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Spatial Competition in Quality, Demand-Induced Innovation, and Schumpeterian Growth*

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Abstract

We develop a general equilibrium model of vertical innovation in which multiple firms compete monopolistically in the quality space. The model features many firms, each of which holds the monopoly to produce a unique quality level of an otherwise homogenous good, and consumers who are heterogeneous in their valuation of the good's quality. If the marginal cost of production is convex with respect to quality, multiple firms coexist, and their equilibrium markups are determined by the degree of convexity and the density of quality-competition. To endogenize the latter, we nest this industry setup in a Schumpeterian model of endogenous growth. Each firm enters the industry as the technology leader and successively transits through the product cycle as it is superseded by further innovations. The intrinsic reason that innovation happens in our economy is not one of displacing the incumbent; rather, innovation is a means to differentiate oneself from existing firms and target new consumers. Aggregate growth arises if, on the one hand, increasingly wealthy consumers are willing to pay for higher quality and, on the other hand, private firms' innovation generates income growth by enlarging the set of available technologies. Because the frequency of innovation determines the toughness of product market competition, in our framework, the relation between growth and competition is reversed compared to the standard Schumpeterian framework. Our setup does not feature business stealing in the sense that already marginal innovations grant non-negligible profits. Rather, innovators sell to a set of consumers that was served relatively poorly by pre-existing firms. Nevertheless, "creative destruction" prevails as new entrants make the set of available goods more differentiated, thereby exerting a pro-competitive effect on the entire industry.

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1 Introduction

Ever since Schumpeter (1942) laid down the mechanisms by which continued innovation of ever higher quality products spurs "creative destruction", economists have viewed quality innovation as a major engine of economic growth. Sparked by Aghion and Howitt (1992), Grossman and Helpman (1991a and 1991b), and Segerstrom et al. (1990), an influential literature has formalized these insights and uses the resulting models to analyze the relationship between market structure and growth.

An equally influential literature deriving from Mussa and Rosen (1978) and Shaked and Sutton (1982 and 1983) argues that product quality is also one of the major determinants of market structure itself, i.e., it is a means by which firms differentiate their products from each other.

The Schumpeterian growth literature has, rather surprisingly, ignored this well-known differentiation motive for quality innovation. Such a limitation is especially striking when one considers that in most vertically differentiated industries, multiple heterogeneous firms compete.[1] In these industries, the growth of the average good quality is also a by-product of entrants' desire to differentiate their products from the competition by inventing higher quality goods.[2]

In this paper, we set up a framework that enables us to analyze how multiple firms compete in the quality space. We then show how firms' innovation decisions determine the density of quality supply, equilibrium markups, and profits in a multi-firm environment of monopolistic competition in quality.

In our model, growth arises through continued quality innovation by firms, just as in Schumpeterian growth models. A key novelty is that firm entry also determines the degree of product market competition (PMC). A high degree of PMC can only arise if entry to the industry is cheap

[1] For example, recent empirical studies in the field of international trade have documented that nearly all manufacturing industries are characterized by many coexisting firms with heterogeneous prices and profits and that this heterogeneity can, to a large extent, be explained by underlying heterogeneity in product quality. See, in particular, Khandelwal (2010) and Kugler and Verhoogen (2010), but also Baldwin and Harrigan (2007), Johnson (2007), Verhoogen (2008), and Hallak and Schott (2009). Moreover, many industry-level studies document the frequent coexistence of a technological leader and multiple laggard firms. For example, Aizcorbe and Kortum (2005) document how in the semiconductor industry, constant innovation of ever more powerful chips coincides with the continued production of less advanced chips. Finally, Bils and Klenow (2001) provide direct evidence that in a cross section of households, different levels of product quality are imperfect substitutes in the eyes of consumers: richer households typically buy more expensive, higher-quality versions of the same goods that poor households also buy.

[2] Starting with von Hippel (1988), numerous empirical studies have shown that in the private sector, much innovational activity is directed towards unserved consumer wants. Indeed, Saha (2007) find that such innovation is empirically more important for firm growth than is process innovation directed toward lowering costs (see also Sutton (1996 and 1998)).

and, therefore, frequent innovation generates a dense quality spacing of firms in equilibrium.[3] This finding is the opposite of the existing literature, where the rate of innovation is strictly decreasing in the degree of PMC, reflecting the lower profit flow of monopolists. We also show that "creative destruction" may work via a pro-competitive effect rather than by displacing the incumbent.

Our analysis proceeds in three steps. In the first, we focus on monopolistic competition in the quality space. More precisely, we develop a model that is suitable for analyzing the density of competition and firm markups in vertically differentiated markets characterized by a large number of active firms. Our model explains how multiple seemingly inferior, low-quality firms can exist alongside the technological leader. The reason for their survival is that, although the highest-quality good is preferred by all consumers, it also carries a higher price tag, which is not worth paying for consumers with relatively low valuation for quality.

We document that market power of such low quality producers arises if the marginal cost of production is convex with respect to quality. Consider, for example, three firms producing $q_1 < q_2 < q_3$ under a marginal cost schedule that is convex in quality (the cost increment per quality between production of q_3 and q_2 is larger than the cost increment between q_1 and q_2). Next, consider the range of consumers whose willingness to pay for additional quality exceeds the first increment but falls short of the second. These consumers receive a surplus by buying q_2 at the marginal production cost instead of buying either q_1 or q_3 at any price exceeding the respective marginal production costs. Because the producers of q_1 and q_3 never sell below their marginal cost, the producer of good q_2 enjoys positive market power. In this way, the convexity of the marginal cost schedule generates market power for individual firms.[4]

In the second part of our analysis, we endogenize the firms' location choice in the quality space and analyze the resulting degree of competition under constantly growing income and valuation for quality. Firms can incur a fixed cost to improve upon the existing qualities, and they are granted a perpetual patent to produce the quality level of their choice. Each firm enters the industry as the technology leader and successively transits through the product cycle as it becomes superseded by further innovations. The benefit of entering with a higher quality good and the cost of doing so both grow at constant rates, so that all entering firms face a scaled but symmetric entry condition.

[3] In this sense, we go beyond Schumpeter's notion that current market structure determines firm profits and thus the incentives to innovate. In addition, we argue that the frequency and magnitude of innovations themselves determine the "toughness" of competition, the market structure, and thus the incentives for further innovation.

[4] Shaked and Sutton (1982 and 1983) and successive work focus on the case where marginal cost is concave in quality, hence implying that only one firm can survive at equilibrium.

We prove that in this setup, the conditions required for the economy to be on a balanced growth path imply that there is a dynamic equilibrium in which each new entrant chooses a quality that is a constant percentage higher than the incumbent technology leader.[5]

Upon market entry, a firm chooses its quality level. In doing so, it aims to distinguish its quality from those of the incumbents, as such isolation in quality increases market power and profits. Higher qualities, however, come at higher fixed and marginal production costs. Whereas the former effect drives firms to pick 'remote' qualities, the latter effect limits quality dispersion.

We also analyze how market size and the underlying technology parameters affect equilibrium quality spacing, prices and quantities. We find that larger markets induce more frequent firm entry and a higher density of quality supply because higher sales and profits allow a faster recovery of setup costs. Markups, in turn, are decreasing in the density of supply and are thus decreasing in the market size.

Surprisingly, a proportional increase in the marginal cost of production for all firms in the industry by the same proportion is associated with a more densely supplied market. The reason is that, in equilibrium, markups are proportional to costs. Thus, when production costs rise for all firms, profits actually increase for any given quality spacing. Excess profits cannot exist in equilibrium, and consequently, firms must exhibit denser quality spacing and "tougher" competition.

The third part of the analysis nests the above-described economy in a dynamic model of endogenous growth with vertical innovation, as described by Aghion and Howitt (1992). We show that long-run economic growth can arise from entrants' desire to differentiate their output in the quality space if, on the one hand, increasingly wealthy consumers are willing to pay for higher quality and, on the other hand, private firms' innovation generates income growth by enlarging the set of technologies available. In this way, the firms' research efforts may generate exactly the income growth that is needed to spur demand for quality.

This paper contributes to two broad literatures. First, it adds to the sizeable literature deriving from Mussa and Rosen (1978) and Shaked and Sutton (1982 and 1983) that focuses on vertically

[5] In our setup, with a clear ranking along the quality line, there is a unique top-quality producer whose first-order condition differs from the first-order conditions of the rest of the firms facing two competitors each. The latter fact substantially complicates our analysis, and therefore, we do not consider a simultaneous entry game as in Vogel (2008). In models based on Hotelling (1929), one can avoid such border conditions because one can think of a circle street or the beach surrounding an island. In our setup, however, any attempt to "close the circle" must fail, as it would amount to identifying the highest quality good with the lowest quality good.

differentiated markets in which natural oligopolies prevail, i.e., markets that are dominated by a limited number of "market leaders".[6]

Our approach differs from this literature only in the underlying production technology. Existing studies assume that the marginal cost of production increases only moderately with quality, which enables high-quality firms to out-price low-quality competitors.[7]Whenever this condition is violated, heterogeneous consumers may differ in their individual ranking of variety-price pairs. Shaked and Sutton (1983) do not analyze this case, which would, given their assumption of costless market entry, imply entry of unaccountably many firms and competitive pricing along a dense set of qualities. In the present paper, we analyze the case where the marginal cost of production does increase sufficiently in quality, while explicitly modeling the firms' quality choice under the standard assumption of costly market entry.

Second, our model is also relevant to the static and the dynamic aspects of the literature analyzing the product market competition (PMC) and growth nexus. The principal difference between the classical Schumpeterian growth models described by Aghion and Howitt (1992) and our approach is that, whereas existing work focuses on the supply side of technical change (i.e., innovation as a means to reduce costs and undercut the competition), we investigate the importance of consumer preferences and the latent demand for new products as a driver of aggregate growth.

In our approach, firm innovation creates growth because innovational efforts are directed toward consumer preferences for higher quality goods. This motive for innovation is akin to the one in the literature on the direction of technical change. For example, Acemoglu (1998, 2002, and 2007) argue that a growing supply of skilled labor generates incentives to invest in technology directed toward that factor of production, and Acemoglu (forthcoming) analyzes the general conditions under which the scarcity of a factor encourages technological progress directed towards it.[8] Similarly, we focus on how the direction of technological advance tracks the evolution of consumer preferences: as consumer valuations grow over time, the market for higher-quality

[6]See also Shaked and Sutton (1984) and Sutton (2007) and (2007a) for the case of one-firm environments, and see Champsaur and Rochet (1989) for the duopolistic case.

[7]See Lahmandi-Ayed (2000 and 2004) for an extensive discussion of the conditions of technology that induce natural oligopolies.

[8]There exists ample empirical evidence for the importance of these considerations for the direction of innovation. Acemoglu and Finkelstein (2008) document that in the case of the health sector, changes in the relative cost of labor had pronounced effects on the adoption of new labor saving technologies. Newell et al. (1999) and Popp (2002) document that energy prices have strong effects on the innovation of energy-saving technologies.

goods expands, creating technological advances directed towards higher-quality goods.

This principal difference in the motivation of firms to conduct costly R&D is associated with two key novel implications of our results compared to the existing literature. First, in our setup, the degree of product market competition (PMC) and the frequency of innovation are jointly determined. Whereas the existing literature introduces PMC via exogenous parameters, in our setup, the degree of PMC is determined endogenously, arising from the entrant's decision to differentiate its product from existing goods.

In the classical Schumpeterian growth models, the rate of innovation is strictly decreasing in PMC, reflecting the lower profit flow of monopolists. In our approach, this finding is reversed: a high degree of PMC can only arise if entry to the industry is cheap and if, therefore, innovation happens frequently. In this sense, the incentives to innovate in our model are related to the "escape competition" motive for R&D in Aghion et al. (2001) (see also Aghion et al. (1997) and Aghion et al. (2005) and in the informal discussion in Boldrin and Levine (2004)), where incumbent firms innovate to increase their cost advantage over lagging imitators.

There exists a fundamental difference, however, between frameworks such as that of Aghion et al. (2001) and our approach. Whereas these existing frameworks focus on cost-innovation in a setup where government policy can directly affect the degree of substitutability between products (for example, the elasticity of substitution between different goods is a direct policy choice in Aghion et al. (2001)), we take the view that the degree of good substitutability is the result of firms' location choice in the quality space. Our main focus, then, is to analyze how post-innovation substitutability itself is shaped by the degree to which innovators distinguish their products from existing ones given a set of entry barriers and input costs.

The second novel implication concerns the nature of "creative destruction" and the mechanisms through which innovating firms create aggregate innovation (see Klette and Kortum (2004)). In existing Schumpeterian growth models, innovation occurs because it allows entrants to displace the incumbent firm. Our setup does not necessarily feature such a "business stealing" effect in the sense that already marginal innovations grant non-negligible profits. Instead, innovators sell only to a set of consumers that was served relatively poorly by pre-existing firms.

Creative destruction does, however, prevail due to the pro-competitive effect of entry. New entrants make the set of available goods more differentiated, which is shown to reduce the market power of all firms so that in equilibrium, firm entry exerts a pro-competitive effect on the entire industry. Margins are thus strictly decreasing in entry.

Our model also features a product life cycle, where each firm enters the industry as the technology leader and successively becomes superseded by further innovations. As it transits through this life cycle, each firm's margins are depressed with every new entry. The latter entry effect, however, also becomes smaller and smaller as the firm becomes sufficiently "backwards" and is no longer much affected by high-quality entry. The firm's markups asymptotically approach a positive value that is determined only by the quality spacing.[9]

The remainder of this paper is structured as follows. In Section, 2, we present some empirical evidence on the relation between patenting and markups. In Section 3, we develop a theoretical model of competition in the quality space. We examine the static predictions of this model in subsection 3.2. We next analyze free entry decisions and the stationary equilibria in 4. Finally, we endogenize the growth rate in Section 5 before describing our conclusions in Section 6.

2 A Brief Look at Innovation and Product Market Competition in Europe

In developing our theory, we are motivated by the fact that in most vertically differentiated industries, a wide set of firms coexist, providing goods of heterogeneous quality. Furthermore, we are able to match the product life cycle that results from firms' continued innovation of higher product qualities. More importantly, however, our model has implications for one of the fundamental questions of the Schumpeterian growth literature: the link between PMC and economic growth.

In this respect, the most salient feature of our theory is that the degree of PMC emerges endogenously via the frequency of entry and, ultimately, through the costs of market entry. This direction of causality implies that a higher frequency of firm entry generates higher PMC, as firms squeeze together more densely in the quality space. In contrast, models in the spirit of Aghion and Howitt (1992) generally explore causality in the reverse direction, predicting that (exogenously

[9] The model also features the substitution and complementarity effects of innovation and the product life cycle first analyzed in Young (1993). In our model, as new innovation occurs, the economy grows, and consumer valuations increase. This growth has two consequences: first, because it raises the average willingness to pay for quality, prices increase. Second, as the support of the valuation distribution grows, its density thins out, and any firm serving a fixed range of consumer valuations thus serves fewer customers.

given) higher profit margins attract more firms, which enter markets at a higher frequency.[10]

Clearly, the correlation between PMC and the frequency of firm entry could empirically discern these two motivations for product innovation. Therefore, we take a brief look at these two variables.

Does PMC relate positively or negatively to the frequency of firm entry? To shed light on this question, we use the demand elasticity estimated by Broda et al. (2006) to measure the degree of equilibrium PMC, exploiting the variation along the country and the industry dimension. On the other side, we use patenting activity as a gauge of the frequency of industry-specific firm entry or, reading our theory a bit more generally, as a measure of product innovation.

For our empirical exercise, we use the patenting data from Johnson (2002), who reports the number of patents issued in 66 manufacturing and commodity sectors for six European countries (Denmark, France, Germany, Italy, Netherlands, and United Kingdom). International patent data is usually categorized according to the International Patenting Classification (IPC) scheme, which the Yale/OECD technology concordance maps into international standards industry codes (ISICs). This concordance table is constructed by Johnson and Evanson (1997) following the methodology of Kortum and Putnam (1994), and it maps IPC codes into the ISIC sectors that actually use the patents ("sectors of end use"). Thus, the resulting dataset of patents at the ISIC level correlates well with the actual adoption of new technologies in these ISIC sectors.

The 66 ISIC sectors from Johnson (2002) are classified at various levels of disaggregation. They include four observations at the "section" level, 31 2-digit industries, 17 3-digit industries, and 14 4-digit industries, hence resulting in fewer than 66 independent observations. Taking this limitation into account, we can concord 38 of the sectors reported by Johnson with the demand elasticities from Broda et al. (2006) that are reported at the 3-digit level of disaggregation in the Harmonized System (HS). Johnson (2002) report patenting data for 1998, which happens to be near the midpoint of the time interval of the trade data (1993 to 2004) used by Broda et al. (2006) to construct their trade demand elasticity estimates.

Having matched demand elasticities and innovation rates for 38 different sectors, it is essential that we apply a consistent rule to classify the industries into two groups, the first of "vertically differentiated" goods and the remainder, which we simply refer to as "horizontally differentiated". The most convincing criterion for vertical differentiation is developed by Bils and Klenow (2001),

[10] Aghion et al. (2001) predict an inverted U-shape relation between the two variables.

who directly estimate the steepness of the Engel curve utilizing the cross section of household incomes and the associated spending for discrete choice durable goods such as cars or television sets in the Bureau of Labor Statistics' "Consumer Expenditure Survey". We match the Harmonized System codes to the Bils and Klenow (2001) set of durable goods.[11]

The disadvantage of following this methodology is that the resulting concordance of Bils and Klenow (2001) goods to HS 3-digit sectors is rather rough because Bils and Klenow (2001) focus on consumption goods such as telephones, TVs, rugs, and multiple apparel goods (e.g., men's suits, women's dresses). In many cases, multiple Bils and Klenow goods fall into the same industry, so that we can identify only 8 of the 38 industries use "vertically differentiated" goods: steel, manufacturing of motor vehicles, precious metals, office machinery, television and radio receivers, wearing apparel, and wood products (see Table A1 in the Appendix for a list of all 38 sectors and their classification).

Figure 1 demonstrates that in these vertically differentiated industries, more patenting is associated with a higher degree of PMC. This figure relates the logarithm of the elasticity of substitution in 48 industry-country pairs (eight industries and six countries) to the logarithm of the number of patents issued in 1998 in the respective country-industry pair.[12]

[11] The 66 durable goods discussed in Bils and Klenow (2001) are not classified by a standard classification scheme. To match this data, we used a text search matching the verbal description of their data with the universe of the 6-digit 1992 HS good classes. In most cases, we coule uniquely match them to one 6-digit class, except for the case of "Hard Flooring", for which we instead searched for "parquet".

[12] There are a few sector-country observations with 0 patents. We therefore define the logarithm of patents as Logarithm of No. Patents= N_Patents=Ln(No. Patents+e^(-1))+1, which equals 0 for sectors with 0 patents and is positive otherwise.

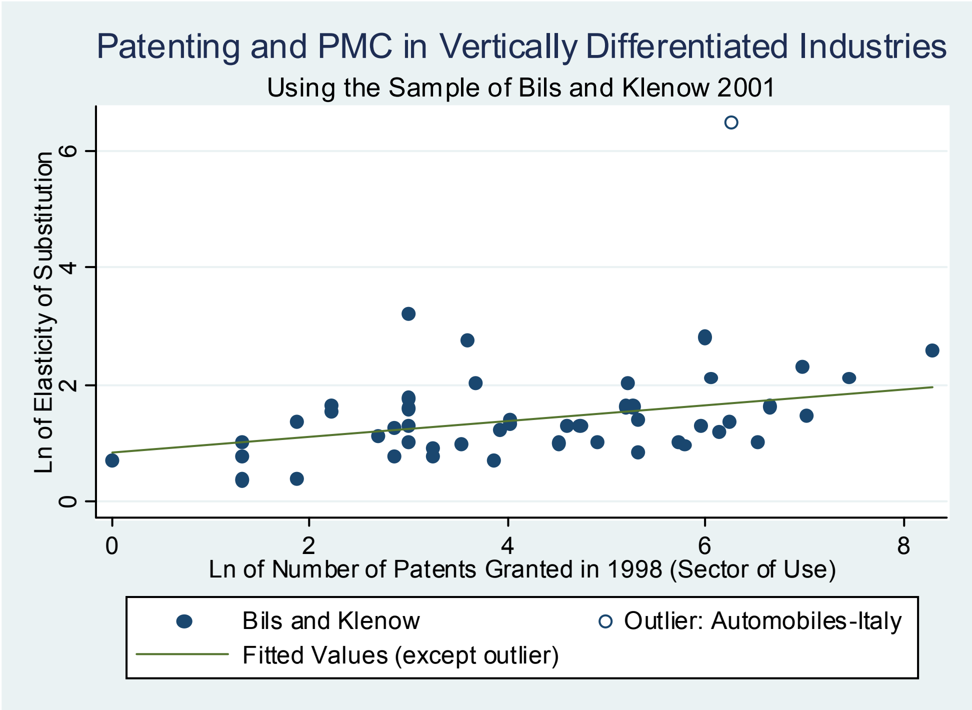


Figure 1

The scatter plot displays a clear positive correlation between these two variables. Because markups are a negative function of the elasticity of substitution, this correlation implies that industries characterized by more product innovation have lower markups, as opposed to the prediction of the standard Schumpeterian model. There is one clear outlier (cars and car parts made in Italy, with an elasticity of 669), but the inclusion or exclusion of this observation does not alter the qualitative picture.

Table 1 examines whether the correlation depicted in Figure 1 is statistically significant and whether it is driven by country-specific factors. In all estimations, the dependent variable is the logarithm of the Broda et al. (2006) demand elasticity. In Column (1), only the logarithm of the number of patents granted in 1998 is added as dependent variable, and the sample includes the 47 (six times eight country-industry pairs minus one outlier) vertically differentiated industries.

The positive raw correlation between these two variables is indeed statistically significant, and the estimated elasticity of the degree of PMC (i.e., the percentage change of the elasticity of substitution) with regard to the frequency of patenting is 0.135. A brief back-of-the-envelope calculation shows that this value is economically quite significant. The median elasticity in this sample of vertically differentiated industries is 3.73, whereas the standard deviation of the logarithm of the patenting frequency is 1.87. Therefore, for the median industry, if the patenting frequency were to increase by one standard deviation, the average industry elasticity would increase from from 3.73 to $3.73 * e^{0.135*1.87} = 4.8$. This, in turn, implies that the average markup would decrease from $3.73/(3.73-1)-1 = 37\%$ to $4.8/(4.8-1)-1 = 26\%$. A one-standard deviation increase in patenting would thus reduce markups from 37% to 26%, or by nearly a third.

It could be the case that patenting is generally higher in larger economies and that markups are generally lower in these larger economies because those markets are more crowded. We want to be sure that the positive correlation between the degree of PMC and the frequency of entry is not driven by this aggregate pattern but rather has industry-specific origins. Therefore, the estimation reported in Column (2) includes country fixed effects that control for the across-country variation. This approach does not alter the results, and the coefficient is still estimated significantly positive. Also, within the countries, the frequency of patenting determines the degree of PMC across the industries. Column (3) repeats the specification of Column (2), but adding the outlier, which leads to a much larger estimated elasticity.

Repeating the regression for the rest of the sample with the "horizontally differentiated" goods, produces no significant correlation between patenting and PMC. Column (4) reports the estimates for the specification with country fixed effects for the 35 sectors (210 country-industry observations) that we cannot match with the vertically differentiated goods of Bils and Klenow (2001). The coefficient is an order of magnitude smaller in size than for the vertically differentiated industries, and it is not significant. It is noteworthy that industries classified as "horizontally differentiated" are, in terms of elasticities, about as differentiated as the "vertically differentiated" ones. With the inclusion of the outlier, the average logarithm of the demand elasticity in the former sample of vertical differentiated industries is 1.524, while it is 1.449 in the latter. This result not only justifies our labeling of "horizontally differentiated" goods but also ensures that our estimation results are not driven by varying degrees of vertical differentiation across the good classes or, similarly, by a different mix of both classes under aggregation of the industries.

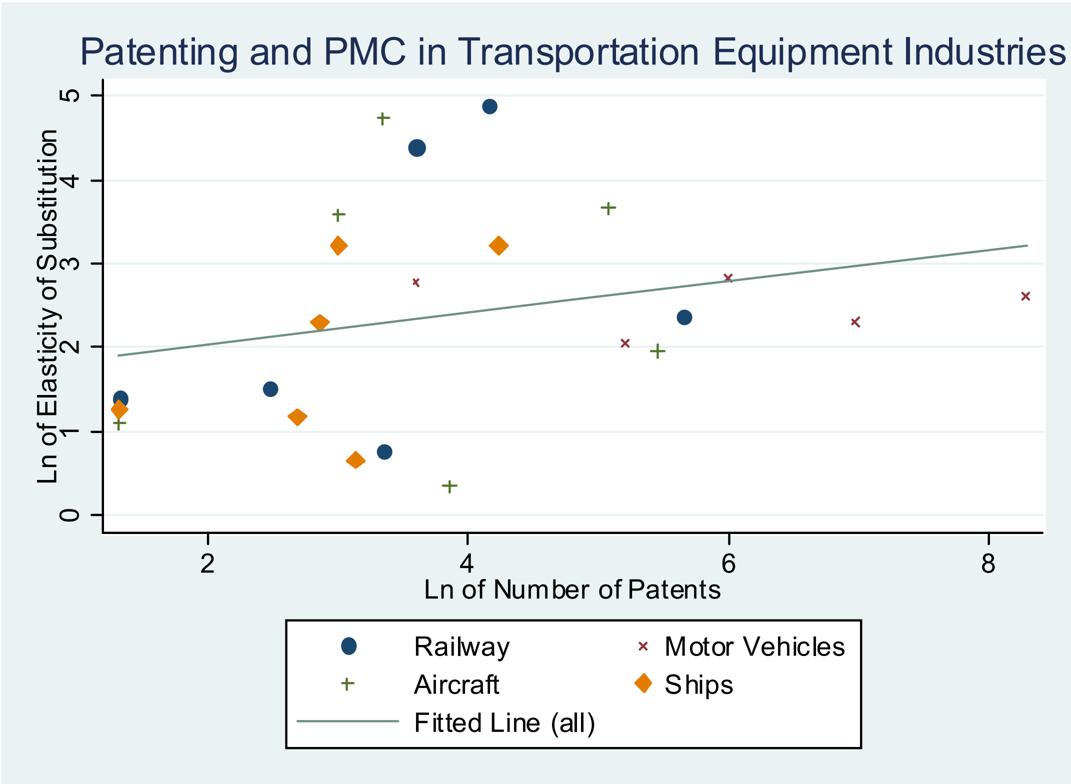
Table 1 - Patenting and Product Market Competition in Vertically Differentiated Industries

	(1)	(2)	(3)	(4)	(5)
Sample:	*Bills and Klenow Vertically Dif. Excl. Cars-Italy*	*Bills and Klenow Vertically Dif. Excl. Cars-Italy*	*Bills and Klenow Vertically Dif. all sectors*	***not** Vertically Dif. in Bills and Klenow*	*Personal Transp. (Cars, Trains, Ships, Aircraft)*
Dependent Variable:	**Ln of the Sector- and Country-Specific Demand Elasticity from Broda et al. (2006)**				
Ln of Patenting Per Sector and Country	0.1345 [0.0399]***	0.1689 [0.0656]**	0.2632 [0.1081]**	0.0401 [0.0291]	0.2977 [0.1515]*
Country Dummies		y	y	y	
Observations	47	47	48	210	24
R-squared	0.16	0.19	0.26	0.1	0.12

Robust standard errors in brackets * significant at 10%; ** significant at 5%; *** significant at 1%

Table 1

While we think that using the Bils and Klenow (2001) classification of vertically differentiated and other sectors is the most objectively consistent role we can follow, it is noteworthy that other reasonable definitions of vertically differentiated goods lead to the same pattern. For example, Figure 2 presents a scatter plot relating the logarithms of patenting and the elasticity of substitution for the four personal transportation equipment industries in our dataset: cars, ships, railway, and aircraft. These industries are also arguably discrete-choice, vertically differentiated industries. Also, within this sample, the relationship between PMC and patenting is negative rather than positive. Due to the small sample size, however, (we have now four industries in six countries minus one outlier) the relationship is only significant at the 10% level (see Column 5 of Table 1).



These patterns in the data lead us to conclude that there is a robust positive correlation between product innovation and PMC in vertically differentiated industries. We therefore believe that the causal link running from innovation activity to larger product differentiation to PMC is particularly strong in the case of vertically differentiated industries, which justifies an effort to develop a model of Schumpeterian growth with firm entry, generating PMC endogenously. In the following theory section, we set out to propose such a model.

3 Spatial Competition in Quality

In the following, we aim to analyze Schumpterian growth in a setup where more than one firm survives. To make this analysis possible, we must first develop a setup, where seemingly inferior, low-quality firms survive along with the technological leader. This task is not trivial, as Hotelling's classic 'location' paradigm, widely used to reflect generic product characteristics, does not apply to competition in quality. By its very definition, quality requires that individuals agree on the ranking of varieties so that, in particular, their individually preferred "ideal variety" coincide.

When it comes to vertical differentiation – or differentiation in quality – only the higher price tag of the universally preferred higher-quality goods causes different consumers to buy distinct qualities.

Our approach is based on Shaked and Sutton (1982, 1983), who pioneered research on vertically differentiated markets in which natural oligopolies prevail, i.e. markets that are dominated by a limited number of "market leaders". The authors call this feature, characterizing vertically differentiated markets, the *finiteness property.* Its key element is that the marginal production costs increase only moderately with quality, which enables high-quality firms to out-price low-quality competitors.[13] Whenever this condition is violated, heterogeneous consumers may differ in their individual ranking of variety-price pairs, and Shaked and Sutton (1983) observe that the competition in quality is "reminiscent of the 'location' paradigm" by Hotelling. The authors do not analyze this case, which would, given their assumption of costless market entry, imply entry of unaccountably many firms and competitive pricing along a dense set of qualities.

In the present section, we analyze the case where the marginal cost of production does increase sufficiently in quality, thus violating the *finiteness property*, while explicitly modeling the firms' quality choice under the standard assumption of costly market entry. We adopt a setup in the spirit of Shaked and Sutton, where consumers prefer quality at a linear rate.[14] We depart from their setup, however, by assuming that the price of a good increases steeply with the good's quality, so that lower-valuation consumers in equilibrium prefer to buy goods other than the one of the current technological leader.

Figure 3 depicts the resulting equilibrium market structure of our approach: higher-valuation consumers tend to buy from high-quality producers. Each firm has two direct competitors (one for the maximum-quality producer) and sells to a range of consumers who, on the one hand, do value quality enough to buy from the firm in question rather than the direct lower competitor but, on the other hand, do not value quality enough to buy from the higher-quality competitor.

[13] See also Shaked and Sutton 1984, Lahmandi-Ayed 2000 and 2004, and Sutton 2007, 2007a.

[14] See also Mussa and Rosen (1978) and Auer and Chaney (2008 and 2009)

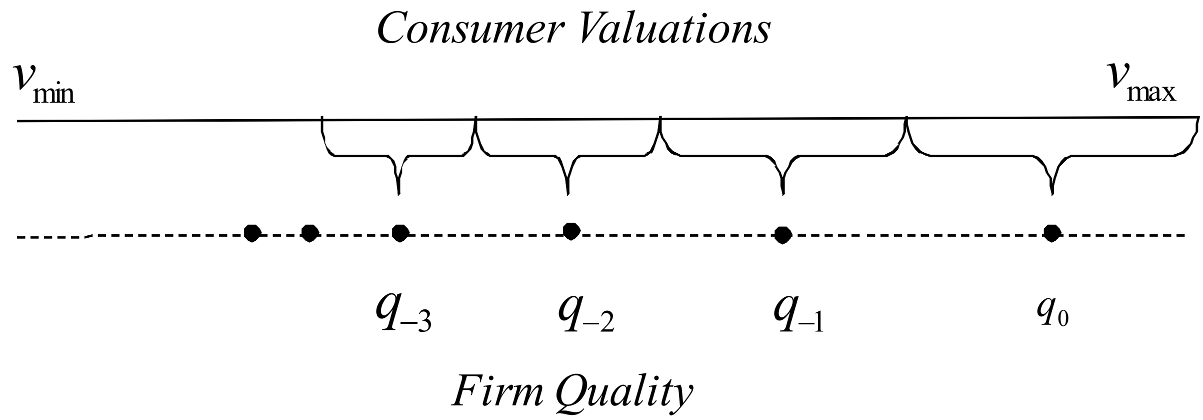


Figure 3: Segmentation of the consumer/valuation space by quality levels.

In the next steps, we formalize this framework and analyze the static determinants of prices and profits for a given quality spacing.

3.1 General Setup

There are one homogeneous good D and a continuum of differentiated goods of total mass one Q_j ($j \in [0,1]$). Each of the differentiated goods comes in a set of different quality levels $\{q_{nj}\}_{n \in S_j}$.

3.1.1 Preferences

From each of the differentiated goods Q_j, consumers consume either one unit or none at all. When consuming the amount d of good D and the vector of qualities $\mathbf{q} = \{q_j\}_{j \in [0,1]}$ of the Q_j-goods, an individual derives utility

$$u_v(\mathbf{q}, d) = v \cdot \left[\int_0^1 q_j \; dj \right] + d \tag{1}$$

The higher the parameter v, the higher is the individual's desire to consume quality. We will, therefore, in the following call v the valuation of quality or simply *valuation*.

Our formulation of preferences slightly modifies the approach from Mussa and Rosen (1978) to a multitude of differentiated goods; it is also close to the standard formulation of Shaked and Sutton (1982), who assume a multiplicative structure between the homogeneous and the differentiated good.

We normalize the price of the homogeneous good O to unity and write $p_i(q_i)$ for the price of quality q_i. The mass of individuals totals L. These individuals value the differentiated good differently, i.e., with different parameters v. We define the resulting cumulative density function of valuations as

$$G(v) : [v_{\min}, v_{\max}] \rightarrow [0,1] \tag{2}$$

where $0 \leq v_{\min} < v_{\max} < \infty$.

3.1.2 Production

The O-type good is produced competitively with constant returns to scale and labor as the only factor. Production technologies of the Q-type goods exhibit increasing returns to scale and depend on the quality level produced. In the following, we consider a representative Q-industry and drop the index j. Firms that enter the Q-market to produce the quality $q \in (0, \infty)$ must acquire a blueprint at the fixed cost of

$$F(q, \bar{q}) = \phi f(q/\bar{q}) \bar{q}^{\theta} \tag{3}$$

effective labor units, where $\bar{q}$ is the maximum quality of the incumbent firms. The function $f(.)$ is differentiable and increasing, while $f(q/\bar{q}) \bar{q}^{\theta}$ is (weakly) decreasing in $\bar{q}$. We thus assume that blueprints of higher qualities are always more expensive, but invention of a given quality is less expensive the more advanced the existing quality frontier.

A firm, having acquired a blueprint for quality q, can produce at the constant marginal cost of

$$c(q) = \varphi q^{\theta} \tag{4}$$

labor units. The parameters $\phi, \varphi > 0$ govern the production cost. We assume that both, the fixed cost of entry, as well as, the marginal cost are increasing and convex in quality ($\theta > 1$).

We characterize the equilibrium in which firms enter production at the optimal quality level and subsequently engage in monopolistic pricing. The equilibrium is solved through backward induction, i.e., we first determine the prices at given quality levels and subsequently analyze entry decisions.

3.2 Optimal Pricing

We begin by characterizing the general pricing solution for an arbitrary distribution of a countable set of qualities. For notational simplicity, we set $p_n = p(q_n)$ and $c_n = c(q_n)$, where q_n is the quality level produced by firm n. We index firms by $n \in \mathcal{N}_0 = \{0, -1, -2, ...\}$ and order firms by their quality level so that firm 0 produces the highest quality level q_0, and all further quality levels satisfy $q_{n-1} < q_n$.[15]

Firms compete in prices, i.e., each firm sets the price of its quality to maximize its operating profits, while taking total demand and the other firms' prices as given. Under preferences (1), a consumer with valuation v is indifferent between two goods q_n and q_{n+1} if and only if their prices p_n and p_{n+1} are such that $vq_{n+1} - p_{n+1} = vq_n - p_n$. Thus, given $G(v)$ from (2) and given the prices $\{p_n\}_{n\leq 0}$, the n^{th} firm sells to all consumers with valuations v in the interval $[v_{n-1},\ v_n]$, where[16]

$$v_n = \begin{cases} v_{\max} & \text{if } n = 0 \\ \dfrac{p_n - p_{n-1}}{q_n - q_{n-1}} & \text{if } n < 0 \\ v_{\min} & \text{if } n = n_{\min} - 1 \end{cases} \tag{5}$$

The firms' market shares are thus $[v_n, v_{n+1}]$, and the market is partitioned as shown in Figure 3. Because each consumer with valuation $v \in [v_n, v_{n+1}]$ demands one unit of the variety produced by firm n, firm n serves the mass of $G(v_{n+1}) - G(v_n)$ consumers and solves the maximization problem

$$\max_{p_n} (p_n - c_n) \left[G(v_{n+1}) - G(v_n)\right] L \qquad s.t.\ (5) \tag{6}$$

The optimality condition of this problem is

$$G(v_{n+1}) - G(v_n) - (p_n - c_n) \left[\frac{G'(v_{n+1})}{q_{n+1} - q_n} + \frac{G'(v_n)}{q_n - q_{n-1}}\right] = 0 \tag{7}$$

where the expressions (5) apply. At $v_{\min}$, $v_{\max}$, the constant limits of the distribution, the derivatives in (7) are set to zero ($G'(v_{\min}) = G'(v_{\max}) = 0$). Firm n's profits are zero at $p_n = c_n$ and at

$$\bar{p}_n = \frac{(q_n - q_{n-1})\, p_{n+1} + (q_{n+1} - q_n)\, p_{n-1}}{q_{n+1} - q_{n-1}}$$

because the latter price implies $v_{n+1} = v_n$ and thus zero market share for the n^{th} firm. Finally, as[17]

$$\bar{p} = \frac{(q_n - q_{n-1})\, p_{n+1} + (q_{n+1} - q_n)\, p_{n-1}}{q_{n+1} - q_{n-1}} \geq \frac{(q_n - q_{n-1})\, c_{n+1} + (q_{n+1} - q_n)\, c_{n-1}}{q_{n+1} - q_{n-1}} > c_n$$

[15] Notice that we implicitly assume the set of firms is countable. By making this assumption, we already anticipate that in the equilibrium of the later entry game, firms need to recoup their setup cost with monopoly rents. Under Bertrand competition and positive setup cost, this assumption implies that firms must be located at positive distance to each other, and the number of firms is necessarily countable.

[16] We rule out undercutting, where firm n sets its quality-adjusted price to take the market share of a directly neighboring firm and compete with second-next firms.

[17] The last inequality holds by convexity of $c(q)$.

and profits are positive for $p_n \in [c_n, \bar{p}]$, there is an interior solution to the profit maximization problem, which necessarily satisfies (7). Generic profits are

$$\pi_n = (p_n - c_n)^2 \left[\frac{G'(v_{n+1})}{q_{n+1} - q_n} + \frac{G'(v_n)}{q_n - q_{n-1}} \right] L \tag{8}$$

With this characterization of prices and operating profits, some regularities of equilibrium prices and profits emerge.

Lemma 1 *Let $\{q_n\}_{n \leq 0}$, $\{c_n\}_{n \leq 0}$ and (7) define a system with the prices $\{p_n\}_{n \leq 0}$ and operating profits $\{\pi_n\}_{n \leq 0}$. For any $\chi > 0$, the following statements hold:*
(i) The transformed system defined by $q'_n = \chi q_n$, $c'_n = \chi^\theta c_n$, $v' = \chi^{\theta-1} v$ and corresponding (7) has the solution $\{p'_n\}_{n \leq 0}$ and $\{\pi'_n\}_{n \leq 0}$ satisfying

$$p'_n = \chi^\theta p_n \qquad \textit{and} \qquad \pi'_n = \chi^\theta \pi_n \quad \forall n.$$

(ii) The transformed system defined by $q''_n = q_n$, $c''_n = \chi c_n$, $v'' = \chi v$ and corresponding (7) has the solution $\{p''_n\}_{n \leq 0}$ and $\{\pi''_n\}_{n \leq 0}$ satisfying

$$p''_n = \chi p_n \qquad \textit{and} \qquad \pi''_n = \chi \pi_n \quad \forall n.$$

Proof. *See Appendix* ■

The first part of the Lemma states that, if quality levels, marginal production costs and valuations increase at the right proportions (according to (3) and (4)), then equilibrium prices and profits are a constant proportion of the marginal production costs. Part (ii) of the Lemma states that if marginal production costs and valuations increase proportionally while quality levels are constant, then prices and profits are a constant proportion of the marginal production costs.

These regularities will lead us to a particularly nice pattern of the firms' quality choice – namely proportional spacing. We turn to this feature next.

4 Endogenous Spacing Under Free Entry

This section shows that in a dynamic version of the general setup described above, free entry supports equilibria with equal relative spacing of firms, endogenously generating quality levels that satisfy

$$\gamma q_{n-1} = q_n \qquad \forall\, n. \tag{9}$$

We introduce a dynamic dimension to our model by assuming that time is continuous and that valuations grow at the constant rate a. The rate a is for now given exogenously and is endogenized in Section 5 below. Indexing each valuation parameter with time subscripts, we can write $v_t = e^{at}v_o$. Consequently, the distribution G is time dependent and satisfies

$$G_t(v) = G_o(e^{-at}v) \tag{10}$$

where G_o is the distribution at the initial date $t = 0$.

Our analysis aims at a stationary equilibrium in which each firm enters the industry as the technological leader and successively transits through the product cycle as it becomes superseded by further innovators. The gain of such a dynamic entry game is that we only need to analyze the entry problem of one firm at a time. In particular, we avoid the problems that arise in a simultaneous entry game, such as in Vogel (2008).[18]

In a dynamic game of this type, the profits of a firm producing q_o evolve as depicted by the bold line segments in Figure 4. Each continuous section represents the profits when no innovation occurs. Innovations occur at regular intervals (depicted by t_1^*, t_2^*...). At these moments, the firm's profit drops by a discrete amount, because the new competitor reduces the incumbents' sales and markups.

The dashed line illustrates the general trend. Two opposing forces are at work that explain why this trend may be first increasing and then decreasing over time. First, for a given set of firms, the profit flow for top-quality firms is increasing as consumer valuations increase over time, driving up market shares and markups. Second, the growing range of consumer valuations also implies that the density of consumer valuations constantly thins out. In particular, firms converge at the limit to serving a fixed interval of valuations, while the density of valuation over this range constantly thins out. Therefore, the profit flow drops to zero at the limit.

[18] In fact, the resulting complications would be tremendous in our setup, because the clear ranking of the quality line prevents us from using the symmetry properties that arise in models based on Hotelling (1929), where one can consider economies formed like a circle street or the beach surrounding an island. In a quality setup, however, any attempt to "close the circle" must fail, as it would amount to identifying the highest quality good with the lowest quality good.

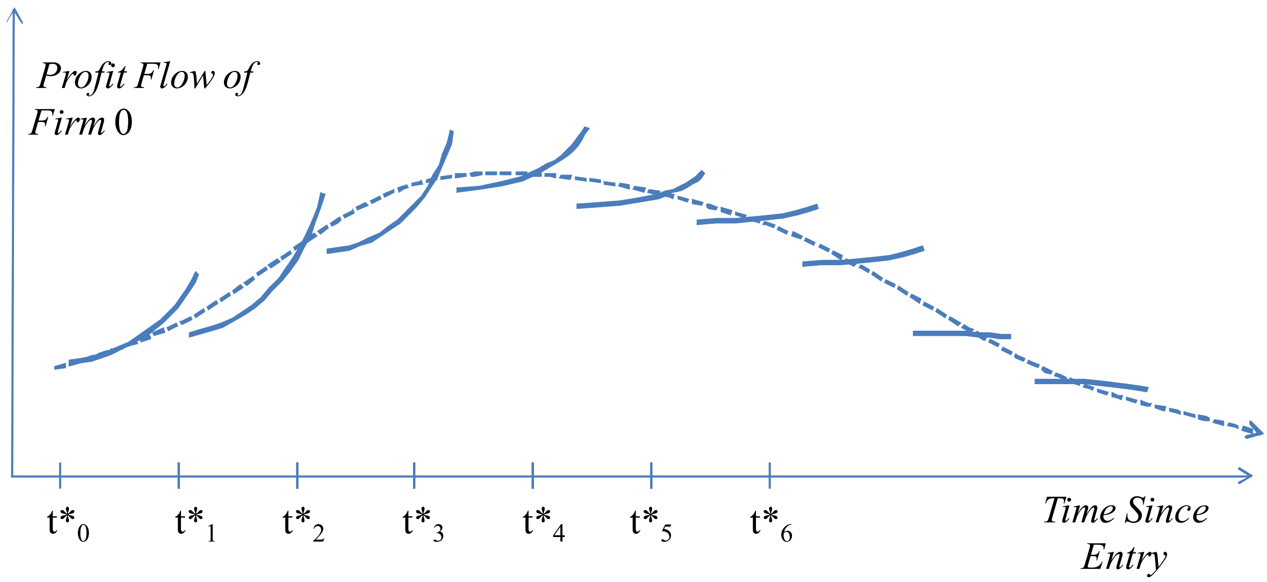


Figure 4: Profit Flow over the Life Cycle of the Firm Entering at t_0^*

In the entry equilibrium depicted in Figure 4, entry is such that any innovating firm immediately starts producing once it enters the market. If entry is sufficiently cheap, the zero profit condition may also force firms to enter the industry pre-emptively: at the moment of entry t_0^*, the good produced by the technological leader is of such a high quality that there would be no demand for the firm's product even if it were sold at marginal costs.

The resulting product life cycle with pre-emptive entry is depicted in Figure 5. In this equilibrium, valuations only catch up after some time and the associated income and valuation growth. In Figure 5, we depict an economy where in equilibrium, consumers' willingness to pay for quality $-v_{max}-$ is such that new firms only sell after two further entries. In such an equilibrium, at the moment when the maximum valuation $v_{max}(t)$ is just high enough so that this consumer buys from the firm that entered at t_0^*, firm 0 sells at marginal costs (because it sells to a set of 0 consumers so that demand is infinitely elastic). Thereafter, the profit flow of firm 0 increases steeply with time: not only does the set of consumers it serves grow rapidly as valuations catch up, but also, as it sells to a growing range of consumers, its demand becomes less elastic and, thus, its markup increases steadily. In the limit, the firm approaches a constant markup. Because the firm converges at the limit to serving a fixed interval of valuations, while the density of valuation over this range constantly thins out, later in the product life cycle the profit flow drops and asymptotes to 0.

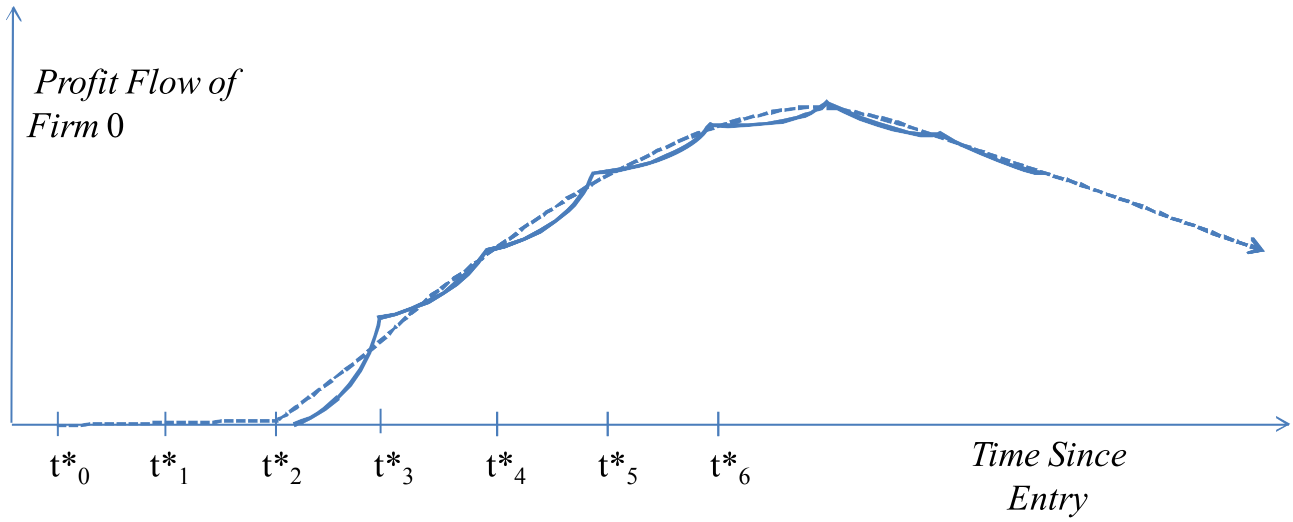


Figure 5: Profit Flow of the Firm Entering at t_0^* under Pre-Emptive Entry

In the entry game, firms decide not only which quality to aim for but also when to enter the industry. We assume that at each point in time, there is a mass of potential entrants who could pay a fixed cost $F(q)$ to receive a perpetual monopoly to produce the good of quality level q. With this setup, the potential entrants start innovating as soon as innovation generates a profit flow with a net present value as least as big as the innovation cost.

Initially, the set of active firms is $\{0, -1, -2, ...\}$. Firms are ordered according to ascending qualities, so that a higher firm index corresponds to a higher quality. These initially active firms produce qualities $\{q_n\}_{n \leq 0}$, which satisfy (9). As demand grows for goods at the top end of the quality spectrum, new firms gradually establish at the upper end of the quality spectrum.

We assume that a plant established at quality level q_m automatically holds the blueprints for all qualities between q_{m-1} and q_m, where q_{m-1} is the next lower quality level. This assumption restricts entry of additional firms to quality levels above the pre-existing ones ($q_{m+1} \geq q_m$).

Now, for $m \geq 1$, let t_m denote the entry date of the m^{th} *additional* firm (implying $0 \leq t_1 \leq t_2 \leq ..$), and further let q_m stand for its quality level ($q_0 \leq q_1 \leq q_2 \leq ..$). It will prove convenient to express the quality choice of the m^{th} entrant relative to the highest quality of all incumbents (q_{m-1}) as

$$\gamma_m = q_m / q_{m-1} \qquad\qquad m \geq 1.$$

At time $\tau \in [t_{m+k}, t_{m+k+1})$ the set of quality levels supplied to the market is $\{q_n\}_{n \leq m+k}$. Current prices are determined implicitly by (7) and depend on all currently produced quality levels as well as on all current valuations $v_\tau = e^{a\tau}v$. Consequently, at time $\tau \in [t_{m+k}, t_{m+k+1})$, the operating profits (8) of the m^{th} additional firm are a function of qualities $\{q_n\}_{n\leq m+k}$ and time τ. We can express this time dependence as dependence on the factor $e^{a\tau}$, which multiplies all valuation parameters v. Formally, operating profits of the firm m at time τ are thus

$$\pi_m\left(e^{a\tau}, q_{m+k}, \gamma_{m+k}, \gamma_{m+k-1}, \gamma_{m+k-2}, ..., \gamma_1, \gamma\right) \qquad \tau \in [t_{m+k}, t_{m+k+1}).$$

Defining now the product

$$\Gamma_{m,k} = \prod\nolimits_{j=1}^{k} \gamma_{m+j} \tag{11}$$

we have $q_{m+k} = \Gamma_{m,k} q_m$ so that at time t_m, the present value of the flow of operating profits for a potential entrant is

$$\Pi(\gamma_m, t_m) = \sum_{k\geq 0} \int_{t_{m+k}}^{t_{m+k+1}} e^{-r(\tau - t_m)} \pi_m\left(e^{a\tau}, \Gamma_{m,k}\gamma_m\Gamma_{0,m-1}q_0, \gamma_{m+k}, \gamma_{m+k-1}, ..., \gamma_1, \gamma\right) d\tau. \tag{12}$$

The parameter r is the constant rate at which firms discount future profits.

We are now ready to formulate the entry decision of firms. The m^{th} firm chooses its entry date (t_m) and its location on the quality line (γ_m). With the second choice, it maximizes the present value of profits at time t_m (12) net of costs (3). Given the spacing $\gamma_{m-1}, \gamma_{m-2}, ..., \gamma_1, \gamma$, and conditional on the entry date t_m, the m^{th} optimal quality choice is

$$\begin{aligned}\hat{\gamma}_m\left(\gamma_{m-1}, ..., \gamma_1, \gamma\right) = \underset{\tilde{\gamma}\geq 1}{\arg\max} \Bigg\{ \sum\nolimits_{k\geq 0} \int_{t_{m+k}}^{t_{m+k+1}} e^{-r(\tau - t_m)} \pi_m \Big(& e^{a\tau}, \tilde{\Gamma}_{m,k}\tilde{\gamma}\Gamma_{0,m-1}q_0, \hat{\gamma}_{n+k}, \hat{\gamma}_{n+k-1} \cdots \\ & \cdots, \hat{\gamma}_{n+1}, \tilde{\gamma}, \gamma_{n-1}, ..., \gamma_1, \gamma \Big) d\tau - F(\tilde{\gamma}, \Gamma_{0,m-1}q_0) \Bigg\}\end{aligned} \tag{13}$$

Here, $\tilde{\Gamma}_{m,k}$ stands, similar to (11), for the product of the k future optimal relative spacing parameters, given that the m^{th}-entrant plays $\tilde{\gamma}$:

$$\tilde{\Gamma}_{m,k} = \prod\nolimits_{j=1}^{k} \hat{\gamma}_{m+j}\left(\hat{\gamma}_{m+j-1}, \hat{\gamma}_{m+j-2}, ...\tilde{\gamma}_m, \gamma_{m-1}, .., \gamma_1, \gamma\right).$$

Note that all future location choices $\hat{\gamma}_{m+j}$ (and $\tilde{\Gamma}_{m,j}$) as well as future entry dates t_{m+j} are functions of the m^{th} firm's choice. For expositional purposes, however, the arguments $\hat{\gamma}_{m+j}(\tilde{\gamma})$, $\tilde{\Gamma}_{m,j}(\tilde{\gamma})$, $t_{m+j}(\tilde{\gamma})$ are suppressed in (13) and further down. The m^{th} firm's entry date is determined by the free entry condition, *i.e.*, the requirement $\Pi(\gamma_m, t_m) \geq F(\gamma_m, q_{m-1})$. Formally, we write

$$\begin{aligned} t_m = \inf \Bigg\{ t \geq t_{m-1} \;\Bigg|\; \sup_{\tilde{\gamma}\geq 1} \Bigg[\sum\nolimits_{k\geq 0} \int_{t_{m+k}}^{t_{m+k+1}} e^{-r(\tau - t_m)} \pi_m \Big(& e^{a\tau}, \tilde{\Gamma}_{m,k}\tilde{\gamma}\Gamma^*_{0,m-1}q_0, \hat{\gamma}_{m+k}, \hat{\gamma}_{m+k-1}, \cdots \\ & ..., \hat{\gamma}_{m+1}, \tilde{\gamma}, \gamma^*_{m-1}, \gamma^*_{m-2} \cdots, \gamma^*_1, \gamma \Big) d\tau - F(\tilde{\gamma}, \Gamma^*_{0,m-1}q_0) \Bigg] \geq 0 \Bigg\} \end{aligned} \tag{14}$$

where γ_m^* denotes the equilibrium locations

$$\gamma_1^* = \hat{\gamma}_1(\gamma) \qquad \text{and} \qquad \gamma_k^* = \hat{\gamma}_k(\gamma_{k-1}^*, \gamma_{k-2}^*, ..., \gamma_1^*, \gamma) \tag{15}$$

and $\Gamma_{0,k}^*$ is defined in parallel to the above definitions as the product of the equilibrium γ_j^*

$$\Gamma_{0,k}^* = \prod\nolimits_{j=1}^{k} \gamma_j^*.$$

Optimal quality choices (13) and the free entry conditions (14) of all entrants ($m \geq 1$) determine the equilibrium of the entry game. The first important result of this section concerns the solution of the system (13) - (14) and is formulated in the following Proposition.

Proposition 1 *For any combination of positive parameters $(\theta, \phi, \varphi, L, r, a)$, there exists a $\bar{\gamma} > 1$ so that the equilibrium of the entry game (13) - (14) sustains*

$$q_m = \bar{\gamma} q_{m-1} \qquad\qquad m \in \mathbb{Z}.$$

In this equilibrium, the time intervals between consecutive entries are constant and equal to

$$\Delta = (\theta - 1)\, a^{-1} \ln(\bar{\gamma}). \tag{16}$$

Proof. See Appendix. ■

For the parameters $(\theta, \phi, \varphi, L, r, a)$, we label the corresponding equilibrium the Equal Relative Spacing Equilibrium (ERSE). Notice that the proposition establishes existence of the ERSE but is silent about its uniqueness. We therefore restrict all further considerations to the one ERSE (out of possibly many) with the minimal spacing $\bar{\gamma}$. Now, (as argued in the proof of Proposition 1), under a preexisting spacing parameter equal to one ($\gamma = 1$), the optimal spacing of the first entrant $\gamma^*(\gamma)$ from (15) satisfies $\gamma^*(\gamma) > \gamma$ for all $\gamma \in (1, \bar{\gamma})$. Consequently, we conclude that at the minimal symmetric $\bar{\gamma}$, characterized by $\gamma^*(\bar{\gamma}) = \bar{\gamma}$

$$\left.\frac{d\gamma^*(\gamma)}{d\gamma}\right|_{\gamma=\bar{\gamma}} < 1. \tag{17}$$

holds. For this ERSE with the smallest γ, we can show the following Lemma.

Lemma 2 *Let $\bar{\gamma}$ be the spacing parameter of the ERSE. Then,*

(i) $\bar{\gamma}$ depends on $\phi/(\varphi L)$ only and can be written as $\bar{\gamma}(\phi/(\varphi L))$.

(ii) $\bar{\gamma}$ is constant under the transformation $(r, a, L) \to \chi \cdot (r, a, L)$, where $\chi > 0$.

Proof. *(i)* Operating profits π are linear in L; setup costs F are linear in ϕ. Thus, when replacing $\phi' = \phi/L$, population L factors out of the slanted brackets in (13) and the square brackets in (14). Consequently, the solution to problem (13) - (15) and thus $\bar{\gamma}$ depends on $\phi' = \phi/L$ only. Similarly, operating profits are, by Lemma 2 (ii), linear in φ under the transformation $v'(t) = v(t)/\varphi$ (or $t' = t - \ln(\varphi^{1/a})$). Hence, replacing $\phi' = \phi/\varphi$ in (13) and (14), $\bar{\gamma}$ depends on ϕ' only.

(ii) Net present profits (12) are constant under the time transformation $t \to \chi t$, given that locations are constant and firm entry dates transform by $t_m \to \chi t_m$. Under this condition, the firm entry remains unchanged. By (13), firm entries are transformed accordingly. Finally, the time transformation is equivalent to $(r, a, L) \to \chi \cdot (r, a, L)$. ■

Technically, the Lemma shows that we can choose the notation $\bar{\gamma}(\phi)$ to reflect the functional dependence of $\bar{\gamma}$ on all three parameters ϕ, φ and L. Economically, it says that the density of quality supply (and of competition) is equally affected by a doubling of the market size or the marginal costs or by a reduction of setup cost by half.

It would be premature, however, to infer welfare consequences based on the parameter $\bar{\gamma}$ (and its impact on markups) alone, conjecturing, e.g., that an equal increase of the setup costs ϕ and the operating costs φ leaves the economy unchanged. In fact it does not. Such a change in technology actually postpones innovation (reflected in the time transformation in the Lemma's proof) so that more time elapses until a given quality is on the market. This delay means that individuals purchase lower qualities because each quality is more expensive, and fewer high-quality goods are available on the market. Both effects have a negative impact on consumer surplus. It is straight forward, however, to show that an increase in setup cost that is entirely offset by an increase in the size of the workforce L, preserving not only the relative spacing $\bar{\gamma}$ but also the timing of innovations, and thus leaving the quality spectrum at each point in time unchanged.[19]

This section has derived a novel result about the regularity of spacing (Proposition 1) and the relative impact of the model's key parameters (Lemma 2). These findings hold in a relatively general setup, which includes, in particular, a non-degenerate distribution of valuations (2). This generality, however, comes at a price. In particular, we were unable to show uniqueness of

[19] It is interesting to note that our model exhibits a monopoly distortion that is new to the endogenous growth literature: positive markups lead consumers to choose a quality that differs from the socially optimal one. In equilibrium, each consumer compares the increase in the goods' quality to the increase in the good's price. Because markups are generally increasing along the quality dimension, the increase in price from one good to the other is higher than the cost increase. Consumers, therefore, tend to choose too low a quality.

equilibria – neither of the entry game nor, in fact, of the pricing game (determined by (7)). We solved the first of these uniqueness problems by restricting our analysis to the equilibrium with the highest density of quality and simply ignored the second.[20] In the important case of a uniform distribution of valuations, also the second of the ambiguities luckily vanishes. We next turn to this case.

4.1 Uniform Distribution

We analyze the special case when valuations are distributed uniformly as

$$G_o(v) = U([0, v_{\max}]). \tag{18}$$

In this case, which also appears in Auer and Chaney (2007), the optimality conditions (7) give rise to the system

$$p_n = \begin{cases} \frac{1}{2}\left[c_0 + (q_0 - q_{-1})\, v_{\max} + p_{-1}\right] & \text{if } n = 0 \\ \frac{1}{2}\left[c_n + \frac{q_n - q_{n-1}}{q_{n+1} - q_{n-1}} p_{n+1} + \frac{q_{n+1} - q_n}{q_{n+1} - q_{n-1}} p_{n-1}\right] & \text{if } n < 0 \end{cases} \tag{19}$$

which implies that equilibrium prices are determined as follows.

Proposition 2 *Assume equal relative spacing in quality, i.e., (9) holds. Then prices are*

$$p_n = \left(A\left(\lambda/\gamma^\theta\right)^n + \alpha\right) c_n \qquad \forall \;\; n \le 0 \tag{20}$$

where

$$\alpha = \frac{\gamma + 1}{2(\gamma + 1) - \gamma^\theta - \gamma^{1-\theta}} \tag{21}$$

$$\lambda = \gamma + 1 + \sqrt{\gamma^2 + \gamma + 1} \tag{22}$$

$$A = \frac{\lambda}{2\lambda - 1}\left(1 - \alpha\left(2 - \gamma^{-\theta}\right) + \frac{\gamma - 1}{\gamma}\frac{q_0 v_{\max}}{c_0}\right). \tag{23}$$

Proof. See Appendix ■

Proposition 2 not only provides a closed-form solution but, in addition, establishes uniqueness of the pricing equilibrium. Notice finally that, while the term α from (20) may be negative, markups are always positive.[21]

[20] In fact, it is easy to remedy this problem by either assuming that economic agents correctly anticipate one stable pricing equilibrium or by introducing expectations, of profits in particular, when realizations of equilibria are identically and independently distributed over time.

[21] Inequalities $2(\gamma+1) > \gamma^\theta + \gamma^{1-\theta}$ and $\lambda/\gamma^\theta > 1$ hold (are violated) for $\gamma \to 1$ ($\gamma \to \infty$) and $\lambda = \gamma^\theta$ if and only if γ solves $\gamma^\theta + \gamma^{1-\theta} = 2(\gamma+1)$. Thus, α is negative iff $\lambda/\gamma^\theta > 1$. Now, distinguish two cases. First, if $\alpha < 0$ holds,

Proposition 2 also allows us to make an intuitive and simple statement regarding the effect of entry on the markups of existing firms.

Lemma 3 *For a given set of firms that satisfies (9), entry of an additional firm at the top end of quality at* $q_1 = \gamma q_0$ *weakly decreases the markup of any preexisting firm.*

Proof. Denote the parameter from (23) before (after) entry of the additional firm with A ($\tilde{A}$) and notice that by (4), $A > \tilde{A}$ holds. Consider now the case where $\tilde{A} + \alpha < 1$. In this case, the additional firm does not produce, and the pricing of producing firms does not change. Consequently, markups stay constant. Consider next $\tilde{A} + \alpha > 1$. Denoting the price of firm k before and after entry of the additional firm with p_k, we have

$$\frac{p_k}{\tilde{p}_k} = \frac{A\left(\lambda/\gamma^\theta\right)^k + \alpha}{\widetilde{A}\left(\lambda/\gamma^\theta\right)^{k-1} + \alpha}.$$

There are now two cases to distinguish. First, $\lambda/\gamma^\theta > 1$, implying $\alpha > 1$. In this case, $p_k/\tilde{p}_k > 1$ holds by $A > \tilde{A}$. Second, $\lambda/\gamma^\theta > 1$, implying $\alpha < 0$. In that case, we write with (23)

$$\begin{aligned}
\frac{p_k - \tilde{p}_k}{c_k\left(\lambda/\gamma^\theta\right)^k} &= A - \widetilde{A}\left(\lambda/\gamma^\theta\right)^{-1} \\
&= \frac{\lambda}{2\lambda - 1}\left\{\frac{\lambda - \gamma^\theta}{\lambda}\left[1 - \alpha\left(2 - \gamma^{-\theta}\right)\right] + \frac{\gamma - 1}{\gamma}\frac{q_0 v_{\max}}{c_0}\left[\frac{\lambda - \gamma}{\lambda}\right]\right\}
\end{aligned}$$

The expression on the right is positive, proving $p_k/\tilde{p}_k > 1$ in the second case, too. ■

The Lemma distinguishes two cases. First, the additional firm engages in production and impacts the whole market by depressing markups. Second, it does not pay for the additional firm to produce and sell its goods, and consequently leaves the market unaffected. As this second case is a possibility, the entry of additional firms decreases the markup of any preexisting firm only weakly.

Our next aim is to conduct comparative statics with regard to the model's parameters. As a preparatory step, we write for the relative markup of the highest quality firm

$$A + \alpha - 1 = \frac{\lambda}{2\lambda - 1}\left\{\frac{\gamma - 1}{\gamma}\frac{q_0 v_{\max}}{c_0} + \left(\frac{\alpha}{\gamma^\theta} - 1\right) - \frac{\alpha - 1}{\lambda}\right\}.$$

---

we have $A > 0$ and $\lambda/\gamma^\theta < 1$ so that $A\left(\lambda/\gamma^\theta\right)^n + \alpha > A + \alpha$ for all $n < 0$. Second, if $\alpha > 0$, we verify $\alpha > 1$, so that all markups are positive if $A > 0$ holds. If, instead, $A < 0$ then $A\left(\lambda/\gamma^\theta\right)^n + \alpha > A + \alpha$ holds again for all $n < 0$ by $\lambda/\gamma^\theta < 1$. It is thus sufficient to show $A + \alpha > 1$. By (23), this condition is satisfied as long as $v_{\max} q_0/c_0$ is large enough. Obviously, it may happen that $v_{\max} q_0/c_0$ is very small. In that case, however, top-quality firms do not sell at all, and we can renumber firms, indexing the highest quality firm with *positive output* with $n = 0$. This renumbering increases A up to the point where $A + \alpha > 1$ holds.

With the explicit formula for the prices (20), the operating profits from (8) are thus

$$\pi_n = \begin{cases} \gamma\left(\gamma-1\right)\left(\dfrac{A+\alpha-1}{\gamma-1}\right)^2 \dfrac{c_0^2}{q_0}\dfrac{L}{v_{\max}} & \text{if } n=0 \\[2ex] \left(\gamma^2-1\right)\left(\dfrac{A\left(\lambda/\gamma^{\theta}\right)^n+\alpha-1}{\gamma-1}\right)^2 \dfrac{c_n^2}{q_n}\dfrac{L}{v_{\max}} & \text{if } n<0 \end{cases} \tag{24}$$

Observe with (21) and (23) that the limit

$$\lim_{\gamma\to 1}\frac{A+\alpha-1}{\gamma-1}=\frac{1}{\sqrt{3}}\left\{\frac{q_0 v_{\max}}{c_0}-\theta\right\}$$

is finite. Thus, in the case of equal relative spacing, the operating profits are, by (21) - (23) and the limit above, continuously differentiable for all $\gamma \geq 1$ and satisfy, moreover

$$\pi_n \to 0 \qquad (\gamma \to 1).$$

Moreover, and very importantly, we can sign the slope of the ERSE's location, i.e., the function $\bar{\gamma}(\phi)$.

Proposition 3 *Let* $0<\underline{\phi}<\bar{\phi}<\infty$. *Then, the following statements hold.*

(i) There is a $r_0>0$ *so that* $\bar{\gamma}(\phi)$ *is weakly increasing on* $[\underline{\phi},\bar{\phi}]$ *for all* $r\in[0,r_0]$.

(ii) There is a $\phi_0\geq 0$ *so that* $\bar{\gamma}(\phi)$ *is constant if and only if* $\phi\leq\phi_0$.

Proof. See Appendix. ■

In combination with the proof of the existence of an equilibrium (see Proposition 1), Proposition 3 represents the main result of our analysis. It establishes the comparative statics of the equilibrium degree of spacing with regard to the entry cost.[22]

This proposition shows that higher setup costs increase the relative spacing between quality levels. Intuitively, firms must be compensated for increases in setup costs by increased profits. The latter profit increases are brought about by larger market shares and by higher markups and, ultimately, by a wider spacing parameter $\bar{\gamma}$. Larger markets induce more frequent firm entry and a higher density of quality supply, because higher sales and profits allow a faster recovery of setup costs. Markups, in turn, are decreasing in the density of supply and are thus decreasing in the market size.

[22] The condition on the interest rate r is a technical restriction without an economic interpretation.

In our modeling strategy, the direction of technological advance follows from firms' desire to differentiate their products from the competition. This causality contrasts with the motivation for research in most existing Schumpeterian growth models, where the motive for innovation is to undercut the competition in effective per-unit costs. This principal difference in the motivation of firms to conduct costly R&D is associated with the key implications that the degree of product market competition (PMC) depends positively on the rate of entry, as only frequent entry can give rise to dense competition in the quality space, and thus, to a high degree of PMC. In our setup, the degree of product market competition (PMC) and the frequency of innovation are thus increasing, whereas this relation is reversed in models a là Aghion and Howitt (1992).

Together with Lemma 2, Proposition 3 also determines the impact of market size (L) and marginal production costs (φ) on the spacing $\bar{\gamma}$ of the ERSE. In particular, increases in L and φ have a similar effect on $\bar{\gamma}$ as reductions in setup costs – all of them decreasing the equilibrium spacing $\bar{\gamma}$. Clearly, a larger market induces, *certeris paribus*, higher profits and allows firms to generate more profits. At constant setup costs, larger markets therefore experience more frequent entry of firms at closer distances: the competitive pressure among firms rises.

Surprisingly, productivity growth at the margin (a decrease in marginal production costs φ) *increases* relative spacing and *reduces* the toughness of competition. This adverse effect of marginal productivity growth on competitive pressure may appear somewhat puzzling. To understand the forces operating to this effect, observe that the preference specification generates, just as ordinary CES preferences, relative firm markups $p_n/c_n - 1$ that are independent of costs (see prices (20)). Put differently, at a given relative spacing, operating profits constitute a constant share of revenues. Hence, when quality levels are constant, an increase in marginal productivity (a drop in marginal costs) tends to curb revenues and thereby depresses operating profits.[23] As firms must cover their setup costs, however, the productivity gains that curb profits per consumer must come about with increases in market share, *i.e.*, with a wider equilibrium spacing. This widening of relative spacing does, at the same time, increase relative markups. Hence, competitive pressure decreases as marginal productivity grows.

Notice that for this effect to play a crucial role requires the assumption that demand does not react along an intensive margin. In particular, consumers do not react to price changes by consuming more or less but by switching to other firms.

[23] This aggregate relationship does not, of course, mean that each single firm can raise its profit by decreasing its productivity.

5 Endogenous Growth

Up to this point, we have treated the growth of valuations (10) and interest rates as exogenous and have neglected, moreover, resource constraints. In this section, we repair these shortcomings. In particular, we show that the partial equilibrium model above is compatible with individual optimization under balanced growth. In doing so, we postulate spillover effects of innovation and solve for endogenous growth rates.

The balanced growth path, if it exists, is characterized by a constant growth rate of income, g, as well as a constant rate of innovation. Under the simplifying assumption $f(\gamma) = 1$ regarding setup costs from (3), constant expenditure on innovation implies that income grows at the same pace as the top quality, $q_{\max}$, raised to the power of θ. Thus, $q_{\max}$ grows at the rate g/θ. Constant innovation also implies that, within each industry, the top quality increases by the factor $\bar{\gamma}$ each period of length Δ (compare (16)) – or $e^{\Delta g/\theta} = \bar{\gamma}$ so that $g/\theta = a/(\theta - 1)$. This identity determines the relation between the growth rate of income, g, and that of valuations, a.

Concerning the growth of valuations (10), we simply assume that the individual valuation v is proportional to a power of income, I. By the above identity relating growth rates, this assumption leads to

$$v = I^{(1-1/\theta)}. \tag{25}$$

Together with the utility (1) this specification implies that income and quality are complementary. The higher a consumer's income, the higher is v and thus her willingness to pay for quality. Our specific functional form (25) substantially simplifies the closing of the model, but at the cost of departing from the standard theory. More precisely, the literature following Shaked and Sutton (1982) postulates that v is a function of residual income (income minus the expenditure on the differentiated good). While this standard approach can be read as a shortcut for the consumption level of the homogeneous numeraire, no such interpretation is available for our specification above.[24] Nevertheless, we stress that any previous result is independent of our peculiar assumption, and only the endogeneity of interest and growth rates rely on it.

To identify a path of balanced growth, we return to the notation of many identical industries, indexed by $j \in [0, 1]$. Within industry j, a set of qualities S_{jt} is produced. The set S_{jt} expands over time and is therefore indexed by t.

[24] In fact, we leave the ground of classical consumer theory here.

Technology Spillovers. The structure of the model implies that, in addition to endogenous innovation through market entry, more general efficiency gains must be generated in order to sustain positive growth. More precisely, as an increasing number of high-quality goods are produced at higher unit labor requirements, the effective labor supply must increase at a corresponding rate. We therefore define a time-dependent parameter of labor productivity by B_t, which multiplies individual labor endowments. Further, we postulate that B_t depends positively on all qualities produced up to time t in the following way

$$B_t = \int_0^1 \sum\nolimits_{n \in S_j} [q_{nj}]^\mu \, dj.$$

Identifying a variety with the date at which it has been invented, we write q_t and compute B_t as the integral

$$B_t = \int_{-\infty}^{t} (q_\tau)^\mu \;\; \Psi(\tau) \; d\tau.$$

where Ψ is the density or rate at which the q are invented.

Constant innovation activity requires that the average rate of innovation be constant over time. Thus, we assume that in any two time intervals of equal length, the same number of additional qualities are invented. Consequently, the rate at which qualities appear on the timeline is constant: each dt, there are $\Delta^{-1}dt$ qualities invented (Δ from (16)), and the density Ψ of qualities is Δ^{-1}.

As argued above, the growth rate of the maximal q_t equals g/θ or $a/(\theta-1)$. Together, this argument implies that labor productivity becomes

$$B_t = \frac{q_t^\mu}{\Delta} \int_{-\infty}^{t} e^{-\mu a/(\theta-1)(t-\tau)} \; d\tau = \frac{1}{\mu \ln(\bar{\gamma})} q_t^\mu.$$

Finally, as income is proportional to labor productivity, $g = \dot{B}/B$ must hold so that balanced growth requires $\mu = \theta$ and

$$B_t = \frac{1}{\theta \ln(\bar{\gamma})} q_t^\theta \tag{26}$$

In sum, when production costs, preferences and spillover effects are governed by θ in the way specified by (1), (3), (4), (25) and (26), the model may generate balanced growth. An additional condition, however, is that individuals save at constant rates. We next propose a setup of individual optimization that generates constant savings rates.

Consumer Optimization. Nesting the paper's model in a standard dynamic setting with infinitely lived consumers and dynamic optimization is tricky for the following reason. The somewhat peculiar instantaneous utility (1) implies that the composite of differentiated goods is a non-linear function of expenditure. More precisely, the relative price between the quality aggregate and the homogeneous good is not constant in expenditure. A consumer reacts to these price effects when trading off a marginal increase of consumption today against consumption tomorrow. This reaction distorts the standard intertemporal optimality conditions. Worse yet, the price effect of a marginal quality upgrading of the bundle (1) is different for rich and for poor consumers, due to the varying markups from (20). We cannot hope to easily bring the resulting heterogeneous savings rates to a constant aggregate one while preserving the expenditure patterns that generate valuations (10) and (18).

To resolve these difficulties, we turn to a setting of overlapping generations, assuming that at each infinitesimal time interval dt, the constant mass $L/T \cdot dt$ of individuals is born, which constitutes a fraction of a continuum of overlapping generations. Individuals live for T time units so that, at each point in time, L individuals populate the economy. An individual born at t is endowed with l_{it} labor units, where l_{it} is distributed as

$$l_{it}^{(\theta-1)/\theta} \sim G(./const) \tag{27}$$

(G from (2)). Individuals save their labor income to consume at the end of their lifes. To avoid the difficulties of intertemporal expenditure allocation sketched above, the final consumption period has length zero. Lifetime wealth W is proportional to labor productivity l_{it}. With the adequate choice of the constant in (27), this implies that $v = W^{(\theta-1)/\theta}$, and hence valuations v are distributed according to (2).[25]

Aggregate Savings. Consider the cohort born at t_0, which is endowed with labor L/T. At time $t \in (t_0, t_0 + T)$ the cohort earns $B_t L/T$ from labor income, which it saves. Consequently, at period t of its life (we use that B_t grows at rate g, keeping in mind that $g = a\theta/(\theta - 1)$), its wealth amounts to

$$w_{t,t_0} = \int_{t_0-t}^{0} e^{-r\tau} B_{t+\tau} L/T \; d\tau = \frac{1 - e^{(g-r)(t_0-t)}}{g-r} \frac{B_t L}{T}.$$

Hence, the wealth of the oldest living cohort, which equals aggregate consumption E_t, is

$$E_t = w_{t,t-T} = \frac{e^{(r-g)T} - 1}{r-g} \frac{B_t L}{T}. \tag{28}$$

The total wealth of all living cohorts, expressed in terms of E_t, is

$$W_t = \int_0^T w_{t,t-\tau} d\tau = \frac{E_t - B_t L}{r-g}. \tag{29}$$

[25] The choice of the constant is not essential, as, by (10), a change in the constant amounts to a re-normalization of time.

Investment. Total investment goes to the invention of blueprints. Because each Δ time units the qualities of all industries are upgraded exactly once (see (16)), $\Delta^{-1} \cdot dt$ new blueprints appear in each infinitesimal time interval dt, generating the flow of investment costs

$$IN_t = \Delta^{-1} \phi q_t^{\theta}. \tag{30}$$

(Recall that $f(\gamma) \equiv 1$.)

Resource Constraint. We write Y_t for the value of the total output produced at time t. As every dollar produced ultimately ends up in the pockets of individuals, and individual income consists of returns to savings plus labor income, we have

$$Y_t = rW_t + B_t L \tag{31}$$

In the firms' books, the total value of output appears as the wage bill plus the flow of operating profits. Summing the value of all firms implies

$$Y_t = B_t L + P_t \tag{32}$$

where we have set $P_t = \int_0^1 \sum_{n \in S_j} \pi_{nj} \; dj$.[26] With these equations, we are ready to pin down the evolution of the economy and the interest rate.

Market Clearing. Capital market clearing requires that total investment equals output minus consumption expenditure ($Y_t - E_t = IN_t$). Using expression (31) and collecting E_t and W_t from (28) and (29), this condition yields

$$\frac{1}{r-g} \left[\frac{e^{(r-g)T} - 1}{(r-g)\,T} - 1 \right] = \frac{\phi}{L}. \tag{33}$$

Observing that the expression $((e^x - 1)/x - 1)/x = \sum_{k=2}^{\infty} k! x^k$ is increasing in x, we know that condition (33) uniquely determines the difference between interest rate and growth rate, $r - g$. Thus, (33) pins down expenditure on consumption (28) and total wealth (29).

To close the model, we finally combine (31) and (32) to observe that at each point in time, the total returns to savings equal the total operating profits. Writing this condition in per capita terms renders

$$r \frac{\frac{e^{(r-g)T} - 1}{(r-g)T} - 1}{r - g} B_t = \frac{P_t}{L}. \tag{34}$$

[26] We avoid the Greek Π to avoid confusing total instantaneous profits with the discounted flow of profits.

Because $g = a\theta/(\theta - 1)$, we can apply Lemma 2 (ii) to the transformation $(r, g, L) \to (\chi r, \chi g, \chi L)$. Hence, this transformation leaves the relative spacing γ unchanged, which implies that per capita profits, on the right of (34), remain constant. Further, the expression on the left is increasing in χ, thus determining the unique equilibrium interest rate χr. Finally, (33) determines the growth rate of the economy, g, and thus the growth rate of valuations is $a = g(\theta - \theta)/\theta$.

In summary, this last part of the analysis has demonstrated that long-run economic growth can arise from entrants' desire to differentiate their output in the quality space if, on the one hand, increasingly wealthy consumers are willing to pay for higher quality and, on the other hand, private firms' innovation generates income growth by enlarging the set of technologies available. In this way, the firms' research efforts generate exactly the income growth that is needed to spur demand for ever higher-quality goods.

6 Conclusion

The "Schumpeterian" class of endogenous growth models has focused almost exclusively on industries where the technological leader takes over the entire market.[27] A major shortcoming of this modeling strategy is that only one firm is active at a time, so that the degree of product market competition has to be introduced via exogenous parameters. In particular, the toughness of product market competition does not arise from firms' decisions to differentiate their products from those of their competitors.

In this paper, we address this shortcoming by developing a new model suitable for analyzing the competition, innovation, and growth nexus in vertically differentiated markets featuring a large number of firms and an endogenous degree of product differentiation. We focus on one-time innovation decisions and examine how the demand parameters themselves are shaped by the degree to which innovators distinguish their products from existing ones.

Our model helps to understand how firm profits, firm innovation, and the toughness of competition emerge endogenously. This understanding enables us to analyze how market characteristics influence product market competition and how, in turn, the toughness of competition affects investments in innovation and economic growth.

[27] Aghion et al. (1997) and Aghion et al. (2005) analyze an economy with two firms in a setup where demand parameters are fixed, but firms can innovate repeatedly to "escape" their competition.

Our work points out that creative destruction may work through pro-competitive effects rather than through the "business stealing" mechanism found in the existing literature. In our setup, business stealing by new entrants is limited in the sense that innovators only sell to a set of consumers whose demand was relatively poorly matched by the supply of pre-existing firms. Consequently, it is not warranted that innovation occurs too often in the decentralized economy. On the contrary, new entrants make the set of available goods more differentiated, which is shown to exert a pro-competitive effect on *all* firms, leading to a reduction of firm markups by all firms. This reduction is strongest for those firms closest to the technological frontier.

A Appendix

Proof: Lemma 1. (i) Under $q_n' = \chi q_n$, $v' = \chi^{\theta-1} v$ and $p_n' = \chi^{\theta} p_n$, the cutoffs from (5) become

$$v_n' = \chi^{\theta-1} v_n \qquad \text{and} \qquad v_{n+1}' = \chi^{\theta-1} v_{n+1}.$$

Now, based on (2), the transformation $v' = \chi^{\theta-1} v$ induces a new cdf $\tilde{G}$ with

$$\tilde{G}(v') = G(v). \tag{35}$$

With this identity and $v' = \chi^{\theta-1} v$, compute

$$\tilde{G}'(v') = \lim_{\delta \to 0} \frac{\tilde{G}(v') - \tilde{G}(v' - \delta)}{\delta} = \lim_{\delta \to 0} \frac{G(v) - G(v - \chi^{1-\theta}\delta)}{\delta} = \chi^{1-\theta} G'(v)$$

Hence, (7) is satisfied, since

$$\tilde{G}\left(v_{n+1}'\right) - \tilde{G}\left(v_n'\right) = G\left(v_{n+1}\right) - G\left(v_n\right)$$

and

$$\left(p_n' - c_n'\right)\left[\tilde{G}'\left(v_{n+1}'\right)\frac{dv_{n+1}'}{dp_n'} - \tilde{G}'\left(v_n'\right)\frac{dv_n'}{dp_n'}\right] = \left(p_n - c_n\right)\left[G'\left(v_{n+1}\right)\frac{dv_{n+1}}{dp_n} - G'\left(v_n\right)\frac{dv_n}{dp_n}\right]$$

where $dv_n'/dp_n' = \chi^{-1} dv_n/dp_n$ and $dv_{n+1}'/dp_n' = \chi^{-1} dv_{n+1}/dp_n$ has been used. This shows that $p_n' = \chi^{\theta} p_n$ solves the transformed pricing system. By (4), (8) and (35) $\pi_n' = \chi^{\theta} \pi_n$ follows, completing the proof of (i).

(ii) Under $q_n'' = q_n$, $v'' = \chi v$ and $p_n'' = \chi p_n$ the cutoffs from (5) become

$$v_n'' = \chi v_n \qquad \text{and} \qquad \overline{v}_n'' = \chi \overline{v}_n.$$

As in (i), the transformation $v'' = \chi v$ induces a new cdf $\hat{G}$ with

$$\hat{G}(v'') = G(v) \qquad \text{and} \qquad \hat{G}'(v'') = \chi^{-1} G'(v)$$

This implies that (7) is satisfied, since

$$\hat{G}\left(v''_{n+1}\right) - \hat{G}\left(v''_{n}\right) = G\left(v_{n+1}\right) - G\left(v_{n}\right)$$

holds and with $c''_n = \chi c_n$ and the above

$$\left(p''_n - c''_n\right)\left[\frac{d\hat{G}}{dv''}\left(v''_{n+1}\right)\frac{d\overline{v}''_n}{dp''_n} - \frac{dG}{dv''}\left(v''_n\right)\frac{d\underline{v}''_n}{dp''_n}\right] = \chi\left(p_n - c_n\right)\chi^{-1}\left[\frac{dG}{dv}\left(v_{n+1}\right)\frac{d\overline{v}_n}{dp_n} - \frac{dG}{dv}\left(v_n\right)\frac{d\underline{v}_n}{dp_n}\right]$$

where $dv''_n/dp''_n = dv_n/dp_n$ and $dv''_{n+1}/dp''_n = dv_{n+1}/dp_n$ has been used. This shows that $p''_n = \chi p_n$ solves the transformed pricing system. By (8) and $\hat{G}(v'') = G(v)$, $\pi''_n = \chi \pi_n$ follows, completing the proof of (ii). ■

Proof: Proposition 1. Consider the location choice of the first entrant ($n = 1$), given $\{q_n\}_{n \le 0}$ satisfying (9) with prevailing γ. Observe that $\gamma_1 = 1$ is not optimal since Bertrand competition would imply $\Pi_{t_1}(\gamma_1) = 0$ (regardless of t_1) in this case, thus violating the free entry condition. Hence, $\gamma = 1$ implies $\gamma_1 > \gamma$.

We show next that $\gamma_1 < \gamma$ holds for some γ large enough. Assume not: $\gamma_1 \ge \gamma$ for all $\gamma \ge 1$. This implies $\gamma_1 \to \infty$ as $\gamma \to \infty$. Consider this limit case, where $q_n = 0$ for $n \le 0$ and $q_n = \infty$ for $n > 1$ must hold. In this case firm 1 is effectively the monopolist in the market at all times after the entry date t_1. Consider a time $t' > t_1$ with $\pi_1(t') > 0$ and denote the optimal price with p'. A consumer with valuation v purchases q_1 at price p' if and only if $vq_1 \ge p'$. The cutoff at the lower bound is thus $\underline{\text{v}} = \max\{v_{\min}, p'/q\}$. If firm 1 follows the pricing strategy $p_1(t) = e^{at}p'$ for $t > t'$, the dynamics from (10) imply that $G_t(p_1(t)/q) = G_{t'}(p'/q)$. Therefore, we have

$$\pi_1(t) = \left[1 - G_{t'}(p'/q)\right](e^{at}p' - c)L \ge e^{a(t-t_1)}\pi_1(t')$$

Hence, the discounted value of profits at time t'

$$\int_0^\infty e^{rt}\pi_1(t)dt = \pi_1(t')\int_0^\infty e^{(a-r)t}dt$$

is unbounded. This implies that $\pi_1(t_1) = 0$ at entry date t_1. (It implies even more: production only becomes active at infinite time after entry). But at the entry date, the discounted flow of profits just covers the setup cost:

$$\int_{t_1}^\infty e^{rt}\pi_1(t)dt - F(q) = 0$$

must hold. Now, consider a potential entry at time t_1 but at location $\tilde{q}_1 = e^{-a\delta}q_1$ ($\delta > 0$). For this strategy (whose payoff is denoted by a~), Lemma 1 implies

$$\int_{t_1}^{\infty} e^{rt}\tilde{\pi}_1(t)dt - F(\tilde{q}_1) = e^{ra\delta(\theta-1)} \int_{t_1}^{\infty} e^{rt}e^{-a\delta\theta}\pi_1(t)dt - e^{-a\delta\theta}F(q_1)$$

The factor $e^{ra\delta(\theta-1)}$ stems from the time shift of profits associated with $\tilde{v} = e^{-a\delta(\theta-1)}v$. Total profits of this alternative strategy are thus positive

$$\int_{t_1}^{\infty} e^{rt}\tilde{\pi}_1(t)dt - F(\tilde{q}_1) > e^{-a\delta\theta}\left(\int_{t_1}^{\infty} e^{rt}\pi_1(t)dt - F(q_1)\right) = 0$$

contradicting the initial assumption of q's optimality. This shows that $\gamma_1 < \gamma$ holds for γ large enough.

Together, with $\gamma_1 > \gamma$ at $\gamma = 1$ continuity, we conclude that there is a $\gamma > 1$ so that $\gamma_1 = \gamma$. Denote this by $\bar{\gamma}$. At this $\gamma = \bar{\gamma}$, the firm $n = 1$ locates in the quality space, extending equal relative spacing (9) to all $n \leq 1$.

Take the case of $\gamma = \gamma_1 = \bar{\gamma}$ and call the spacing problem of the remaining additional firms ($n = 2, 3, ...$) the *residual spacing problem.* With the notation

$$\gamma'_n = \gamma_{n+1} \qquad (n \geq 1) \qquad q'_0 = \bar{\gamma}q_0 = q_1 \qquad \text{and} \qquad \tau' = \tau + a^{-1}(\theta - 1)\ln\bar{\gamma} \qquad \text{(A1)}$$

the residual spacing problem solves the corresponding system (13) - (15) above, where now all state and choice variables bear a prime (q', v', γ'). Apply Lemma 1 (i), (10) and (A1) to verify that

$$\pi_n\left(e^{a\tau}, \tilde{\Gamma}'_{n,k}\hat{\gamma}'\Gamma'^{(*)}_{0,n-1}q'_0, \gamma'_m, \gamma'_{m-1}, ..., \gamma'_1, \bar{\gamma}\right) = \bar{\gamma}^{\theta}\pi_n\left(e^{a\tau'}, \tilde{\Gamma}'_{n,k}\hat{\gamma}'\Gamma'^{(*)}_{0,n-1}q_0, \gamma'_m, \gamma'_{m-1}, ..., \gamma'_1, \bar{\gamma}\right)$$

Notice that the setup cost (3) satisfies $F(\Gamma'^{(*)}_{0,n-1}, q'_0) = F(\Gamma'^{(*)}_{0,n-1}\bar{\gamma}, q_0) = \bar{\gamma}^{\theta}F(\Gamma'^{(*)}_{0,n-1}, q_0)$. Hence, $\bar{\gamma}^{\theta}$ factors out of the right hand side of (13) and of the square brackets in (14). Consequently, the solution of the residual spacing problem coincides with the original problem, implying $\gamma'_1 = \gamma_2 = \gamma_1 = \bar{\gamma}$. A simple induction argument completes the proof that $\gamma_n \equiv \bar{\gamma}$ for all $n \geq 1$.

Finally, (10) and the transformation (A1) show that two consecutive entries occur at dates satisfying $v_{\max}(t_n) = \bar{\gamma}^{\theta-1}v_{\max}(t_{n+1})$. With (10), this is $e^{a(t_{n+1}-t_n)} = \bar{\gamma}^{\theta-1}$ and proves the second statement. ■

Proposition 2. Substitution $u_n = p_n - \alpha c_n$ and recursive formulation (19) of the prices gives

$$2\left[u_n + \alpha c_n\right] = c_n + \frac{1}{\gamma+1}\left[u_{n+1} + \alpha c_{n+1}\right] + \frac{\gamma}{\gamma+1}\left[u_{n-1} + \alpha c_{n-1}\right]$$

for $n > 1$. With $\alpha = (1+\gamma)/\left[2(1+\gamma) - \gamma^{\theta} - \gamma^{1-\theta}\right]$ this is $2(\gamma+1)u_n = u_{n+1} + \gamma u_{n-1}$. The equation

$$X^2 - 2(\gamma+1)X + \gamma = 0 \tag{36}$$

has two roots, $\lambda = \left[\gamma + 1 + \sqrt{\gamma^2+\gamma+1}\right]$ larger than unity and $\mu = \left[\gamma + 1 - \sqrt{\gamma^2+\gamma+1}\right]$, smaller than unity. The general solution to the recursive series is thus

$$p_n = \tilde{A}\lambda^n + \tilde{B}\mu^n + \alpha c_n \tag{37}$$

where $\tilde{B} = 0$ because of $\mu < 1$ and the transversality condition $\lim_{n \to -\infty} p_n = 0$. Equation (19) for $n = 0$ is $2p_0 = c_0 + (q_0 - q_{-1})\, v_{\max} + p_{-1}$ and implies

$$2\left[\tilde{A} + \alpha c_0\right] = c_0 + q_0\,(1 - 1/\gamma)\, v_{\max} + \tilde{A}/\lambda + \alpha c_{-1}.$$

Solving for $\tilde{A}$ and replacing $A = \tilde{A}/c_0$ proves (23). ■

Proof: Proposition 3. As a preparatory step, define net profits of the first entrant as a function of existing spacing γ, setup costs ϕ, entry date t and location choice $\hat{\gamma}$, while suppressing dependence of Ψ on parameters other than ϕ and normalizing $q_0 = 1$:

$$\Psi(\gamma, t, \hat{\gamma}, \phi) = \Pi(\hat{\gamma}, t) - \phi f(\hat{\gamma})$$

Free entry implies that equilibrium entry date and location $t^*(\gamma)$ and $\gamma^*(\gamma)$ satisfy

$$\Psi(\gamma, t^*(\gamma), \gamma^*(\gamma), \phi) = 0. \tag{38}$$

for all γ and ϕ and optimal location choice implies

$$\Psi_{\gamma^*}(\gamma, t^*(\gamma), \gamma^*(\gamma), \phi) = 0. \tag{39}$$

Taking derivatives of (38) w.r.t. γ and using (39) yields

$$\Psi_\gamma + \Psi_{t^*}\frac{dt^*}{d\gamma} = 0. \tag{40}$$

At the ERSE, (38) is $\Psi(\bar{\gamma}(\phi), t^*(\bar{\gamma}(\phi), \phi), \bar{\gamma}(\phi), \phi) = 0$. Taking derivatives w.r.t. ϕ yields

$$0 = \left[\Psi_\gamma + \Psi_{t^*}\frac{dt^*}{d\gamma}\right]\frac{d\bar{\gamma}}{d\phi} + \Psi_{t^*}\frac{\partial t^*}{\partial \phi} + \Psi_\phi = \Psi_{t^*}\frac{\partial t^*}{\partial \phi} + \Psi_\phi$$

where equation (40) has been used. With $\Psi_\phi = -f(\bar{\gamma})$ this implies

$$\frac{\partial t^*}{\partial \phi} = \frac{f(\bar{\gamma})}{\Psi_{t^*}}. \tag{41}$$

Now, taking derivatives of (39) w.r.t. γ leads to

$$0 = \Psi_{\gamma^*\gamma} + \Psi_{\gamma^* t^*}\frac{dt^*}{d\gamma} + \Psi_{\gamma^*\gamma^*}\frac{d\gamma^*}{d\gamma}. \tag{42}$$

At the ERSE, (39) is $\Psi_{\gamma^*}(\bar{\gamma}(\phi), t^*(\bar{\gamma}(\phi), \phi), \bar{\gamma}(\phi), \phi) = 0$. Taking derivatives w.r.t. ϕ yields

$$0 = \left[\Psi_{\gamma^*\gamma} + \Psi_{\gamma^* t^*}\frac{dt^*}{d\gamma} + \Psi_{\gamma^*\gamma^*}\right]\frac{d\bar{\gamma}}{d\phi} + \Psi_{\gamma^* t^*}\frac{\partial t^*}{\partial \phi} + \Psi_{\gamma^*\phi}$$

or, with (42),

$$\Psi_{\gamma^*\gamma^*}\left[1 - \frac{d\gamma^*}{d\gamma}\right]\frac{d\bar{\gamma}}{d\phi} = -\Psi_{\gamma^* t^*}\frac{\partial t^*}{\partial \phi} - \Psi_{\gamma^*\phi}$$

Equations (3) and (12) imply $\Psi_{t^*}|_{\gamma=\bar{\gamma}} = r\phi f(\bar{\gamma}) - \pi(t^*)$ and $\Psi_{t^*\gamma^*}|_{\gamma=\bar{\gamma}} = r\phi f'(\bar{\gamma}) - \pi_{\gamma^*}(t^*)$ and $\Psi_{\gamma^*\phi} = -f'(\gamma^*)$ so that we have with (41)

$$\Psi_{\gamma^*\gamma^*}\left[1 - \frac{d\gamma^*}{d\gamma}\right]\frac{d\bar{\gamma}}{d\phi} = \left\{\pi_{\gamma^*}(t^*) - \pi(t^*)\frac{f'(\bar{\gamma})}{f(\bar{\gamma})}\right\}\frac{f(\bar{\gamma})}{\Psi_{t^*}} \tag{43}$$

The second order condition of (43) and the firm's optimization yields $\Psi_{\gamma^*\gamma^*} < 0$, while (17) implies that the term in the square brackets is positive. Moreover, by definition of t^*, $\Psi_{t^*} > 0$ holds. Consequently, $\bar{\gamma}(\phi)$ is increasing (constant) in ϕ if and only if the expression in the slanted brackets on the right is negative (zero).

(i) Whenever $\pi(t^*) = 0$ the expression on the right is zero and thus $\bar{\gamma}$ is constant in ϕ. We thus need to show that at the ERSE $d\ln(\pi(t^*)/f(\gamma^*))/d\gamma^* < 0$ holds for $\pi(t^*) > 0$. To this aim, recall that $f(.)$ from (3) was assumed to satisfy $d\left[f(q/\bar{q})\bar{q}^{\theta}\right]/d\bar{q} \leq 0$, or, with $\gamma = q/\bar{q}$, $\theta \leq \gamma f'(\bar{\gamma})/f(\bar{\gamma})$. It is thus suffices to show (remember $q_0 = 1$ so that $q_1 = \gamma^*$)

$$\frac{d}{dq_1}\ln(\pi(t^*)) < \theta/q_1$$

With profits $\pi_1 = L(p_1 - c_1)(v_{\max} - v_1)/v_{\max}$ and $v_1 = (p_1 - p_0)/(q_1 - q_0)$ (compare (5) and (6), shifting up indices) and the envelope theorem the condition above is equivalent to

$$\frac{-\dot{c}_1}{p_1 - c_1} - \frac{1}{v_{\max} - v_0}\left(\frac{-\dot{p}_0}{q_1 - q_0} - \frac{p_1 - p_0}{(q_1 - q_0)^2}\right) < \theta$$

where $\dot{x} \equiv dx/dq_1$. With (5) and (19) (shifting up indices) this condition is

$$\dot{p}_0 + v_1 - \dot{c}_1 < \theta\,(p_1 - c_1) \tag{44}$$

To compute $\dot{p}_0$ write the system (19) as

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -(q_0 - q_{-1}) & 2(q_1 - q_{-1}) & -(q_1 - q_0) & 0 \\ 0 & -(q_{-1} - q_{-2}) & 2(q_0 - q_{-2}) & \ldots \\ \ldots & 0 & \ldots & \ldots \end{pmatrix} p = \begin{pmatrix} c_1 + (q_1 - q_0)v_{\max} \\ (q_1 - q_{-1})c_0 \\ (q_0 - q_{-2})c_{-1} \\ \ldots \end{pmatrix} \tag{45}$$

where $p \equiv (p_1, p_0, ...)^t$. Taking derivatives w.r.t. q_1 yields

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -(q_0-q_{-1}) & 2(q_1-q_{-1}) & -(q_1-q_0) & 0 \\ 0 & -(q_{-1}-q_{-2}) & 2(q_0-q_{-2}) & ... \\ ... & 0 & ... & ... \end{pmatrix} \dot{p} + \begin{pmatrix} 0 & 0 & 0 & 0 & ... \\ 0 & 2 & -1 & 0 & \\ 0 & 0 & 0 & 0 & \\ ... & & & & ... \end{pmatrix} p = \begin{pmatrix} \dot{c}_1 + v_{\max} \\ c_0 \\ 0 \\ ... \end{pmatrix}$$

and evaluating at the ERSE leads to

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2(\gamma+1) & -\gamma & 0 \\ 0 & -1 & 2(\gamma+1) & -\gamma \\ ... & 0 & ... & ... \end{pmatrix} \dot{p} = \begin{pmatrix} \dot{c}_1 + v_{\max} \\ \gamma \frac{-2p_0+p_{-1}+c_0}{q_1-q_0} \\ 0 \\ ... \end{pmatrix} \tag{46}$$

Replicating the proof of Proposition 2, we obtain that $\dot{p}_n$ satisfies $\dot{p}_n = \lambda^n \dot{p}_0$ with $\lambda = \gamma + 1 + \sqrt{\gamma^2 + \gamma + 1}$ for $n \leq 0$. The second row of (46) thus becomes

$$-\dot{p}_1 + [2(\gamma+1) - \gamma/\lambda]\, \dot{p}_0 = -\dot{p}_1 + \lambda \dot{p}_0 = -\gamma \frac{2p_0 - p_{-1} - c_0}{q_1 - q_0}$$

where we have used that λ solves (36). Combining this equation with the first row of (46) ($2\dot{p}_1 - \dot{p}_0 = \dot{c}_1 + v_{\max}$) leads to

$$[2\lambda - 1]\, \dot{p}_0 = \dot{c}_1 + v_{\max} - 2\gamma \frac{2p_0 - p_{-1} - c_0}{q_1 - q_0} = \dot{c}_1 + v_{\max} - 2\frac{p_1 - 2p_0 + c_0}{q_1 - q_0}$$

where we used the second row of (45) in the last step. With $v_1 = (p_1 - p_0)/(q_1 - q_0)$ and $v_{\max} - v_1 = (p_1 - c_1)/(q_1 - q_0)$ (compare (5) and (19)), we have

$$\dot{p}_0 = \frac{1}{2\lambda - 1} \left\{ 3v_{\max} - 6v_1 + \dot{c}_1 + 2\frac{c_1 - c_0}{q_1 - q_0} \right\}$$

and hence

$$\dot{p}_0 + v_1 - \dot{c}_1 = \frac{1}{2\lambda - 1} \left\{ 3\,(v_{\max} - v_1) - 2\left(\dot{c}_1 - \frac{c_1 - c_0}{q_1 - q_0} \right) - [2\lambda - 4]\,(\dot{c}_1 - v_1) \right\} \tag{47}$$

We show next that (44) holds for all $t \in [t^{**}, t^{**} + \delta]$ with $\delta > 0$ small enough and t^{**} defined as the date where $v_{\max} = v_1$ holds. At this date we have $p_1 = c_1$ by (19) so that

$$\dot{p}_0 + v_1 - \dot{c}_1 = \frac{-2}{2\lambda - 1} \left[\left(\dot{c}_1 - \frac{c_1 - c_0}{q_1 - q_0} \right) + [\lambda - 2]\,(\dot{c}_1 - v_1) \right] < 0$$

where the last inequality holds by $\lambda > 2$ and

$$v_1 = \frac{c_1 - p_0}{q_1 - q_0} < \frac{c_1 - c_0}{q_1 - q_0} < \dot{c}_1$$

for all $\gamma > 1$, showing (44). By continuity, there is a $\delta > 0$ so that (44) holds for all $t \in [t^{**}, t^{**} + \delta]$.

Now, since π_1 from (24) is increasing in t, there is an $\varepsilon > 0$ so that $t \in (t^{**}, t^{**} + \delta)$ holds whenever $\pi_1 < \varepsilon$. This last condition holds for $r > 0$ small enough as $\Psi_{t^*} > 0$ implies $0 < -\pi_1 + r\Pi$ or

$$\pi_1 < r\phi f(\bar{\gamma}) \tag{48}$$

Finally, we restrict the pair of parameters (r, ϕ) to the compact set $[0, r_1] \times [\underline{\phi}, \bar{\phi}]$. Hence, there are $\gamma_{\min}$ and $\gamma_{\max}$ with $1 < \gamma_{\min} < \gamma_{\max} < \infty$ so that $\bar{\gamma}$ is restricted to the compact set $[\gamma_{\min}, \gamma_{\max}]$. Consequently, there is a uniform $r_0 \leq r_1$ so that for all $(r, \phi) \in [0, r_1] \times [\underline{\phi}, \bar{\phi}]$ we have $\pi_1 < \varepsilon$ and (44) holds uniformly. This proves the statement.

(ii) First notice with (43) that $\bar{\gamma}$ is constant if $\pi(t^*) = 0$. If $\pi(t^*) = 0$, (40) and (41) imply

$$\frac{dt^*(\bar{\gamma}(\phi), \phi)}{d\phi} = \frac{\partial t^*}{\partial \phi} = -\frac{\Psi_\phi}{\Psi_{t^*}} = \frac{f(\bar{\gamma})}{r\Pi} = \frac{1}{r\phi}$$

and we have $t^*(\phi) = \mathit{const} + \ln(\phi)$. As profits from (24) are increasing in t, this implies that, if $\pi(t^*)|_{\phi=\phi_1} = 0$ for $\phi_1 > 0$ decreases in ϕ leave $\bar{\gamma}$ unchanged and decrease t^*. Consequently, $\pi(t^*) = 0$ holds for all $\phi < \phi_1$.

Using $t^*(\phi) = \mathit{const} + \ln(\phi)$ and rescaling time we can write $t^*(\phi) = \ln(\phi)$. But by (24) there is a $\tilde{v}_{\max} \in (0, \infty)$ so that $A + \alpha - 1 = 0$, implying that $\pi_1 = 0$ marginally, and $\pi_1 > 0$ if $v_{\max} > \tilde{v}_{\max}$. Therefore, at entry cost $\tilde{\phi} \equiv (\tilde{v}_{\max}/v_{\max}(0))^{1/a}$ we have $t^*(\tilde{\phi}) = a^{-1} \ln(\tilde{v}_{\max}/v_{\max}(0))$ and

$$v_{\max}(t^*(\tilde{\phi})) = \tilde{v}_{\max}$$

Hence, $\pi_1 = 0$ holds for all $\phi \leq \tilde{\phi}$ and $\pi_1 > 0$ else. Together with (i) and (43) this proves the statement. ∎

A.1 Table A1: List of data and Constructed Concordance

Table A1 - Patenting, Demand Elasticity and Vertical Differentiation in 38 Industries

Sector Name in Johnson (2002)	*1998 Patents (in Thousands)*						*Concorded to Bils*	*Concorded to Broda et al. 2006*	
	DEN	*FRA*	*DEU*	*ITA*	*NLD*	*UK*	*and Klenow (2001)*	*Elasticity of Subst.*	*HS Code*
Fishing, operation of fish hatcheries	1	8	11	4	8	6	0	8.924688	30
Growing of vegetables, horticultural	1	7	14	4	5	4	0	3.674227	70
Growing of fruit, nuts, beverage	0	1	1	0	1	0	0	8.507714	80
Growing of cereals and other	7	32	77	18	22	19	0	4.692318	100
Manufacture of food products	22	88	231	66	123	84	0	3.923592	160
Manufacture of tobacco products	0	4	20	8	1	7	0	5.284598	240
Other mining and quarrying	0	2	6	1	1	1	0	2.304289	250
Mining of metal ores	1	6	22	3	2	4	0	25.0319	260
Mining of uranium and thorium	0	0	1	0	0	0	0	1.162808	261
Mining of coal and lignite	0	1	4	1	1	1	0	3.555673	270
Manufacture of coke, refined petroleum	4	41	68	11	18	15	0	3.555673	270
Extraction of crude petroleum, natural	2	57	96	14	24	36	0	3.733456	271
Manufacture of chemical products	81	491	1411	170	254	427	0	1.276282	280
Manufacture of pharmaceuticals	47	187	432	62	92	196	0	2.649367	300
Manufacture of paints	1	20	91	6	14	15	0	5.003189	320
Manufacture of soap	2	25	93	9	32	40	0	5.418962	340
Manufacture of pesticides	4	25	68	11	9	20	0	6.257432	380
Manufacture of rubber and plastics	12	114	436	78	67	74	0	1.340096	390
Tanning and dressing of leather	0	23	25	29	5	4	0	10.12166	410
Manufacture of wood products	3	20	74	17	7	9	1	5.239073	441
Manufacture of paper products	4	32	229	31	21	35	0	3.417688	481
Publishing, printing and reproduction	3	33	174	23	27	27	0	1.788363	490
Manufacture of man-made fibers	1	12	45	8	6	8	0	6.020334	540
Manufacture of textiles	3	34	156	33	16	23	0	6.020334	540
Manufacture of wearing apparel	1	12	36	18	2	6	1	2.182875	610
Manufacture of basic precious metals	0	14	49	7	3	7	1	2.029337	711
Manufacture of basic iron and steel	1	25	101	24	5	13	0	2.955521	720
Manufacture of fabricated metal	6	94	335	60	32	57	0	5.893758	730
Casting of iron and steel	1	9	33	7	2	5	1	1.472366	732
Casting of non-ferrous metals	0	0	0	0	0	0	0	2.878988	741
Manufacture of machinery	48	737	2281	426	363	398	0	6.14374	840
Manufacture of electrical machinery	1	18	40	6	7	7	0	9.124066	844
Manufacture of office machinery	6	187	407	70	169	118	1	3.567555	847
Manufacture of television, radio recvr.	1	65	157	20	112	41	1	2.844963	852
Manufacture of radio, television	7	281	632	74	249	139	1	2.844963	852
Manufacture of insulated wire, cable	18	40	6	7	7	1	0	4.764942	854
Manufacture of electronic valves	95	248	35	60	37	3	0	4.764942	854
Manufacture of railway	23	105	13	4	10	1	0	4.018861	860
Manufacture of motor vehicles	391	1461	192	66	147	13	1	16.19744	870

Table A1

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