for different values of \( \delta \).

![Figure 5: The Share of Domestic Equities as a Function of Tradables in the Aggregate Consumption Basket](image)

An increase in the preference for tradables generally lowers the optimal portfolio share of domestic equities, irrespective of whether distribution costs are accounted for or not. The reason for the negative impact of \( \gamma \) on \( \phi \) is easy to see: The higher \( \gamma \), i.e. the higher the importance of tradables relative to non-tradables in households’ consumption baskets, the weaker the response of the real exchange rate to changes in the relative nontraded price, and, thus, the lower the importance of real exchange rate risk as compared to labor income risk. If the hedging of real
exchange rate risk does not play an important role for the portfolio choice of households, it is very likely that households choose to predominantly hold foreign equities. Interestingly, the decline of $\phi$ is much steeper for $\delta = 0$. With $\delta = 0.8$, $\phi$ does not take on values below zero. Even more importantly, as long as there are some nontraded goods in the overall consumption basket households will go long in domestic equities. By contrast, when $\delta$ is zero, households start to go short in domestic equities for relatively small values of $\gamma$, and a home bias in equity holdings only emerges if the share of nontraded goods in agents’ consumption basket is assumed to be unrealistically large (greater than 90 percent). These observations demonstrate that the role of distribution costs goes beyond just increasing the de-facto share of nontraded goods in an economy. As argued above, the existence of distribution costs changes firms’ pricing behavior and the relative importance of financial and nonfinancial income. As a result, the variable $\phi$ is greater than 0.5 even if the share of nontraded goods in GDP is a (reasonable) 60 percent.

Figure 6 demonstrates the impact of variations in $a$ on $\phi$. We allow $a$ to vary in a range between 0.5 and 1, as a foreign consumption bias is rather unrealistic.

![Figure 6: The Share of Domestic Equities as a Function of Domestic Tradables in the Tradables Consumption Basket](image)

Apparently, $\phi$ is extremely sensitive to variations in $a$ when $\delta = 0$, while it is almost invariant and greater than 0.5 once $\delta$ is set to 0.8. With a greater bias in traded goods consumption, terms of trade fluctuations play an increasingly important role in the context of real exchange rate hedging. The higher $a$, the stronger the (negative) impact of a “terms-of-trade improvement” on

\[12\text{Distribution costs obviously do not play a role, when } \gamma \text{ is zero, as in this case there are no tradables. It is then optimal for agents to invest all their wealth in domestic equities.}\]
the real exchange rate. Generally, if $a$ is greater than 0.5, relative profits respond to price changes of both tradables and non-tradables, with the direction of the effect crucially depending on the elasticities of substitution $\eta$ and $\theta$. For our benchmark parameterization and $\delta = 0$, a higher value of $a$ first enhances the incentive to purchase foreign assets, leading to unrealistically large short positions in domestic equities as $a$ approaches 0.9. For larger values of $a$, the covariance of financial income and the real exchange rate turns strongly negative and $\phi$ takes on an implausibly high value. With a (reasonable) distribution margin of around 40 per cent, by contrast, terms of trade movements play a significantly smaller role for both the dynamics of the real exchange rate and of relative profits. Hence, the extent of the bias is less decisive for portfolio choice and, for given parameter values, $\phi$ is almost invariant in $a$.

Figure 7 displays equilibrium domestic equity shares as a function of $\eta$. $\phi$, again, is subject to dramatic swings for the case without distribution costs.

![Graph showing the Share of Domestic Equities as a Function of the Elasticity of Substitution Between Tradables](image)

Figure 7: The Share of Domestic Equities as a Function of the Elasticity of Substitution Between Tradables

For $\eta = 1$, domestic households are extremely biased towards domestic equities, while they choose to hold a large short position in domestic equities when $\eta$ is slightly above unity and higher. This result reflects the importance of labor income risk in the tradables sector for the portfolio choice of households. Remember from (35) that, when $\delta = \lambda = 0$, relative profits in the tradables sector are extremely dependent on the elasticity of substitution between domestic and foreign tradables. By contrast, $\eta$ does not show up in equations (36) and (38), which characterize the reaction of profits in the non-tradables sector and of the real exchange rate. Furthermore
remember from (37) that profits and labor incomes are perfectly correlated in the tradables sector if $\delta = \lambda = 0$. Obviously, the degree of labor income risk arising in the tradables sector depends on $\eta$, while the real exchange rate is independent from $\eta$. Now consider the case, where $\eta$ is unity. Equations (35) and (37) show that relative profits and relative labor incomes (in the traded goods sector) are not responsive to terms of trade fluctuations. This is, essentially, the finding in Cole and Obstfeld (1991) who highlighted the role of relative price adjustment as a potential risk-sharing mechanism. It follows that the position households choose to hold in this special case is purely motivated by their desire to hedge against real exchange rate risk and labor income risk that arise from fluctuations in the relative nontraded goods price. The resulting portfolio choice crucially depends on the value of $\theta$. Since our benchmark value of $\theta$ is smaller than one, households find it optimal to hold a long position in domestic equities. As $\eta$ takes on values below or above unity, the picture changes dramatically. This rapid change in the sign of the domestic equity position is only due to the arising labor income risk in the tradables sector which motivates households to purchase foreign equities. As profits and labor incomes are affected in the same way for any value of $\eta$, while the real exchange rate is not influenced by the value for $\eta$, the optimal portfolio share is almost invariant to values of $\eta$ different from unity.

For $\delta = 0.8$, the range of values $\phi$ takes on is not as broad as for $\delta = 0$. However, there is only a rather small range of $\eta$ values for which the model can generate a realistic home bias in equity holdings. This range includes the value we assigned to $\eta$ in our benchmark calibration and it should be emphasized that this value is not arbitrarily chosen but rather commonly used in the literature. If there are distribution costs, the vulnerability of relative labor incomes in the tradables sector with regards to terms of trade fluctuations also depends on $\eta$, as shown by (37). This link, however, is dampened because terms of trade fluctuations become less important for the dynamics of relative labor incomes if $\Omega < 1$. As a consequence, the position in foreign equities held to hedge against labor income risk (in the tradables sector) is smaller than before.

In Figure 8 we vary the elasticity of substitution between tradables and non-tradables ($\theta$). For $\delta = 0.8$, households exhibit a realistic equity home bias for values of $\theta$ between 0.7 and 1, as can be seen in Figure 8. By contrast, without distribution costs households choose to go short in domestic equities for any value of $\theta$. The short position becomes extremely large with high values of $\theta$. This is due to the declining hedging quality of domestic equities with regard to real exchange rate risk: the (negative) correlation between relative nontraded goods profits and the real exchange rate becomes weaker as $\theta$ increases.

If $\delta$ is 0.8, households hold considerably larger positions in domestic equities, for a given value of $\theta$ and $\phi$ increases as $\theta$ grows larger. The reason for this result - which is in stark contrast to the scenario without distribution costs - is the fact that $\omega$ increases in $\theta$ if $\delta > 0$. This raises the
steady state ratio of nontraded to traded firms’ profits and reduces $\Lambda_T$ (see section 2.5), which, in turn, reinforces the correlation between domestic returns and the real exchange rate. As a consequence, the equilibrium value of $\phi$ increases.

Figure 8: The Share of Domestic Equities as a Function of the Elasticity of Substitution between Tradables and Nontradables

4 Conclusion

The goal of this paper was to explore the potential role of goods market frictions in explaining the observed home bias in equity portfolios. In doing so, we focused on distribution costs as a source of international price differences and real exchange rate fluctuations. Moreover, instead of assuming that equity payoffs originate in exogenous endowment shocks, we related these payoffs to agents’ labor supply.

Our analysis yielded the following results: First, distribution costs do, indeed, help to explain the high share of domestic equity in agents’ portfolios. When feeding our model with plausible parameter values, we are able to replicate observed portfolio shares, i.e. a weight of domestic equity in the range of 70 to 80 percent. Interestingly, this home bias emerges although a non-diversifiable labor income raises agents’ incentive to purchase foreign equities. Moreover, our setup does not exhibit the dramatic sensitivity to small parameter variations that characterizes models which trace international price differences back to iceberg-type trading costs.

We conclude that distribution costs are an important part of the picture when it comes to explaining international portfolio choice. More importantly, their effect goes beyond just increas-
ing the de-facto share of nontraded goods in GDP: by influencing effective demand elasticities and thus firms’ pricing behavior as well as the relative importance of profit and labor income in financing individuals’ consumption, the existence of distribution costs alters the covariances between profits, the real exchange rate and labor income that determine agents’ portfolio choices.

Of course, this paper represents only a first step towards a more comprehensive analysis of goods market structure and international portfolio choice. Our approach rested on a number of simplifying assumptions, in particular with respect to the set of assets that are available to investors. While we followed Coeurdacier (2009) in focusing on agents’ choice of equity ownership, it remains to be shown how distribution costs affect portfolio choices when agents have access to a broader menu of assets - in particular when, as emphasized by Coeurdacier and Gourinchas (2009), internationally traded bonds offer an alternative means to hedge real exchange rate risk.

References


Technical Appendix

Part 1: Important steady state relationships

As the system is approximated around the steady state, it is useful to first clarify some steady relationships between variables frequently utilized throughout the remainder of the appendix.

Goods Prices

An important assumption we make concerning steady state goods prices is that steady-state technology parameters are identical across sectors and across countries ($\tilde{A}_N = \tilde{A}^*_N = \tilde{A}_H = \tilde{A}^*_F = \tilde{A}$). This assumption has far-reaching consequences. First, as elasticities of substitution between varieties ($\sigma$) are generally assumed to be identical in our model, prices for non-tradables are equal across borders. Similarly, since markup prices for tradables ($\tilde{P}^{MC}_H, \tilde{P}^{MC}_F$) are also identical for both types of tradables, the law of one price holds for traded goods on the wholesale and on the retail level. Further note that, since we assume symmetric preferences, this implies that consumer demand for domestic tradables in Home equals the demand of foreign consumers.
for tradables produced in their country. It follows that the representative consumption baskets (aggregate and tradables only) in Home and Foreign are identical. Furthermore, the price index for tradables equals the retail price for either home or foreign tradables and is identical in both countries. The real exchange rate, finally, is equal to one. All these results are summarized by the following equations:

\[
\begin{align*}
\tilde{P}_N &= \frac{\sigma}{(\sigma - 1)} \frac{\bar{W}}{\bar{A}} = \tilde{S} \tilde{P}_N^* = \tilde{P}_H^{MC} = \tilde{S} \tilde{P}_F^{MC} \\
\tilde{P}_H &= \left(1 + \frac{\delta}{\sigma - 1}\right) \frac{\sigma}{(\sigma - 1)} \frac{\bar{W}}{\bar{A}} = \tilde{S} \tilde{P}_H^* \\
\tilde{P}_H &= \left(1 + \frac{\sigma \delta}{\sigma - 1}\right) \frac{\bar{W}}{\bar{A}} \\
\tilde{P}_H &= \tilde{P}_F = \tilde{P}_T = \tilde{S} \tilde{P}_T^* \\
\tilde{C}_H &= \tilde{C}_F^* \\
\tilde{C}_F &= \tilde{C}_H^* \\
\tilde{C}_T &= \tilde{C}_T^* \\
\tilde{C} &= \tilde{C}^* \\
\tilde{P} &= \tilde{S} \tilde{P}^*
\end{align*}
\]

### Profits, Revenues, and Labor Incomes

If we define revenues in the domestic traded goods sector as \( \tilde{R} \tilde{e}v = \tilde{P}_H \tilde{Y}_H \), make use of equation (49) and take into account the production function of domestic tradable goods firms, we get their steady state profit as a function of revenues

\[
\tilde{\Pi}_H = \frac{1}{\sigma (1 - \delta \Omega)} \tilde{R} \tilde{e}v
\]  

(57)

By analogous reasoning, profits in the domestic nontraded goods sector can also be expressed as a function of steady state revenues:

\[
\tilde{\Pi}_N = \frac{1}{\sigma} \tilde{P}_N \tilde{Y}_N
\]  

(58)

A further important steady-state relationship is the one between revenues and nominal aggregate spending. For this purpose, first recall that revenues in the domestic sector can also be defined as follows:

\[
\tilde{R} \tilde{e}v = a \left( \tilde{P}_H / \tilde{P}_T \right) \left( \tilde{P}_H / \tilde{P}_T \right)^{(1 - \eta)} \gamma \left( \tilde{P}_T / \tilde{P} \right)^{(1 - \theta)} \tilde{P} \tilde{C}^* \\
+ (1 - a) \left( \tilde{S} \tilde{P}_H^* / \tilde{S} \tilde{P}_T^* \right) \left( \tilde{S} \tilde{P}_H^* / \tilde{S} \tilde{P}_T^* \right)^{(1 - \eta)} \gamma \left( \tilde{S} \tilde{P}_T^* / \tilde{S} \tilde{P}^* \right)^{(1 - \theta)} \tilde{S} \tilde{P}^* \tilde{C}^*
\]  

(59)

30
Taking into account that $\tilde{PC} = \tilde{P}^* \tilde{C}^*$ as well as the following two steady state price ratios and (51):

$$\frac{\tilde{P}_H}{P_H} = \frac{\tilde{P}_H^*}{P_H^*} = 1 - \delta \Omega$$

(60)

$$\left(\frac{\tilde{P}_T}{P}\right)^{(1-\theta)} = \frac{1}{\gamma + (1-\gamma)\Omega^{(1-\theta)}},$$

(61)

yields

$$\tilde{Rev} = \frac{\gamma(1 - \delta \Omega)}{\gamma + (1 - \gamma)\Omega^{(1-\theta)}} \tilde{PC}.$$  

(62)

Recalling that we defined $\omega \equiv \frac{1-\gamma}{\gamma} \Omega^{(1-\theta)}$, we can rewrite equation (62) as

$$\tilde{Rev} = \frac{(1 - \delta \Omega)}{1 + \omega} \tilde{PC}.$$  

(63)

Combining equation (63) with equation (59), we get domestic profits in the tradables sector as a function of aggregate nominal spending:

$$\tilde{\Pi}_H = \frac{1}{\sigma(1 + \omega)} \tilde{PC}.$$  

(64)

In order to derive steady state profits in the domestic non-tradables sector, we need to recall how total demand for this type of goods is defined:

$$P_N Y_N = (1 - \gamma) \left(\frac{P_N}{P}\right)^{1-\theta} \tilde{PC} + \delta \Omega \gamma \left(\frac{P_T}{P}\right)^{1-\theta} \tilde{PC}.$$  

(65)

Using

$$\left(\frac{\tilde{P}_N}{P}\right)^{(1-\theta)} = \frac{1}{\gamma \Omega^{(\theta-1)} + (1 - \gamma)},$$

(66)

we get

$$\tilde{P}_N \tilde{Y}_N = \frac{(1 - \gamma)\Omega^{(1-\theta)} + \delta \Omega \gamma}{\gamma + (1 - \gamma)\Omega^{1-\theta}} \tilde{PC} = \frac{(\omega + \delta \Omega)}{1 + \omega} \tilde{PC}.$$  

(67)

Combining (67) with (58), we get profits in the domestic non-tradables sector as a function of steady state aggregate nominal spending:

$$\tilde{\Pi}_N = \frac{\omega + \delta \Omega}{\sigma(1 + \omega)} \tilde{PC}.$$  

(68)

Equations (63), (67), (64), and (68) display revenues and profits in terms of steady state nominal spending. Steady state labor incomes (in terms of consumption) can be easily derived by making use of these equations as well as of the fact that labor incomes represent, per definition, the difference between revenues and profits:

$$\tilde{W} L_T = \frac{(\sigma - 1)}{\sigma} \Omega \Lambda_T \tilde{PC},$$  

(69)

$$\tilde{W} \tilde{L}_N = \frac{(\sigma - 1)}{\sigma} \Lambda_N,$$  

(70)

(71)
Note that, due to our symmetry assumptions, the same relationships between profits, revenues, labor incomes and nominal spending hold for the Foreign country.

**Part 2: Matching \( \gamma \) and \( \alpha \) to observables**

We use the share of services in the steady state GDP as a basis for computing \( \gamma \). Here, the term 'services' is used as an equivalent for non-tradables goods. The service share in the domestic country \( s \) can be defined as follows:

\[
s = \frac{\bar{P}_N \bar{C}_N + \delta \bar{P}_N (\bar{C}_H + \bar{C}_F)}{\bar{P}_H (\bar{C}_H + \bar{C}_H) + \bar{P}_N \bar{C}_N + \delta \bar{P}_N (\bar{C}_H + \bar{C}_F)}. \tag{72}
\]

The following relationships hold in the steady state:

\[
\bar{C}_H = \bar{C}^*_F, \\
\bar{C}_F = \bar{C}^*_H, \\
\frac{\bar{P}_H}{\bar{P}_N} = \frac{(\sigma - 1 + \delta)}{\sigma - 1} = 1 + \frac{\delta}{\sigma - 1}.
\]

Consequently we can write (72) as follows:

\[
s = \frac{\bar{C}_N}{\bar{C}_H + \bar{C}_H} + \delta \frac{\bar{C}_N}{\sigma - 1 + \frac{\delta}{\sigma - 1}}. \tag{73}
\]

In addition note that:

\[
\frac{\bar{C}_N}{\bar{C}_H + \bar{C}_H} = \frac{(1 - \gamma)}{\gamma} \left( \frac{\bar{P}_N}{\bar{P}_T} \right)^{\theta} \left( \frac{\bar{P}_H}{\bar{P}_T} \right)^{\eta} = \frac{(1 - \gamma)}{\gamma} \Omega^{-\theta}.
\]

The service share can, thus, be written as follows:

\[
s = \frac{(1 - \gamma)}{\gamma} \Omega^{-\theta} + \delta \frac{\delta}{\sigma - 1 + \frac{\delta}{\sigma - 1}} + \frac{(1 - \gamma)}{\gamma} \Omega^{-\theta} \frac{(1 - \gamma)}{\gamma} + \delta \frac{\delta}{\sigma - 1 + \frac{\delta}{\sigma - 1}} \Omega^{-\theta}. \tag{74}
\]

Solving this expression for \( \gamma \) yields:

\[
\gamma = \frac{(1 - s)}{s(\sigma\delta) + 1 - \delta)}\Omega^{\theta} + (1 - s). \tag{75}
\]
Our choice of $a$ – the preference for domestically produced tradables – is based on the share of import expenditures in GDP ($e$). In our model, this ratio is defined as:

$$e = \frac{\bar{P}_F \bar{C}_F}{\bar{P}_H (\bar{C}_H + \bar{C}_H') + P_N \bar{C}_N + \delta P_N (\bar{C}_H + \bar{C}_F')}. \quad (76)$$

Using the steady state price ratios and relative consumption demands:

$$\frac{\bar{P}_F}{\bar{P}_N} = 1 + \frac{\sigma \delta}{\sigma - 1},$$
$$\frac{\bar{C}_H}{\bar{C}_F} = \frac{a}{1 - a},$$
$$\frac{\bar{C}_N}{\bar{C}_F'} = \frac{1 - \gamma}{\gamma} (1_a \Omega^{-\theta}),$$

yields:

$$e = \frac{1 + \frac{\sigma \delta}{\sigma - 1}}{1 - a} (1 + \frac{1 - \gamma}{\gamma} \Omega^{-\theta}). \quad (77)$$

Solving for $a$ gives the parameter as a function of the share of imports in expenditure to GDP and of $\gamma$:

$$a = 1 - e(1 + \frac{1 - \gamma}{\gamma} \Omega^{-\theta}). \quad (78)$$

**Part 3: Log-linear approximation of the second-period budget constraint ($\Delta \hat{PC}$)**

Domestic and foreign second-period budget constraints are given by equation (6) in the main text. Using equations (64) and (68) from part 1 of the appendix we can write

$$\frac{\bar{P}_H + \bar{P}_N}{PC} = \frac{1 + \delta \Omega \lambda_T}{\sigma} = \frac{\bar{P}_F + \bar{P}_N'}{SP^*C^*}.$$  

Furthermore, using equations (69) and (70) we can write:

$$\frac{\bar{W}_L + \bar{W}_L}{PC} = \frac{\sigma - 1}{\sigma} (\Omega \lambda_T + \Lambda N) = 1 - \frac{\bar{P}_H + \bar{P}_N}{PC} = \frac{\bar{S}W^*L_T + \bar{S}W^*L_N}{SP^*C^*}.$$  

This leads to the log-linear approximation of relative budget constraints as in equation (32) in the paper, denoted as $\Delta \hat{PC}$:

$$\Delta \hat{PC} = (2\phi - 1) \left( \frac{1 + \delta \Omega \lambda_T}{\sigma} \right)(\hat{P} - \hat{S} \hat{P}^*) + (1 - 1) (\bar{W}_L - \bar{S} \bar{W}^* \bar{L}). \quad (79)$$

It is convenient to split this term into two components which can be separately analyzed: the financial income part, denoted by $\hat{Y}_{Fin}$, and the labor income part, denoted by $\hat{Y}_{Fin}^{Non}$:

$$\hat{Y}_{Fin} = \frac{1 + \delta \Omega \lambda_T}{\sigma} (\bar{P} - \bar{S} \bar{P}^*) \quad (80)$$
$$\hat{Y}_{Fin}^{Non} = (1 - 1) (\bar{W}L - \bar{S} \bar{W}^* \bar{L}). \quad (81)$$
Part 4: Relating $\hat{Y}^{\text{Fin}}_F$ and $\hat{Y}^{\text{Non}}_F$ to $\hat{\tau}$, $\hat{P}_N - \hat{S} \hat{P}_N$ and relative budget constraints

Equation (79) defines relative nominal consumption as a function of relative aggregate profits and labor incomes. The goal in this section is to express $\hat{Y}^{\text{Fin}}_F$ and $\hat{Y}^{\text{Non}}_F$ as functions of the terms of trade (defined as $\tau = \frac{P^M_H}{P^M_F}$) and relative nontraded goods prices (as well as to relative nominal spending which equals relative incomes per definition). In a second step, we will then make use of the link between these relative prices and exogenous productivity shocks.

We start with $\hat{Y}^{\text{Fin}}_F$. The first step is to approximate aggregate profits. For this purpose, we again make use of results from section 1 of the appendix. First, taking into account equations (57) and (58), we can write

$$\hat{\Pi}_H = \frac{\hat{\Pi}_H}{\hat{\Pi}_H + \hat{\Pi}_N} = \frac{1}{1 + (1 - \delta \Omega)} \frac{\hat{P}_N \hat{Y}_N}{\hat{\Pi}_N}.$$  
(82)

By making use of (63) and (67) this can be rearranged:

$$\frac{\hat{\Pi}_H}{\hat{\Pi}_H} = \frac{1}{1 + \omega + \delta \Omega} = \Lambda_T^{\Pi - \Pi^*}.$$  
(83)

Due to symmetry it follows that:

$$\frac{\hat{\Pi}_N}{\hat{\Pi}_H} = 1 - \Lambda_T^{\Pi - \Pi^*} = \Lambda_N^{\Pi - \Pi^*}.$$  
(84)

Since the foreign country is characterized by the same structure, we have

$$(\hat{\Pi} - \hat{\Pi}^*) = \Lambda_T^{\Pi - \Pi^*} (\hat{\Pi}_H - \hat{\Pi}_F^*) + \Lambda_N^{\Pi - \Pi^*} (\hat{\Pi}_N - \hat{\Pi}_N^*).$$  
(85)

Now we link relative profits in the traded and nontraded sector to the terms of trade and relative nontraded goods prices. For this purpose, we start by expressing domestic profits in the traded goods sector in terms of revenues and labor costs. Using equation (49) and taking into account the definition of profits (revenues-labor costs) as well as the production technology we get

$$\hat{\Pi}_H = \sigma(1 - \delta \Omega) \hat{\Pi}^{\text{Rev}} - (\sigma - 1) \Omega \hat{W} \hat{L}_T.$$  
(86)

As the foreign country is characterized similarly, we can write

$$\hat{\Pi}_H - \hat{S} \hat{\Pi}^*_H = \sigma(1 - \delta \Omega) \left( \hat{\Pi}^{\text{Rev}} - \hat{S} \hat{\Pi}^{\text{Rev}}^* \right) - (\sigma - 1) \Omega \left( \hat{W} \hat{L}_T - \hat{S} \hat{W}^* \hat{L}_T^* \right).$$  
(87)

We proceed by relating relative revenues and relative labor costs to relative prices. First consider the total consumer demand function for the domestically produced tradable good. Log-linearizing around the steady state delivers

$$\hat{\text{Rev}} = a \hat{P}_H + (1 - a) \left( \hat{S} \hat{P}_H^* - \hat{P}_H^* \right) + a \hat{C}_H + (1 - a) \hat{C}_H^*.$$  
(88)
Taking into account that
\[ a\hat{C}_H + (1-a)\hat{C}^*_H = \hat{Y}_H, \]  
(89)
as well as equation (49) plus definition of profits (= revenues-labor costs), and the production technology we can rewrite (88) as
\[ \hat{R}ev = \left( \hat{P}_H - \hat{P}^*_{\text{MC}} \right) + (1-a) \left( S\hat{P}^*_H - \hat{P}_H \right) + \hat{W}L_T. \]  
(90)
Rearranging (90) and combining it with the analogous expression for the foreign country we get
\[ \hat{W}L_T - \hat{W}^*L^*_T = \hat{R}ev - S\hat{R}ev^* - \left( \hat{P}_H - S\hat{P}_F^* \right) + \left( \hat{P}^*_{\text{MC}} - \hat{P}^*_{H}^{*\text{MC}} \right) \]
\[ - (1-a) \left( S\hat{P}^*_H - \hat{P}_H \right) + (1-a^*) \left( \hat{P}_F - S\hat{P}_F^* \right). \]  
(91)
Now recall the following relationships between the markup price of the representative domestic traded good, its wholesale price and its retail price:
\[ \hat{P}^*_{\text{MC}} = \frac{\sigma}{\sigma - 1} \frac{W}{A_H}, \]  
(92)
\[ \hat{P}_H = \hat{P}^*_{\text{MC}} + \frac{\delta}{\sigma - 1} P_N, \]  
(93)
\[ P_N = \frac{\sigma}{\sigma - 1} \frac{W}{A_N}, \]  
(94)
\[ \hat{P}_H = \hat{P}^*_H + \frac{\delta}{\sigma - 1} P_N. \]  
(95)
Also recall that, in the steady state, we have \(\frac{W}{A_H} = \frac{W}{A_N}\) so that \(\hat{P}^*_{H}^{*\text{MC}} = \hat{P}_N\). We can thus write
\[ \hat{P}_H - S\hat{P}_F^* = \frac{\Omega}{1 - \delta \Omega} \left( \hat{P}^*_{\text{MC}} - S\hat{P}^*_F^{*\text{MC}} \right) + \left( 1 - \frac{\Omega}{1 - \delta \Omega} \right) \left( \hat{P}_N - S\hat{P}_N^* \right), \]  
(96)
\[ \hat{P}_H - S\hat{P}_H^* = \left( 1 - \frac{\Omega}{1 - \delta \Omega} \right) \left( \hat{P}_N - S\hat{P}_N^* \right), \]  
(97)
\[ \hat{P}_F - S\hat{P}_F^* = \left( 1 - \frac{\Omega}{1 - \delta \Omega} \right) \left( \hat{P}_N - S\hat{P}_N^* \right). \]  
(98)
In the paper, the terms of trade (\(\tau\)) are defined as the domestic traded good’s marginal cost price relative to the foreign traded good’s marginal cost price (denominated in a common currency). The log-linear version of \(\tau\) is thus:
\[ \hat{\tau} = \frac{\hat{P}^*_{\text{MC}}}{S\hat{P}^*_{F}^{*\text{MC}}}. \]  
(99)
Substituting (96)-(98) into (91) yields:
\[ \hat{W}L_T - \hat{W}^*L^*_T = \hat{R}ev - S\hat{R}ev^* + \left( 1 - \frac{\Omega}{1 - \delta \Omega} \right) \left[ \hat{\tau} + \lambda \left( \hat{P}_N - S\hat{P}_N^* \right) \right]. \]  
(100)
By substituting equation (100) into (87) we get
\[ \hat{\Pi}_H - \hat{\Pi}_F = \hat{R}ev - S\hat{R}ev^* - (\sigma - 1)\Omega \left( 1 - \frac{\Omega}{1 - \delta \Omega} \right) \left[ \hat{\tau} + \lambda \left( \hat{P}_N - S\hat{P}_N^* \right) \right]. \]  
(101)
What is now left to do is to relate relative revenues to $\tau$ and $\Delta P_N$ (and the nominal consumption differential). To achieve this, we log-linearize the following definition of relative revenues, while bearing in mind that $N^*(1-a^*) = 1-a$:

$$\tilde{Rev} - \tilde{SRev}^* = a\left(\tilde{P}_H + \tilde{C}_H\right) + (1-a)\left(\tilde{SP}_H^* + \tilde{C}_H^*\right)$$
$$- a^*\left(\tilde{SP}_F^* + \tilde{C}_F^*\right) + (1-a^*)\left(\tilde{P}_F + \tilde{C}_F\right). \quad (102)$$

Using the log-linearized consumer demand for domestic tradables we get:

$$a\tilde{C}_H = -a(1-a)\eta\left(\tilde{P}_H - \tilde{SP}_F^*\right) + a\tilde{C}_T, \quad (103)$$
$$\phantom{a\tilde{C}_H} = -a^*(1-a)\eta\left(\tilde{P}_F - \tilde{SP}_H^*\right) + (1-a)\tilde{C}_T^*. \quad (104)$$

The log-linear approximations of consumer demand functions for foreign tradables are analogue. Inserting the consumer demand functions into (100) and recalling that $\lambda \equiv 1 - a - a^*$ finally yields the revenue differential in the traded sector as a function of relative retail prices, relative wholesale prices, and relative consumption of tradables:

$$\tilde{Rev} - \tilde{SRev}^* = -\eta(1-\lambda^2)\left(\tilde{P}_H - \tilde{SP}_F^*\right) + \eta(1+\lambda)\left[a^*\left(\tilde{P}_H - \tilde{SP}_H^*\right) + a\left(\tilde{P}_F - \tilde{SP}_F^*\right)\right]$$
$$\phantom{\tilde{Rev} - \tilde{SRev}^* =} = -\lambda\left(\tilde{C}_T - \tilde{C}_T^*\right) + \left(\tilde{P}_H - \tilde{SP}_F^*\right) + (1-a)\left(\tilde{SP}_H^* - \tilde{P}_H\right) - (1-a^*)\left(\tilde{P}_F - \tilde{SP}_F^*\right). \quad (105)$$

However, as our ultimate goal is to link the profit differential to relative nontraded goods prices and to the terms of trade, we need to express relative retail and wholesale prices in terms of relative nontraded goods prices, terms of trade, and the nominal consumption differential. For this purpose, we first log-linearize the domestic demand for traded goods:

$$\tilde{C}_T = -\theta\tilde{P}_F + (\theta - 1)\tilde{P} + \tilde{PC}. \quad (106)$$

The domestic aggregate price index is approximated by:

$$\tilde{P} = \frac{1}{1+\omega}\tilde{P}_T + \frac{\omega}{1+\omega}\tilde{P}_N. \quad (107)$$

Finally, the traded goods price index can be approximated as follows:

$$\tilde{P}_T = a\tilde{P}_H + (1-a)\tilde{P}_F. \quad (108)$$

Substituting (107) and (108) into (106), then computing the same expression for the foreign country and substracting foreign consumption of tradables from domestic consumption yields:

$$\tilde{C}_T - \tilde{C}_T^* = \tilde{PC} - \tilde{SP}_C^* + (\theta - 1)\frac{\omega}{1+\omega}\left(\tilde{P}_N - \tilde{SP}_N^*\right) - a\frac{1+\theta\omega}{1+\omega}\left(\tilde{P}_H - \tilde{SP}_H\right)$$
$$\phantom{\tilde{C}_T - \tilde{C}_T^* =} + a^*\frac{1+\theta\omega}{1+\omega}\left(\tilde{SP}_F - \tilde{P}_F\right) + \lambda\frac{1+\theta\omega}{1+\omega}\left(\tilde{SP}_H^* - \tilde{P}_F\right). \quad (109)$$
Recalling the relationship between retail prices, wholesale prices and marginal cost prices of traded goods, we can write
\[
\hat{P}_H - \bar{S}
\hat{P}_F = \Omega \hat{\tau} - (1 - \Omega) \left( \hat{P}_N - \bar{S}
\hat{P}_N \right),
\]
(110)
\[
\hat{P}_H - \bar{S}
\hat{P}_H = (1 - \Omega) \left( \hat{P}_N - \bar{S}
\hat{P}_N \right),
\]
(111)
\[
\bar{S}
\hat{P}_F = \bar{S}
\hat{P}_F = (1 - \Omega) \left( \hat{P}_N - \bar{S}
\hat{P}_N \right).
\]
(112)
Substituting (96)-(98) and (110)-(112) into (105) yields
\[
\hat{\text{Rev}} - \hat{\text{Rev}}^* = \Omega \left[ -\eta (1 - \lambda^2) + \frac{1}{1 - \delta \Omega} - \lambda \frac{1 + \theta \omega}{1 + \omega} \right] \hat{\tau}
\]
\[
- \lambda \left[ (1 - \frac{\Omega}{1 - \delta \Omega}) + \frac{1 + \theta \omega}{1 + \omega} \right] \left( \hat{P}_N - \bar{S}
\hat{P}_N \right)
\]
\[
- \lambda \left( \frac{\theta}{1 + \omega} \right) \left( \hat{P}_N - \bar{S}
\hat{P}_N \right).
\]
(113)
Combining equation (113) with equation (87) yields the traded goods differential as a function of relative prices and relative nominal spending:
\[
\hat{\Pi}_H - \bar{S}
\hat{\Pi}_H = \Omega \left[ 1 - \eta \left( 1 - \lambda^2 \right) - \lambda \frac{1 + \theta \omega}{1 + \omega} \right] \hat{\tau}
\]
\[
- \lambda \left[ \frac{\theta}{1 + \omega} \right] \left( \hat{P}_N - \bar{S}
\hat{P}_N \right)
\]
\[
- \lambda \left( \frac{\theta}{1 + \omega} \right) \left( \hat{P}_N - \bar{S}
\hat{P}_N \right).
\]
(114)
The correspondent expression for the profit differential in the nontraded sector can be derived much easier. From (58) it follows that
\[
\hat{\Pi}_N - \bar{S}
\hat{\Pi}_N = \hat{P}_N \hat{Y}_N - \bar{S}
\hat{P}_N \hat{Y}_N.
\]
(115)
In equilibrium, the output of nontraded goods equals the demand for nontraded goods from consumers plus the demand for retail services, i.e.
\[
P_N Y_N = (1 - \gamma) \left( \frac{P_N}{P} \right)^{(1 - \theta)} PC + \delta \gamma \left( \frac{P_T}{P} \right)^{(1 - \theta)} \frac{P_N}{P_T} PC.
\]
(116)
Log-linearizing this market clearing condition and using (107),(108) as well as \( \Omega \equiv \frac{\hat{P}_N}{P_T} \) yields
\[
\hat{\Pi}_N - \bar{S}
\hat{\Pi}_N = \frac{(\omega + \delta \Omega) - \theta \omega (1 - \delta \Omega)}{(\omega + \delta \Omega) (1 + \omega)} \Omega \hat{\tau}
\]
\[
+ \frac{(\omega + \delta \Omega) - \theta \omega (1 - \delta \Omega)}{(\omega + \delta \Omega) (1 + \omega)} \Omega \left( \hat{P}_N - \bar{S}
\hat{P}_N \right)
\]
\[
+ \left( \frac{\theta}{1 + \omega} \right) \left( \hat{P}_N - \bar{S}
\hat{P}_N \right).
\]
(117)
With profits differentials for both sectors expressed as functions of the two relative prices and the consumption differential we can now relate the aggregate profit differential to these three
terms. By substituting (114) and (117) into (85) we can eventually write \( \hat{Y}_{Fin} \) as follows:

\[
\hat{Y}_{Fin} = \Lambda_T \Omega \left\{ \left( 1 - \lambda^2 \right) \left( 1 - \eta \right) + \lambda \Lambda_T \left\{ \delta \Omega (1 + \theta \omega) + \omega (1 + \lambda) (1 - \theta) \right\} \right\} \hat{r} + \frac{\Lambda_T \left\{ \delta \Omega (1 + \theta \omega) + \omega (1 + \lambda) (1 - \theta) \right\}}{\sigma} \Delta \hat{P}_N + \frac{(\Lambda_N - \Lambda_T \lambda)}{\sigma} \Delta \hat{P}_C. \tag{118}
\]

Relating \( \hat{Y}_{Fin}^{Non} \) to relative prices and the consumption differential is somewhat easier. Simply substitute (113) into (100) to get

\[
\tilde{W}_L - \tilde{W}^s L_T = \left[ 1 + \Omega \left( -\eta (1 - \lambda^2) - \lambda^2 \frac{1 + \theta \omega}{1 + \omega} \right) \right] \hat{r} - \lambda \left( \Omega - 1 \right) \frac{(1 + \theta \omega) + (\theta - 1) \omega}{1 + \omega} \left( \tilde{P}_N - \tilde{S}P^*_N \right) - \lambda \left( \tilde{P}_C - \tilde{S}P^* C^* \right). \tag{119}
\]

From (58) and \( \tilde{Y}_N = \tilde{P}_N \tilde{Y}_N - \tilde{W}_T \tilde{L}_N \) it follows that

\[
\tilde{W}_L - \tilde{SWL}_T = \tilde{P}_N \tilde{Y}_N - \tilde{S}P^*_N \tilde{Y}_N = \tilde{Y}_N - \tilde{S}Y^*_N.
\]

Hence, we can use (117) to write

\[
\tilde{W}_L - \tilde{SWL}_T^* = \frac{(\omega + \delta \Omega) - \theta \omega (1 - \delta \Omega)}{(\omega + \delta \Omega) (1 + \omega)} \Omega \hat{r} + \frac{(\omega + \delta \Omega) - \theta \omega (1 - \delta \Omega)}{(\omega + \delta \Omega) (1 + \omega)} \Omega \left( \tilde{P}_N - \tilde{S}P^*_N \right) + \left( \tilde{P}_C - \tilde{S}P^* C^* \right). \tag{120}
\]

**Part 5: The Complete-Markets Case for \( \lambda < 0 \)**

Once \( \lambda < 0 \) the "real exchange rate risk" becomes

\[
E_0 [\Delta \hat{Y}_{Fin} Q] / E_0 [\Delta \hat{Y}_{Fin}^2] = \Theta_{\hat{Q}} / \Theta_{\Delta \hat{Y}_{Fin}}.
\]

The non-diversifiable labor income hedge is

\[
E_0 [\Delta \hat{Y}_{Fin} \Delta \hat{Y}_{Fin}^{Non}] / E_0 [\Delta \hat{Y}_{Fin}^2] = \Theta_{\Delta \hat{Y}_{Fin}^{Non}} / \Theta_{\Delta \hat{Y}_{Fin}}
\]

with

\[
\Theta_{\hat{Q}} = - (1 - (\lambda + \Omega) \Lambda_T),
\]

\[
\Theta_{\Delta \hat{Y}_{Fin}} = \frac{\Lambda_T \Omega \left\{ \left( 1 - \lambda^2 \right) \left( 1 - \eta \right) + (1 - \theta) (1 - \Lambda_T) (1 + \lambda)^2 \right\}}{\sigma} - \frac{1 - \rho \left( \Lambda_T \lambda - \Lambda_N \right)}{\sigma} \Theta_{\hat{Q}} + \frac{\Lambda_T \delta \Omega^2 (1 + \lambda) (1 - (1 - \theta) (1 - \Lambda_T))}{\sigma (\sigma - 1)^{-1}},
\]

\[
\Theta_{\Delta \hat{Y}_{Fin}^{Non}} = \frac{\Lambda_T (\Omega - 1) \Omega \left\{ \left( 1 - \lambda^2 \right) \left( 1 - \eta \right) + (1 - \theta) \lambda^2 (1 - \Lambda_T) - (1 + \lambda) \right\}}{\sigma (\sigma - 1)^{-1}} + \frac{\Theta_{\Delta \hat{Y}_{Fin}}}{\sigma (\sigma - 1)^{-1}}
\[
+ \frac{\Lambda_T^2 (\Omega - 1) \Omega \left\{ \delta \Omega (1 + \theta) (1 - \delta \Omega) (1 - \Lambda_T) \right\} \lambda}{\sigma (\sigma - 1)^{-1}} - \frac{\Lambda_T (\Omega - 1 - 1 - \rho) \Theta_{\hat{Q}}}{\sigma (\sigma - 1)^{-1}}.
\]

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Under this more complex parameter constellation the conclusion we have drawn from above remain valid. Then, under complete financial markets the solution to \( \phi \) (28) is given by
\[
\phi = \frac{1}{2} \left( 1 + \frac{\frac{1}{\rho} \hat{Q} - \Theta \Delta \hat{Y}_{F_{in}}}{\Theta \Delta \hat{Y}_{F_{in}}} \right).
\]

**Part 6: The incomplete markets solution**

Up to now we have related relative \( \hat{Y}_{F_{in}} \) and \( \hat{Y}_{F_{in}}^{Non} \) to terms also affected by exogenous productivity shocks. The ultimate goal, however, is to express the endogenous terms contained in the optimal portfolio choice condition as functions of the exogenous productivity shocks as well as of the expression \( \hat{Y}_{F_{in}}^{Ex} = (2\phi - 1) \hat{Y}_{F_{in}} \), which is initially treated as exogenous. For our purpose it is convenient to rewrite the portfolio choice condition as follows:
\[
E_0 [\hat{Y}_{F_{in}} \left( \frac{1}{\rho} \hat{Q} - \Delta \hat{P}C \right)] = 0. \tag{121}
\]

with
\[
\Delta \hat{P}C = \hat{P}C - \hat{S} \hat{P}^* \hat{C}^*.
\]

We start by linking \( \hat{Y}_{F_{in}} \) to productivity shocks and to \( \hat{Y}_{F_{in}}^{Ex} \). First, let us define relative budget constraints as the sum of relative financial and non-financial income:
\[
\Delta \hat{P}C = (2\phi - 1) \hat{Y}_{F_{in}} + \hat{Y}_{F_{in}}^{Non} = \hat{Y}_{F_{in}}^{Ex} + \hat{Y}_{F_{in}}^{Non}, \tag{122}
\]

with
\[
\hat{Y}_{F_{in}} = b' \hat{\pi}, \tag{123}
\]
\[
\hat{Y}_{F_{in}}^{Non} = \frac{\sigma - 1}{\sigma} c' \hat{w}, \tag{124}
\]
\[
\hat{Y}_{F_{in}}^{Ex} = (2\phi - 1) b' \hat{\pi}, \tag{125}
\]

where
\[
\hat{\pi} = \begin{pmatrix}
\hat{\pi}_H - S \hat{\pi}_F^* \\
\hat{\pi}_N - S \hat{\pi}_N^*
\end{pmatrix},
\]
\[
\hat{w} = \begin{pmatrix}
\hat{W}L_T - SW * L^*_T \\
\hat{W}L_N - SW * L^*_N
\end{pmatrix},
\]
\[
b = \begin{pmatrix}
\frac{\Lambda_T}{\sigma} \\
\frac{\Lambda_N}{\sigma}
\end{pmatrix},
\]
\[
c = \begin{pmatrix}
\Omega \Lambda_T \\
\Lambda_N
\end{pmatrix}.
\]

Next, recall from (114) and (117) that \( \hat{\pi} \) is a function of \( \hat{\pi}, \hat{P}_N - S \hat{P}^*_N \) and \( \Delta \hat{P}C \):
\[
\hat{\pi} = D \hat{\pi} + d \Delta \hat{P}C, \tag{127}
\]

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with
\[
\hat{p} = \left( \Delta \hat{P}_N \right),
\]
and
\[
\Delta \hat{P}_N = \hat{P}_N - \hat{SP}_N^*.
\]
Relative wage incomes (\( \hat{w} \)) can also be related to these terms (see (119) and (120)):
\[
\hat{w} = E\hat{p} + e\Delta \hat{PC}.
\]
(128)
The real exchange rate, finally, can also be linked to the terms of trade and relative nontraded goods prices:
\[
\hat{Q} = q' \hat{p}.
\]
(129)
Now recall from equation (32) in the paper that the terms of trade and relative nontraded goods prices are functions of the productivity shocks, relative budget constraints, and the real exchange rate. We can, thus, express \( \hat{p} \) as follows:
\[
\hat{p} = \Phi^{-1} \left[ \rho i \Delta \hat{PC} - F \hat{A} \right].
\]
(130)
where
\[
F = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix},
\]
\[
\hat{A} = \begin{pmatrix}
\hat{A}_H \\
\hat{A}_F \\
\hat{A}_N \\
\hat{A}_N^* \\
\end{pmatrix},
\]
\[
\Phi = [I - (\rho - 1) iq'],
\]
\[
i = \begin{pmatrix}
1 \\
1 \\
0 \\
1 \\
\end{pmatrix},
\]
\[
I = \begin{pmatrix}
1 & 0 \\
0 & 1 \\
\end{pmatrix}.
\]
Substituting (130) into (127) and the result into (123) yields
\[
\hat{Y}_{Fin} = b' \left[ \rho D \Phi^{-1} i + d \right] \Delta \hat{PC} - b' D \Phi^{-1} F \hat{A}.
\]
Using (122) as well as (125) we get
\[
\hat{Y}_{Fin} = \theta_x \left( \hat{Y}_{Fin}^E + \hat{Y}_{Fin}^N \right) + \Theta_x F \hat{A}.
\]
(131)
with
\[
\theta_x = b' \left[ \rho D \Phi^{-1} i + d \right],
\]
\[
\Theta_x = -b' D \Phi^{-1}.
\]
What remains to be done is to relate $\hat{Y}_{NonFin}$ to $\hat{A}$ and $\hat{Y}_{ExFin}$. From (124) and (128) it follows that

$$\hat{Y}_{NonFin} = \frac{\sigma - 1}{\sigma} c' \left[ E\hat{p} + e\Delta \hat{PC} \right].$$

Using (130) as well as (122) and (127) this becomes

$$\hat{Y}_{NonFin} = \frac{\sigma - 1}{\sigma} c' \left( \rho E\Phi^{-1} i + e \right) \left( \hat{Y}_{ExFin} + \hat{Y}_{NonFin} \right) - \frac{\sigma - 1}{\sigma} c' E\Phi^{-1} F\hat{A},$$

which can be rearranged to

$$\xi \hat{Y}_{NonFin} = \frac{\sigma - 1}{\sigma} c' \left( \rho E\Phi^{-1} i + e \right) \hat{Y}_{ExFin} - \frac{\sigma - 1}{\sigma} c' E\Phi^{-1} F\hat{A},$$

with

$$\xi = 1 - \frac{\sigma - 1}{\sigma} c' \left( \rho E\Phi^{-1} i + e \right).$$

Dividing by $\xi$ yields

$$\hat{Y}_{NonFin} = \theta_w \hat{Y}_{ExFin} + \Theta_w F\hat{A},$$

with

$$\theta_w = \frac{\sigma - 1}{\sigma \xi} c' \left( \rho E\Phi^{-1} i + e \right),$$

$$\Theta_w = -\frac{\sigma - 1}{\sigma \xi} c' E\Phi^{-1}.$$

Substituting (133) into (131) and rearranging yields

$$\hat{Y}_{Fin} = R_1 \hat{Y}_{Fin} + R_2 F\hat{A},$$

with

$$R_1 = \theta \left( 1 + \theta_w \right),$$

$$R_2 = \theta \Theta_w + \Theta_\pi.$$

In order to link the second term in the portfolio choice equation, here termed $D$, to $\hat{A}$ and $\hat{Y}_{ExFin}$ we use (129) and (130) to derive

$$D = -\Delta \hat{PC} + \frac{1 - \rho}{\rho} q'\Phi^{-1} \left( \rho \hat{\Delta} \hat{PC} - F\hat{A} \right).$$

This can be rearranged by using (122) and (125):

$$D = \left[ -1 + (1 - \rho) q'\Phi^{-1} i \right] \left( \hat{Y}_{ExFin} + \hat{Y}_{NonFin} \right) - \frac{1 - \rho}{\rho} q'\Phi^{-1} F\hat{A}. \quad (135)$$

When substituting (133) into (135) we get

$$D = \left[ -1 + (1 - \rho) q'\Phi^{-1} i \right] \left( \hat{Y}_{Fin} + \theta_w \hat{Y}_{ExFin} \Theta_w F\hat{A} \right) - \frac{1 - \rho}{\rho} q'\Phi^{-1} F\hat{A}. \quad (136)$$

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Rearranging this equation finally yields

$$D = D_1 \dot{Y}_{in}^E + D_2 F \hat{A},$$  \hspace{1cm} (137)

with

$$D_1 = \left[ -1 + (1 - \rho) q' \Phi^{-1} i \right] (1 + \theta_w),$$

$$D_2 = \left[ -1 + (1 - \rho) q' \Phi^{-1} i \right] \Theta_w - \frac{1 - \rho}{\rho} q' \Phi^{-1}.$$