Compulsory Voting and Public Finance

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Abstract

Conventional wisdom suggests that compulsory voting lowers the influence of special-interest groups and leads to policies that are better for less privileged citizens, who often abstain when voting is voluntary. To scrutinize this conventional wisdom, I study public goods provision and rents to special-interest groups in a probabilistic voting model with campaign contributions in which citizens can decide how much political information to acquire, and whether to vote or abstain. I find that compulsory voting, modeled as an increase in abstention costs, raises the share of poorly informed and impressionable voters, thereby making special-interest groups more influential and increasing their rents. Total government spending and taxes increase as well, while the effect on public goods provision is ambiguous. Compulsory voting may thus lead to policy changes that harm even less privileged citizens.

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Keywords: Compulsory Voting, Special-Interest Politics, Fiscal Policies.

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1 Introduction

It is well known that voluntary voting leads to unequal turnout as rich and well educated citizens are more likely to participate in elections than their less privileged compatriots.\(^1\) Lijphart (1997) and many others worry that this unequal turnout translates into fiscal policies that are biased towards privileged citizens; and they argue that compulsory voting could solve or, at least, lessen the problems of unequal turnout and biased policies. They also reckon that compulsory voting may lower the influence of special-interest groups, thereby “reducing the role of money in politics” (Lijphart, 1997, p. 10). Hence conventional wisdom can still be summarized by Gosnell’s notion (1930, p. 185) that elections would be “less costly, more honest, and more representative” with compulsory voting.

In this paper I scrutinize the conventional wisdom. I base my analysis on a probabilistic voting model with campaign contributions similar to the models of Baron (1994), Grossman and Helpman (1996, 2001), and Persson and Tabellini (2000). In this model political candidates can choose their policy platform, which consists of public goods provision and rent payments to lobby groups. These groups can make campaign contributions to political candidates. Informed voters base their decision primarily on policy platforms, while uninformed or impressionable citizens base their decision primarily on political advertisements paid for by campaign contributions. Unlike in existing voting models with campaign contributions, in my model citizens can decide how much political information to acquire, and whether or not to participate in the election. I assume that the costs of acquiring political information are lower for citizens with good education and high incomes. Further, citizens also have to bear costs when voting or abstaining, respectively; and I follow Matsusaka (1995) in assuming that the citizens’ benefit from voting are the higher, the more confident they are of their vote choice.

\(^1\)Tingsten (1937, p. 155) was one of the first to provide systematic evidence that “the voting frequency rises with rising social standard.” Lijphart (1997) reviews many studies that document unequal turnout.
In this model citizens with good education and high incomes are more likely to take informed decisions when voting, and they are also more likely to participate in the election. New compulsory voting laws or stricter enforcement of such laws increase abstention costs. Thereby they increase electoral participation as well as the share of impressionable voters whose vote choice depends on campaign contributions rather than policy platforms. As a consequence of this latter change, candidates raise tax rates and total government spending to increase rent payments to lobby groups.\(^2\) The effect of higher abstention costs on public goods provision is ambiguous in general, and negative with Cobb-Douglas preferences. These changes in fiscal policies harm citizens with high incomes, and possibly also less privileged citizens. In addition, all citizens who did not vote before the increase in abstention costs suffer from these higher costs. Therefore, in contrast to what conventional wisdom suggests, my model shows that compulsory voting benefits special-interest groups, but may well harm all other citizens in society.

In my model new technologies that reduce voting costs, such as internet voting, have the same effect on fiscal policies as higher abstention costs.\(^3\) Hence they also lead to higher rents and higher taxes, with the effect on public goods provision being ambiguous. But at least they have the advantage of lowering the voters' costs on election day.

This paper contributes to three different strands of the political economy literature. First, it builds on the contributions of Baron (1994), Grossman and Helpman (1996, 2001), and Persson and Tabellini (2000) on the role of campaign contributions in elections. Because of its focus on fiscal policies, my model is probably closest to Persson and Tabellini (2000). The main differences to all these contributions are that I deviate from the assumption of full (or random) voting participation, and that I do not take the share of informed

\(^2\)This result is consistent with the finding of Wegenast (2010) that interest groups are less influential in US states with highly educated and well informed citizens.

\(^3\)Internet voting trials have been conducted in various countries, including France, the Netherlands, Switzerland, the United Kingdom, and the United States. In Estonia all voters could use Internet voting in the national election in 2007 (Alvarez et al., 2009).
voters as exogenous. This allows me to show that compulsory voting makes campaign contributions more important and, consequently, special-interest groups more powerful.\footnote{Strömberg (2004) endogenizes the share of informed voters in a probabilistic voting model with profit-maximizing media. Other recent contributions building on Baron (1994), Grossman and Helpman (1996, 2001), and Persson and Tabellini (2000) provide a micro-foundation for the effect of political advertisement on voting decisions of imperfectly informed voters. In Prat (2002a, 2002b) political ads are non-informative, but the amount spent on political ads serves as a signal of the candidates’ quality. In Coate (2004a, 2004b) political ads are directly informative and the probability that the voters understand the information increases in the amount spent on political ads. As I focus on the effects of compulsory voting on fiscal policies rather than on why and how political ads work, I build my model directly on Baron (1994), Grossman and Helpman (1996, 2001), and Persson and Tabellini (2000).}

Second, my paper contributes to the literature on the advantages and disadvantages of compulsory voting. So far, there have been surprisingly few theoretical contributions to this literature. Crain and Leonard (1993) consider the effect of compulsory voting on government spending in two distinct political economy models. In line with conventional wisdom they hypothesize that compulsory voting would lead to higher public goods provision in a median voting model in which public goods provision is the only type of public spending, and to less rents to special-interest groups in pressure groups theories of government. I improve upon Crain and Leonard (1993) by studying the effects of compulsory voting on public goods provision and rents in a formal and unified model. When looking separately at public goods provision and rents, my model also suggests that compulsory voting raises public goods provision in the absence of rents, and it is straightforward to show that compulsory voting raises rents in the absence of public goods.\footnote{When abstracting from rents to special-interest groups (or taking them as exogenous), my model further relates to Larcinese (2005) and Lind and Rohner (2008). They find that public spending is biased towards the rich because the poor are politically less informed. Uninformed citizens decide to abstain from voting in Larcinse (2005), and they make more voting mistakes in Lind and Rohner (2008). In my model uninformed citizens are both more likely to abstain and to make voting mistakes.} More importantly, my model shows that when studying public goods provision and rents in a unified framework, compulsory voting increases rents to special-interest groups while its effect on public goods provision is ambiguous.

Börgers (2004), and Krasa and Polborn (2009) compare welfare under compulsory and voluntary voting in costly voting models in which voters only benefit from voting if they
are pivotal. These models focus on the voters’ participation decision and their choice between two fixed alternatives, thereby abstracting from the way candidates choose their policy platforms and the role of special-interest groups, which are both at the heart of my paper.

Third, this paper contributes to the literature on the effects of constitutions and electoral rules on fiscal policies. Persson et al. (2000) and Persson and Tabellini (2003, 2004) focus primarily on the effects of presidential versus parliamentary forms of government, and proportional versus majoritarian electoral rules. I study an additional set of important electoral rules, namely compulsory versus voluntary voting.

The remainder of the paper is structured as follows: Section 2 first presents and then discusses the setup of my model. Section 3 derives the equilibrium and discusses the effects of changes in abstention and voting costs on fiscal policies and the citizens’ welfare. Section 4 concludes. All proofs are in the Appendix.

2 The Model

There are two candidates, a lobby group, and a measure-one continuum of citizens. Each candidate \( P \in \{A, B\} \) is office-motivated and chooses his policy platform to maximize his winning probability \( p_P \), where \( p_A + p_B = 1 \). Platforms consist of public goods provision \( g_P \geq 0 \) and rent payments to the lobby group \( r_P \geq 0 \). These two components of government spending are financed with a linear income tax, and the government budget must be balanced. Hence \( g_P \) and \( r_P \) determine the tax rate \( \tau_P = \frac{g_P + r_P}{y} \), where \( y \) denotes average income. Candidates may differ in their policy platforms \( (g_P, r_P) \) as well as in some predetermined, i.e., exogenous, positions.

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Börgers (2004) mentions that such models are best suited to study elections with small electorates as the probability that a particular voter is pivotal is close to zero in large electorates, e.g., in national elections.
The lobby group can make campaign contributions $C_A \geq 0$ and $C_B \geq 0$ to candidates $A$ and $B$ at increasing marginal costs, and it receives rents $r_P$ from the elected candidate $P$. Its utility is $\Pi(r_P, C_A, C_B) = J(r_P) - \frac{(C_A + C_B)^2}{2}$, where $J'(r_P) > 0$ and $J''(r_P) < 0$.

Citizens differ in their skills $\alpha^i$, which may represent educational attainments or innate abilities. The distribution of $\alpha^i$ is given by $F(\alpha^i)$, with continuous density $f(\alpha^i)$ and mean $\alpha$. For simplicity I assume $F(0) = 0$, $F(1) = 1$, and $f(\alpha^i) > 0$ for all $\alpha_i \in [0, 1]$. Skills $\alpha^i$ have two effects: First they determine citizen $i$’s income $y^i = \alpha^i$. Second they determine how costly it is for citizen $i$ to acquire political knowledge $q_i \in [0, 1]$. I assume that a citizen’s political knowledge $q_i$ measures the probability that she is informed rather than impressionable, thus understanding the candidates’ platforms $(g_P, r_P)$ and their predetermined positions.

If candidate $P$ is elected, the utility of citizen $i$ is

$$W_{i,P} = W(g_P, r_P, \alpha^i, \sigma^i_P, p_i) = U(c^i_P) + H(g_P) + \sigma^i_P + I_i(\beta q_i - \gamma) - (1 - I_i)\delta - \frac{q_i^2}{2\alpha^i}. \quad (1)$$

The first two terms on the right-hand side reflect citizen $i$’s utility from private consumption $c^i_P = (1 - \tau_P)\alpha^i$ and public goods provision $g_P$, respectively. I assume $U'(c^i_P) > 0$, $U''(c^i_P) \leq 0$, $R_R(c^i_P) \equiv -\frac{c^i_P U''(c^i_P)}{U'(c^i_P)} < 1$, $H'(g_P) > 0$, and $H''(g_P) < 0$. The third term, $\sigma^i_P$, represents her utility from the predetermined positions of the elected candidate $P$. I assume that $\sigma^i = \sigma^i_H - \sigma^i_A$ is uniformly distributed in $[-\frac{1}{2\sigma}, \frac{1}{2\sigma}]$.

The fourth term captures benefits and costs associated with voting. $I_i$ is a dummy variable whose value is 1 if citizen $i$ participates in the election, and 0 if she abstains. Some benefits from voting may well depend on the voter’s political knowledge, like the satisfaction of being confident to have voted in one’s own interest (Matsusaka, 1995). These benefits are $\beta q_i$. For simplicity I set $\beta = 1$. The costs of completing and casting one’s ballot are denoted by $\gamma$. These voting costs are relatively high when ballots must be
cast at a polling station, but they may decrease if postal voting or even Internet voting is introduced. Abstaining from the polls can also be costly: citizens may feel bad when violating social norms and not fulfilling what might be perceived as a civic duty. The costs from abstaining further increase when compulsory voting laws make voting a legal duty, and when abstention may lead to a fine or a request to explain the failure to vote (as in Australia). The fifth term, \((1 - I_i)\delta\), captures these various abstention costs. In our model, new compulsory voting laws or stricter enforcement of such laws are thus represented by an increase in \(\delta\).\(^7\) The last term captures the costs of acquiring political knowledge \(q_i\), which are decreasing in skills \(\alpha_i\).

Timing is as follows: First, the candidates choose their policy platforms \((g_P, r_P)\). Second, the lobby group can make campaign contributions. Third, elections take place. The elected candidate then implements the announced platform.

It remains to describe the voters’ decisions.\(^8\) Informed voters vote for candidate \(A\) if \(W_{i,A} \geq W_{i,B}\), and for candidate \(B\) otherwise. The electoral decisions of impressionable voters are driven by political advertisements and policy irrelevant candidate characteristics. The share of impressionable voters who vote for candidate \(A\) is \(\frac{1}{2} + \psi(\Delta C - \eta)\), where \(\Delta C \equiv C_A - C_B\).\(^9\) The remaining impressionable voters vote for candidate \(B\). Note that \(\psi > 0\) measures the effectiveness of advertisements and, therefore, campaign contributions; and \(\eta\) is a popularity shock that is uniformly distributed in \([-\frac{1}{2\lambda}, \frac{1}{2\lambda}]\).

The appropriate solution concept for this sequential game is subgame perfect Nash equilibrium.

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\(^7\) As shown later, an increase in \(\gamma\) and a decrease in \(\delta\) have the same effects on the citizens’ decisions and equilibrium fiscal policies, but different effects on the citizens’ welfare.

\(^8\) I use the term “voters” to refer to citizens who participate in the election.

\(^9\) I could, e.g., assume that impressionable voter \(i\) votes for \(A\) if and only if \(\Delta C > \varepsilon^i + \eta\), with \(\varepsilon^i\) being uniformly distributed in \([-\frac{1}{2\psi}, \frac{1}{2\psi}]\).
2.1 Discussion

I now discuss some of the assumptions made. Utility function (1) implies that the citizens’ utility from private consumption $c^i_P$ and public goods provision $g_P$ is additively separable. The model could be solved with more general utility functions, but assuming additive separability simplifies the analysis, and still allows for popular specifications such as the quasi-linear preferences used by Persson and Tabellini (2000). What I need and want, however, is for any given $r_P$ a negative relationship between a citizen’s skills $\alpha^i$ and the public goods provision $g^i_P$ that maximizes her utility. In my setting this relationship is strictly negative if and only if $R_R(c^i_P) < 1$.\(^{10}\)

Utility function (1) further implies that political knowledge $q_i$ benefits voters because they value the confidence of having voted in their own interest, as suggested by Matsusaka (1995). I could get very similar results if political knowledge had some direct consumption value (like knowledge about sports), or if it had a positive effect on expected (future) income as in Larcinese (2005).\(^{11}\) In my model higher skilled voters will optimally acquire more political knowledge because of lower information acquisition costs, which is consistent with empirical evidence that voters with better education and higher incomes are better informed (e.g., Lind and Rohner, 2008). Again, other mechanism ensuring that higher skilled citizens acquire more political knowledge would serve my purpose equally well. In Larcinese (2005), for example, the effect of political knowledge on expected (future) income increases in the citizens’ skills and income. Similarly, my results also do not depend on the perfect correlation between incomes and political knowledge. A positive correlation is however necessary.

\(^{10}\)To see this, observe that the first-order condition $-U'(c^i_P) \frac{\partial c^i_P}{\partial \tau^i_P} \frac{\partial g^i_P}{\partial g_P} + H'(g_P) = 0$, where $\frac{\partial c^i_P}{\partial \tau^i_P} = -\alpha^i$ and $\frac{\partial g^i_P}{\partial g_P} = \frac{1}{\alpha}$, determines $g^i_P$ for any given $r_P$. Using the implicit function theorem, it can be shown that $\frac{\partial g^i_P}{\partial \alpha^i} = -\frac{U'(c^i_P) \frac{\partial c^i_P}{\partial \tau^i_P} R_R(c^i_P) - 1}{\alpha H'(g_P)}$, which is strictly negative if and only if $R_R(c^i_P) < 1$.

\(^{11}\)In general, citizens also benefit from political knowledge if they are pivotal with non-zero probability. However, in my model where there is a continuum of voters this probability is always zero.
Voting is probabilistic in my model, such that small changes in policy platforms \((g_P, r_P)\) only lead to small changes in the candidates’ winning probabilities \(p_P\). Following Grossman and Helpman (1996, 2001) and Persson and Tabellini (2000), I model probabilistic voting by assuming that candidates differ in predetermined positions or some other exogenous characteristics, and voters in their evaluation \(\sigma^i\) of these positions; and that a popularity shock \(\eta\) affects all (impressionable) voters.\(^{12}\) I further follow Persson and Tabellini (2000) in assuming that \(\sigma^i\) and \(\eta\) are uniformly distributed with mean zero to get simple and tractable functional forms of the candidates’ winning probabilities.

To capture lobbying in a simple way, I assume that there is only one lobby group, that this lobby group cannot vote (or has measure zero), and that citizens do not benefit from rents \(r_P\). However I could derive similar results in a setting in which a non-negligible share of the citizens belong to lobby groups, and in which all these citizens benefit from rents and can decide whether or not to participate in the election.

3 Equilibrium Analysis

In this section I first derive the decisions of the citizens and the lobby group, which yield the candidates’ objective function. I then study how changes in abstention and voting costs affect the candidates’ policy platforms in two simplified versions of my model – one with exogenous rents, and one with an exogenous tax rate. Finally I look at the complete model introduced above, and I discuss how changes in abstention and voting costs affect the equilibrium policy platforms as well as the welfare of the citizens and the lobby group.

\(^{12}\) Results are virtually the same when \(\eta\) affects the decision of all voters as when it only affects the decision of impressionable voters.


\section*{3.1 Decisions of citizens and lobby group}

I start by looking at the citizens’ decisions of how much political knowledge \( q_i \) to acquire, and whether or not to participate in the election. For citizens who abstain from voting acquiring political knowledge has no benefits. Hence they choose \( q_i = 0 \). Citizens who participate in the election choose \( q_i = \alpha_i \) to maximize \( q_i - \frac{q_i^2}{2\alpha_i} \). Hence they choose \( q_i = \alpha_i \). Citizens therefore acquire political knowledge \( q_i = \alpha_i \) and participate in the election if \( \alpha_i - \gamma - \frac{\alpha_i}{2} \geq -\delta \), i.e., if \( \alpha_i \geq \hat{\alpha} \equiv 2(\gamma - \delta) \), but they acquire no political knowledge and abstain from voting otherwise.\(^{13}\) The election participation threshold \( \hat{\alpha} \) directly determines voter turnout, which is \( 1 - F(\hat{\alpha}) \). Note that voting costs \( \gamma \) and abstention costs \( \delta \) will affect equilibrium policy platforms exclusively through their effects on \( \hat{\alpha} \). For simplicity I subsequently focus on parameter constellation that satisfy \( (\gamma - \delta) \in (0, \frac{1}{2}) \), such that marginal changes in \( \gamma \) and \( \delta \) have an effect on equilibrium policy platforms and voter turnout.

I next derive the expected election outcome as a function of the candidates’ platforms \((g_A, r_A)\) and \((g_B, r_B)\), and the campaign contributions \( C_A \) and \( C_B \). Informed voters vote for candidates \( A \) if \( \Delta V(\alpha^i) \equiv U(c^i_A) - U(c^i_B) + H(g_A) - H(g_B) > \sigma^i \), and for \( B \) otherwise. Among informed voters with skills \( \alpha^i \geq \hat{\alpha} \), the share voting for \( A \) is therefore \( \frac{1}{2} + \phi \Delta V(\alpha^i) \).\(^{14}\) By assumption, the share of impressionable voters voting for \( A \) is \( \frac{1}{2} + \psi(\Delta C - \eta) \) for any \( \alpha^i \geq \hat{\alpha} \). As the share of voters with skills \( \alpha_i \geq \hat{\alpha} \) who is informed equals \( q_i = \alpha_i \), the population share who votes for \( A \) thus adds up to \( \pi_A = \int_{\hat{\alpha}}^{1} \left[ \frac{1}{2} + \alpha_i \phi \Delta V(\alpha^i) + (1 - \alpha^i)\psi(\Delta C - \eta) \right] f(\alpha^i)d\alpha^i \), and the population share who votes for \( B \) to \( \pi_B = 1 - F(\hat{\alpha}) - \pi_A \). Candidate \( A \) therefore wins if and only if

\(^{13}\)As a tie-breaking rule I assume that citizens who are indifferent participate in the election.

\(^{14}\)More generally, this share is \( \min\{\max\{0, \frac{1}{2} + \phi \Delta V(\alpha^i)\}, 1\} \), but for simplicity I assume that it is always strictly between zero and one. I make similar (implicit) assumptions for all vote shares and winning probabilities below.
\[
\int_\alpha^1 [\alpha^i \phi \Delta V(\alpha^i) + (1 - \alpha^i) \psi(\Delta C - \eta)] f(\alpha^i) d\alpha^i \geq 0.
\]
Hence his winning probability is
\[
p_A = \text{prob} \left\{ \eta \leq \frac{\phi \int_\alpha^1 \alpha^i \Delta V(\alpha^i) f(\alpha^i) d\alpha^i}{\psi \int_\alpha^1 (1 - \alpha^i) f(\alpha^i) d\alpha^i} + \Delta C \right\} = \frac{1}{2} + \frac{\lambda \phi \int_\alpha^1 \alpha^i \Delta V(\alpha^i) f(\alpha^i) d\alpha^i}{\psi \int_\alpha^1 (1 - \alpha^i) f(\alpha^i) d\alpha^i} + \lambda \Delta C.
\]  
(2)

I now turn to the lobby group’s decision. The lobby group chooses campaign contributions \( C_A \) and \( C_B \) to maximize its expected utility
\[
p_A J(r_A) + (1 - p_A) J(r_B) - \frac{1}{2} (C_A + C_B)^2,
\]
thereby anticipating the effects of \( C_A \) and \( C_B \) on \( p_A \). The lobby group supports no candidate if rents \( r_A \) and \( r_B \) coincide, and the candidate promising more generous rents otherwise. It is easy to see that the lobby group chooses
\[
C_A = \max\{0, \lambda [J(r_A) - J(r_B)]\}
\]
and
\[
C_B = \max\{0, \lambda [J(r_B) - J(r_A)]\},
\]
such that
\[
\Delta C = \lambda [J(r_A) - J(r_B)].
\]
Inserting this expression for \( \Delta C \) into equation (2) leads to
\[
p_A = \frac{1}{2} + \frac{\lambda \phi \int_\alpha^1 \alpha^i \Delta V(\alpha^i) f(\alpha^i) d\alpha^i}{\psi \int_\alpha^1 (1 - \alpha^i) f(\alpha^i) d\alpha^i} + \lambda^2 [J(r_A) - J(r_B)].
\]  
(3)

Candidate \( A \) anticipates the behavior of the lobby group and the citizens, and chooses his fiscal policy platform \((g_A, r_A)\) to maximize his winning probability \( p_A \). Candidate \( B \) chooses \((g_B, r_B)\) to maximize \( p_B = 1 - p_A \). It follows from equation (3) and the definition of \( \Delta V(\alpha^i) \) that each candidate’s optimal platform is independent of his opponent’s platform, and that each candidate’s maximization problem can be written as
\[
\max_{g_P, r_P} \int_\alpha^1 \left[ \alpha^i U(c_P^i) + \alpha^i H(g_P) + (1 - \alpha^i) \Omega J(r_P) \right] f(\alpha^i) d\alpha^i
\]  
subject to \( g_P \geq 0, r_P \geq 0 \) and \( \tau_P = \frac{g_P + r_P}{\alpha} \leq 1 \), where \( \Omega \equiv \frac{\phi \lambda}{\psi} \). I assume throughout that the solution to this problem is interior. As it is standard in this type of lobbying models, the two candidates’ platforms therefore coincide in equilibrium, such that the lobby group makes no campaign contributions even though the candidates offer rents \( r_P > 0 \).
3.2 Policy platforms when rents are exogenous (or absent)

I now look at a simplified version of my model in which rents $r_P$ are exogenous and equal to $\bar{r} \in [0, \alpha)$. This simplified version includes the special case in which there are no rents and no lobbying.\textsuperscript{15} Hence it may be close to the model that some of the proponents of compulsory voting have in mind, and it indeed helps to understand why compulsory voting could potentially benefit citizens with low incomes.

In this simplified version of the model the two endogenous fiscal policy variables, $g_P$ and $\tau_P$, are tied together by the government budget constraint. Hence candidates have effectively only one choice, which I take to be $g_P$, and the maximization problem (4) reduces to

$$\max_{g_P} \int_{\hat{\alpha}}^{1} \alpha^i \left[ U(c^i_P) + H(g_P) \right] f(\alpha^i)d\alpha^i$$  \hspace{1cm} (5)

with $c^i_P = (1 - \tau_P )\alpha^i$ and $\tau_P = \frac{g_P + \bar{r}}{\alpha}$. It follows:

**Proposition 1** Assume $r_P = \bar{r}$. Then public goods provision $g_P$ and the tax rate $\tau_P$ increase in $\delta$ and decrease in $\gamma$.

The intuition for these results is as follows. Higher abstention costs $\delta$ and lower voting costs $\gamma$ both lower the election participation threshold $\hat{\alpha}$, thereby increasing voter turnout and lowering the average voter’s income as well as the average informed voter’s income. Since voters with lower incomes prefer higher public goods provision $g_P$ (because $R_R(c^i_P) < 1$), the candidates respond to the lower income of the average informed voter by increasing $g_P$. Interestingly, however, even if $\hat{\alpha} \to 0$, policy platforms remain biased towards citizens with high incomes, with $g_P$ and $\tau_P$ still being relatively low. The reason is that candidates only care about informed voters, and that the share of informed voters remains higher among citizens with high incomes.

\textsuperscript{15}Results are identical when assuming $r_P = 0$ as when assuming $\Omega = 0$. In the later case each candidate would choose $r_P = 0$, as rents have no effect on his winning probability $p_P$. 

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I now briefly turn to the welfare effects of changes in $\gamma$ and $\delta$. The higher $g_P$ and $\tau_P$, which follow from an increase in $\delta$ or a decrease in $\gamma$, make citizens with low incomes better off and citizens with high incomes worse off. Further, a decrease in $\gamma$ is welfare improving for all citizens who now decide to vote (no matter whether or not they would have voted in the absence of this decrease), while an increase in $\delta$ is costly for all voters who would have abstained otherwise (no matter whether they now vote or abstain). Loosely speaking, compulsory voting therefore harms the rich who suffer from higher taxes $\tau_P$, while the welfare effect on the poor is ambiguous as they benefit from higher public goods provision $g_P$, but suffer from higher costs on election day, which may include the costs for acquiring political knowledge.

3.3 Policy platforms when the tax rate is exogenous

I now look at a simplified version of my model in which the tax rate $\tau_P$ is exogenous and equal to $\tau \in (0, 1]$. This simplified version may reflect the situation in countries in which governments are substantially less constrained in how they allocate public spending than in the amount they can spend. In addition, it nicely illustrates the main mechanism by which compulsory voting can lead to policy changes that make all citizens worse off.

When $\tau_P$ is exogenous, the two endogenous fiscal policy variables, $g_P$ and $r_P$, are again tied together by the government budget constraint. Hence the candidates face a simple trade-off between high public goods provision $g_P$ and high rents $r_P$. From their perspective, public goods are useful to increase electoral support from informed voters, while rents are useful to increase campaign contributions and, thereby, the electoral support from impressionable voters. The maximization problem (4) reduces to

$$
\max_{g_P} \int_0^1 \left[ \alpha^i H(g_P) + (1 - \alpha^i) \Omega J(r_P) \right] f(\alpha^i) d\alpha^i
$$
with $r_P = \tau \alpha - g_P$. It follows:

**Proposition 2** Assume $\tau_P = \overline{\tau}$. Then public goods provision $g_P$ increases in $\gamma$, but decreases in $\delta$ and $\Omega$, while rents $r_P$ increase in $\delta$ and $\Omega$, but decrease in $\gamma$.

To understand these results note that for a given tax rate, all citizens have the same policy preferences: they want public goods provision $g_P$ to be as high as possible. Hence incentivizing more citizens to go to the polls, e.g., by lowering voting costs $\gamma$ or raising abstention costs $\delta$, would have no effect on equilibrium policies if the new voters were equally well informed as those who participated anyway. However these new voters are less skilled and, therefore, acquire less political knowledge even when they participate in the election. As a consequence the average voter’s political knowledge decreases. The candidates optimally respond by increasing rents $r_P$ and lowering public goods provision $g_P$, as rents serve to win votes from impressionable voters while public goods serve to win votes from informed voters. Not surprisingly, rents $r_P$ also increase in $\Omega$, which measures how sensitive the electoral support from impressionable voters is to changes in campaign contributions relative to how sensitive the electoral support from informed voters is to changes in policy platforms.

Hence, when the tax rate is exogenous, lower voting costs $\gamma$ and higher abstention costs $\delta$ lead to policy changes that benefit the lobby group at the expense of all citizens. An increase in $\delta$ further harms all those citizens who would have abstained in the absence of such an increase, while lowering $\gamma$ makes at least all voters better off. Ironically, compulsory voting therefore harms its supposed beneficiaries, the poor, in multiple ways: it leads to lower public goods provision as well as to higher costs on election day.
3.4 Equilibrium policy platform

In this section we derive the equilibrium of the complete model introduced in section 2 in which public goods provision $g_P$, rents $r_P$ and the tax rate $\tau_P = \frac{g_P + r_P}{\hat{\alpha}}$ are all endogenous. We know that in this case the candidates’ maximization problem is given by (4).

I discuss the effects of voting and abstention costs on the three fiscal policy variables in turn, starting with their effects on the tax rate $\tau_P$, which is proportional to the size of government $g_P + r_P$:

**Proposition 3** The tax rate $\tau_P$ and the size of government $g_P + r_P$ increase in $\delta$ and $\Omega$, but decrease in $\gamma$.

Higher abstention costs $\delta$ and lower voting costs $\gamma$ both lower the election participation threshold $\hat{\alpha}$. There are two reasons why a lower $\hat{\alpha}$ leads to a higher tax rate $\tau_P$. First, as seen in section 3.2, for any given $r_P$, a decrease in $\hat{\alpha}$ and the associated decrease in the average informed voter’s income make it optimal for the candidates to choose a higher tax rate $\tau_P$. This puts some upward pressure on $\tau_P$. Second, a decrease in $\hat{\alpha}$ reduces the share of informed voters among the voting population, because less skilled voters acquire less political knowledge. A higher tax rate $\tau_P$ has the advantage that it allows to increase $g_P$ or $r_P$ and, thereby, to raise electoral support from informed or impressionable voters, respectively. But a higher $\tau_P$ has the disadvantage that it lowers private consumption $c_i^P$ of all citizens. This, however, only reduces the electoral support from informed voters. Hence when the share of informed voters decreases, the candidates become less concerned about the disadvantage of high taxes, while the advantage of high taxes remains similarly attractive. This puts additional upwards pressure on $\tau_P$. Furthermore, the candidates choose a higher tax rate $\tau_P$ when the support from impressionable voters becomes relatively more sensitive to campaign contributions, i.e., when $\Omega$ increases.

I now turn to the effects of voting and abstention costs on the rents $r_P$ paid to the
lobby group:

**Proposition 4** Rents $r_P$ increase in $\delta$ and $\Omega$, but decrease in $\gamma$.

Some previous results are helpful to understand Proposition 4. We know from Proposition 2 that a decrease in $\hat{\alpha}$ and the associated increase in the share of impressionable voters increases rents $r_P$ relative to public goods provision $g_P$ for any $\tau_P$; and from Proposition 3 that a decrease in $\hat{\alpha}$ increases the tax rate $\tau_P$. Hence higher abstention costs $\delta$ and lower voting costs $\gamma$ lead to more generous rents $r_P$, because a higher share of impressionable voters tilts both the size and the composition of public spending to the lobby group’s benefit. Proposition 4 further shows that rents $r_P$ increase in $\Omega$, i.e., when the support from impressionable voters becomes relatively more sensitive to campaign contributions.

I next discuss how voting and abstention costs affect public goods provisions $g_P$. There are two countervailing effects: First, candidates would like to choose higher $g_P$ when the voting participation threshold $\hat{\alpha}$ decreases, because the average informed voter then earns a lower income and, therefore, prefers higher $g_P$ for given $r_P$ (as seen in Proposition 1). Second, candidates would like to choose lower $g_P$ when $\hat{\alpha}$ decreases, because informed voters also care about low tax rates $\tau_P$, with the marginal utility of $\tau_P$ being negative and decreasing, and because the decrease in $\hat{\alpha}$ already puts upwards pressure on $\tau_P$ by increasing rents $r_P$ (as seen in Proposition 4). Any of these two effects may dominate in general. But for some specific utility function the net effect is unambiguous:

**Proposition 5** Public goods provision $g_P$ decreases in $\Omega$. The effects of $\gamma$ and $\delta$ on $g_P$ are ambiguous in general, but it holds:

(i) Assume $U(c^i_P) = \chi c^i_P$ with $\chi > 0$. Then $g_P$ increases in $\delta$ and decreases in $\gamma$.

(ii) Assume $R_R(c^i_P) = \theta$ with $\theta \to 1$ (or $\theta = 1$). Then $g_P$ increases in $\gamma$ and decreases in $\delta$. 

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Assumption (i) in Proposition 5 leads to quasi-linear preferences over $c^i_P$ and $g_P$ as in Persson and Tabellini (2000). With these preferences, the marginal effect of an increase in $\tau_P$ on $U(c^i_P)$ becomes independent of the levels of $c^i_P$ and $\tau_P$. Hence the second of the countervailing effects discussed above disappears, and the candidates choose higher $g_P$ when $\hat{\alpha}$ decreases due to an increase in $\delta$ or a decrease in $\gamma$.

Assumption (ii) in Proposition 5 ensures that the differences between the preferred public goods provision $g^i_P$ of citizens with different incomes converge towards zero. In this case the first of the countervailing effects discussed above becomes negligible, and the candidates choose lower $g_P$ when $\hat{\alpha}$ decreases due to an increase in $\delta$ or a decrease in $\gamma$. The same also holds true when $R_R(c^i_P) = 1$, as is the case with Cobb-Douglas preferences in log form over $c^i_P$ and $g_P$.

Proposition 5 further shows that the candidates choose lower public goods provision $g_P$ when $\Omega$ increases, i.e., when the electoral support from informed voter becomes relatively less sensitive to changes in policy platforms.

Finally, let us look at the welfare of citizens and the lobby group. The lobby group only cares about high rents $r_P$. As higher abstention costs $\delta$ and lower voting costs $\gamma$ both increase $r_P$, these changes make the lobby group better off. Citizens prefer high public goods provision $g_P$ and low tax rates $\tau_P$, and the importance they assign to the former relative to the latter decreases in their income. Higher $\delta$ and lower $\gamma$ always increase $\tau_P$, while the effect on $g_P$ is ambiguous. Hence, when higher $\delta$ and lower $\gamma$ reduce $g_P$, then the policy changes following from these changes in abstention and voting costs make all citizens worse off. But when higher $\delta$ and lower $\gamma$ increase $g_P$, then the welfare effects of the subsequent policy changes depend on the citizens’ income. Citizens with low incomes are better off as they primarily care about high $g_P$, while citizens with high incomes are worse off as they primarily care about low $\tau_P$. As discussed before, changes in abstention and voting costs also have direct effects on the citizens’ welfare: an increase in $\delta$ reduces
welfare of all voters who would have abstained otherwise, and a decrease in $\gamma$ increases welfare of all citizen who now decide to vote. Taking these effects together, it follows that compulsory voting has an unambiguously negative effect on the welfare of all citizens when it reduces public goods provision $g_P$. When it increases $g_P$, then it harms citizens with relatively high incomes, while its effect on citizens with low incomes can be positive or negative.

### 4 Conclusions

Conventional wisdom suggests that compulsory voting lowers the influence of special-interest groups and leads to policies that are better for less privileged citizens, who often abstain when voting is voluntary. To scrutinize this conventional wisdom, I have studied how compulsory voting affects public goods provision and rents to special-interest groups in a probabilistic voting model with campaign contributions. This model is fairly standard except that I allow the citizens to decide how much political knowledge to acquire, and whether or not to participate in the election. I find that compulsory voting increases the share of uninformed voters, thereby making special-interest groups more influential. These groups thus receive more generous rents under compulsory voting. In addition, I find that total government spending and taxes are higher under compulsory voting, while public goods provision may be higher or lower. Compulsory voting may thus well lead to policies that make even less privileged citizens worse off.
Appendix: Proofs

Proof of Proposition 1: The interior solution of maximization problem (5) must satisfy the first-order condition

$$\int_{\hat{a}}^{1} \alpha^i \left[ -\frac{\alpha^i}{\alpha} U''(c^i_P) + H'(g_P) \right] f(\alpha^i) d\alpha^i = 0. \quad (7)$$

It is straightforward to show that the second-order condition holds. Denote the left-hand side of (7) by $k_r$. Note that $\frac{\partial k_r}{\partial g_P} = \int_{\hat{a}}^{1} \alpha^i \left[ -\frac{\alpha^i}{\alpha} U''(c^i_P) + H'(g_P) \right] f(\alpha^i) d\alpha^i < 0$. Further it follows from Leibniz's rule that $\frac{\partial k_r}{\partial \alpha} = -\hat{\alpha} \left[ -\frac{\alpha}{\hat{\alpha}} U'(\hat{c}_P) + H'(g_P) \right] f(\hat{\alpha})$, where (in slight misuse of notation) $\hat{c}_P = (1 - \tau_P)\hat{a}$. Observe that $\frac{\partial}{\partial (\frac{\alpha}{\hat{\alpha}} U'(\hat{c}_P) + H'(g_P))} = -\frac{1}{\alpha} U'(c^i_P) + c^i_P U''(c^i_P)) = \frac{1}{\alpha} U'(c^i_P) [R_R(c^i_P) - 1] < 0$, where the inequality follows from our assumption that $R_R(c^i_P) < 1$ for all $c^i_P$. It follows from condition (7) and $\hat{\alpha} < 1$ that $\frac{\alpha}{\hat{\alpha}} U'(\hat{c}_P) + H'(g_P) > 0$ and, consequently, that $\frac{\partial k_r}{\partial \alpha} < 0$. The implicit function theorem then implies $\frac{\partial k_r}{\partial \gamma} = -\frac{\partial k_r}{\partial g_P} \frac{\partial g_P}{\partial \gamma} < 0$. Further note that $\frac{\partial k_r}{\partial \gamma} > 0$ and $\frac{\partial k_r}{\partial \delta} < 0$. It follows that $\frac{\partial k_r}{\partial \gamma} = \frac{\partial k_r}{\partial g_P} \frac{\partial g_P}{\partial \gamma} < 0$ and $\frac{\partial k_r}{\partial \delta} = \frac{\partial k_r}{\partial g_P} \frac{\partial g_P}{\partial \delta} > 0$, which implies $\frac{\partial r}{\partial \gamma} > 0$ and $\frac{\partial r}{\partial \delta} < 0$. ■

Proof of Proposition 2: The interior solution of maximization problem (6) must satisfy the first-order condition

$$\int_{\hat{a}}^{1} \left[ \alpha^i H'(g_P) - (1 - \alpha^i) \Omega J'(r_P) \right] f(\alpha^i) d\alpha^i = 0. \quad (8)$$

It is straightforward to show that the second-order condition holds. Denote the left-hand side of (8) by $k_r$. Note that $\frac{\partial k_r}{\partial g_P} = \int_{\hat{a}}^{1} \alpha^i H''(g_P) + (1 - \alpha^i) \Omega J''(r_P) \right] f(\alpha^i) d\alpha^i < 0$, and $\frac{\partial k_r}{\partial \alpha} = -\int_{\hat{a}}^{1} (1 - \alpha^i) J'(r_P) f(\alpha^i) d\alpha^i < 0$. Further it follows from Leibniz's rule that $\frac{\partial k_r}{\partial \alpha} = -\hat{\alpha} H'(g_P) - (1 - \hat{\alpha}) \Omega J'(r_P) \right] f(\hat{\alpha})$. Observe that $\frac{\partial}{\partial (\hat{\alpha} H'(g_P) - (1 - \hat{\alpha}) \Omega J'(r_P))} = H'(g_P) + \Omega J'(r_P) > 0$. Therefore it follows from condition (8) and $\hat{\alpha} < 1$ that $\hat{\alpha} H'(g_P) - (1 - \hat{\alpha}) \Omega J'(r_P) < 0$ and, consequently, $\frac{\partial k_r}{\partial \alpha} > 0$. The implicit function theorem then implies
\[ \frac{\partial g_P}{\partial \alpha} < 0 \text{ and } \frac{\partial g_P}{\partial \gamma} > 0. \] Consequently, \[ \frac{\partial g_P}{\partial \alpha} = \frac{\partial g_P}{\partial \alpha} \frac{\partial \alpha}{\partial \gamma} > 0 \text{ and } \frac{\partial g_P}{\partial \alpha} = \frac{\partial g_P}{\partial \alpha} \frac{\partial \alpha}{\partial \gamma} < 0. \] It follows that \[ \frac{\partial f}{\partial \alpha} > 0, \frac{\partial f}{\partial \gamma} < 0 \text{ and } \frac{\partial f}{\partial \gamma} > 0. \]

**Proof of Proposition 3:** The interior solution of maximization problem (4) must satisfy the first-order conditions

\[ \int_{\hat{\alpha}}^{1} \left[ -\frac{(\alpha^{'})^2}{\alpha} U'(\hat{c}_P) + \alpha^{'} H'(g_p) \right] f(\alpha') d\alpha' = 0 \quad (9) \]

and

\[ \int_{\hat{\alpha}}^{1} \left[ -\frac{(\alpha^{'})^2}{\alpha} U'(\hat{c}_P) + \alpha^{'}(1 - \alpha^{'}) \Omega J'(r_P) \right] f(\alpha') d\alpha' = 0. \quad (10) \]

It is straightforward to show that the second-order conditions hold. Denote the left-hand side of (9) by \( k_1 \), and the left-hand side of (10) by \( k_2 \). It follows that \( \frac{\partial k_1}{\partial g_P} = K_U + K_H \), \( \frac{\partial k_1}{\partial f} = K_U \), and \( \frac{\partial k_2}{\partial f} = K_U + K_J \), where \( K_U \equiv \int_{\hat{\alpha}}^{1} \frac{(\alpha^{'})^2}{\alpha^2} U''(\hat{c}_P) f(\alpha') d\alpha' \leq 0 \), \( K_H \equiv H''(g_p) \int_{\hat{\alpha}}^{1} \alpha f(\alpha') d\alpha' < 0 \), and \( K_J \equiv \Omega J''(r_P) \int_{\hat{\alpha}}^{1} (1 - \alpha') f(\alpha') d\alpha' < 0 \). Further it holds that \( \frac{\partial k_1}{\partial \alpha} = 0 \) and \( \frac{\partial k_2}{\partial \alpha} > 0 \); and it follows from Leibniz’s rule that \( \frac{\partial k_1}{\partial \alpha} = -\frac{\alpha^{'}}{\alpha} U'(\hat{c}_P) + \hat{\alpha} H'(g_p) f(\hat{\alpha}) \) and \( \frac{\partial k_2}{\partial \alpha} = -\frac{\alpha^{'}}{\alpha} U'(\hat{c}_P) + (1 - \hat{\alpha}) \Omega J'(r_P) f(\hat{\alpha}) \).

The implicit function theorem states that

\[
\left( \begin{array}{c} \frac{\partial f}{\partial g_P} \\ \frac{\partial f}{\partial r_P} \\ \frac{\partial f}{\partial \gamma} \\
\end{array} \right) = -B \left( \begin{array}{c} \frac{\partial g_P}{\partial \alpha} \\ \frac{\partial g_P}{\partial \gamma} \\ \frac{\partial g_P}{\partial f} \\
\end{array} \right) \left( \begin{array}{c} \frac{\partial g_P}{\partial \alpha} \\ \frac{\partial g_P}{\partial \gamma} \\ \frac{\partial g_P}{\partial f} \\
\end{array} \right)
\]

with \( B \equiv \left[ \frac{\partial g_P}{\partial g_P} \frac{\partial g_P}{\partial r_P} - \frac{\partial g_P}{\partial f} \frac{\partial g_P}{\partial f} \right]^{-1} \). Hence

\[ \frac{\partial g_P}{\partial \hat{\alpha}} = B f(\hat{\alpha}) \left\{ K_U [\hat{\alpha} H'(g_P) - (1 - \hat{\alpha}) \Omega J'(r_P)] + K_J \left[ -\frac{\alpha^{'}}{\alpha} U'(\hat{c}_P) + \hat{\alpha} H'(g_p) \right] \right\}, \quad (11) \]

\[ \frac{\partial r_P}{\partial \hat{\alpha}} = B f(\hat{\alpha}) \left\{ K_U [(1 - \hat{\alpha}) \Omega J'(r_P) - \hat{\alpha} H'(g_p)] + K_H \left[ -\frac{\alpha^{'}}{\alpha} U'(\hat{c}_P) + (1 - \hat{\alpha}) \Omega J'(r_P) \right] \right\}. \quad (12) \]
and, consequently,

\[
\frac{\partial (g_P + r_P)}{\partial \hat{\alpha}} = B f(\hat{\alpha}) \left\{ K_f \left[ -\frac{\hat{\alpha}^2}{\alpha} U'(\hat{c}_P) + \hat{\alpha} H'(g_P) \right] + K_H \left[ -\frac{\hat{\alpha}^2}{\alpha} U'(\hat{c}_P) + (1 - \hat{\alpha}) \Omega J'(r_P) \right] \right\}.
\]

(13)

We know that \( K_f < 0 \) and \( K_H < 0 \), and it is easy to show that \( B > 0 \). Hence it remains to determine whether the two terms in square brackets in (13) are positive or negative. As shown in the proof of Proposition 1, it holds that \( \frac{\partial}{\partial \hat{\alpha}} \left[ -\frac{\hat{\alpha}^2}{\alpha} U'(c_P) + \hat{\alpha} H'(g_P) \right] = \frac{U'(c_P)}{\alpha} [R_R(c_P) - 1] < 0 \). It then follows from condition (9) and \( \hat{\alpha} < 1 \) that \( -\frac{\hat{\alpha}^2}{\alpha} U'(\hat{c}_P) + \hat{\alpha} H'(g_P) > 0 \). It further holds that \( \frac{\partial}{\partial \alpha} \left[ -\frac{\hat{\alpha}^2}{\alpha} U'(c_P) + (1 - \alpha) \Omega J'(r_P) \right] = -\frac{\hat{\alpha}^2}{\alpha} [2U'(\hat{c}_P) + \hat{c}_P \Omega J'(r_P)] - \Omega J'(r_P) < 0 \) since \( R_R(c_P) < 1 \) implies \( U'(c_P) + \hat{c}_P \Omega J'(r_P) > 0 \). Together with (13), these results imply that \( \frac{\partial (g_P + r_P)}{\partial \hat{\alpha}} < 0 \). Consequently, \( \frac{\partial (g_P + r_P)}{\partial \gamma} = \frac{\partial (g_P + r_P)}{\partial \hat{\alpha}} \frac{\partial \hat{\alpha}}{\partial \gamma} < 0 \) and \( \frac{\partial (g_P + r_P)}{\partial \delta} = \frac{\partial (g_P + r_P)}{\partial \hat{\alpha}} \frac{\partial \hat{\alpha}}{\partial \delta} > 0 \), such that \( \frac{\partial r_P}{\partial \gamma} < 0 \) and \( \frac{\partial r_P}{\partial \delta} > 0 \).

The implicit function theorem further implies \( \frac{\partial g_P}{\partial \hat{\alpha}} = B \frac{\partial k_1}{\partial r_P} \frac{\partial k_2}{\partial \delta} \leq 0 \), \( \frac{\partial r_P}{\partial \gamma} = -B \frac{\partial k_1}{\partial g_P} \frac{\partial k_2}{\partial \delta} > 0 \), and \( \frac{\partial (g_P + r_P)}{\partial \hat{\alpha}} = B \left[ \frac{\partial k_1}{\partial r_P} - \frac{\partial k_1}{\partial g_P} \right] \frac{\partial k_2}{\partial \delta} = -BK_H \frac{\partial k_2}{\partial \hat{\alpha}} > 0 \), where all inequalities directly follow from results derived above. Consequently, \( \frac{\partial r_P}{\partial \hat{\alpha}} > 0 \).

**Proof of Proposition 4:** This proof builds on various results derived in the proof of Proposition 3. There I show that \( \frac{\partial r_P}{\partial \hat{\alpha}} < 0 \). Further I show that \( B > 0 \), \( K_U \leq 0 \), \( K_H < 0 \), and \( -\frac{\hat{\alpha}^2}{\alpha} U'(\hat{c}_P) + (1 - \hat{\alpha}) \Omega J'(r_P) > 0 \). Hence it follows from (12) that \( \frac{\partial r_P}{\partial \hat{\alpha}} < 0 \) if \( (1 - \hat{\alpha}) \Omega J'(r_P) - \hat{\alpha} H'(g_P) \geq 0 \). It follows from conditions (9) and (9) that

\[
\int_{\hat{\alpha}}^{1} [(1 - \alpha^i) \Omega J'(r_P) - \alpha^i H'(g_P)] f(\alpha^i) d\alpha^i = 0.
\]

(14)

Observe that \( \frac{\partial (1 - \alpha^i) \Omega J'(r_P) - \alpha^i H'(g_P)}{\partial \alpha^i} = -\Omega J'(r_P) - H'(g_P) < 0 \). Therefore condition (14) and \( \hat{\alpha} < 1 \) imply \( (1 - \hat{\alpha}) \Omega J'(r_P) - \hat{\alpha} H'(g_P) > 0 \). Consequently, \( \frac{\partial r_P}{\partial \hat{\alpha}} < 0 \), \( \frac{\partial r_P}{\partial \gamma} < 0 \) and \( \frac{\partial r_P}{\partial \delta} > 0 \).
Proof of Proposition 5: This proof builds on various results derived in the proof of Proposition 3, where I show that \( \frac{\partial g}{\partial \Omega} \leq 0 \).

Assume for the moment that \( U(c^i_P) = \chi c^i_P \) with \( \chi > 0 \). Then \( U''(c^i_P) = 0 \), such that \( K_U = 0 \). Hence equation (11) reduces to \( \frac{\partial g}{\partial \hat{\alpha}} = Bf(\hat{\alpha})K_J \left[ -\frac{\hat{\alpha}^2}{\alpha}U'(\hat{c}_P) + \hat{\alpha}H'(g_P) \right] \). It is shown in the proof of Proposition 3 that \( B > 0 \), \( K_J < 0 \), and \( -\frac{\hat{\alpha}^2}{\alpha}U'(\hat{c}_P) + \hat{\alpha}H'(g_P) > 0 \). It follows that \( \frac{\partial g}{\partial \hat{\alpha}} < 0 \), \( \frac{\partial g}{\partial \gamma} < 0 \) and \( \frac{\partial g}{\partial \delta} > 0 \).

Assume now that \( R_R(c^i_P) = \theta \) with \( \theta \to 1 \) (or \( \theta = 1 \)). Then \( \frac{\partial}{\partial \alpha^i} \left[ -\frac{\hat{\alpha}^2}{\alpha}U'(\hat{c}_P) + \hat{\alpha}H'(g_P) \right] \to 0 \), such that \( -\frac{\hat{\alpha}^2}{\alpha}U'(\hat{c}_P) + \hat{\alpha}H'(g_P) \to 0 \). Hence it follows from (11) that \( \frac{\partial g}{\partial \hat{\alpha}} \to Bf(\hat{\alpha})K_U [\hat{\alpha}H'(g_P) - (1 - \hat{\alpha})\Omega J'(r_P)] \). It is shown in the proofs of Propositions 3 and 4 that \( B > 0 \) and \( \hat{\alpha}H'(g_P) - (1 - \hat{\alpha})\Omega J'(r_P) < 0 \), respectively. Further, \( R_R(c^i_P) > 0 \) implies \( U''(c^i_P) < 0 \) and, consequently, \( K_U < 0 \). It follows that \( \frac{\partial g}{\partial \hat{\alpha}} > 0 \), \( \frac{\partial g}{\partial \gamma} > 0 \) and \( \frac{\partial g}{\partial \delta} < 0 \). ■
References


