Offshoring and Sequential Production Chains: A General-Equilibrium Analysis

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Working Paper 14.01

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This version: January 5, 2014§

Abstract

In this paper, we develop a two-sector general equilibrium trade model which includes offshoring, sequential production, and endogenous market structures. We analyze how relative factor endowments and various forms of globalization and technological change affect equilibrium offshoring patterns. We show that, against common belief, a reduction in trade costs lowers the range of tasks offshored even though the aggregate volume of offshoring may increase.

Keywords: Offshoring, sequential production, global production chain, task trade.

JEL Classification: D24, F10, F23, L23.

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§For helpful comments and suggestions on a preliminary version of this paper, we thank Frank Stähler and participants at the ETSG 2012 Leuven conference, at the ATW 2013 Melbourne workshop, and at seminars in Hannover, Heidelberg, Mainz, and Tuebingen.

1 Introduction

Over the past years a lot of attention has been devoted to the determinants and consequences of the "second unbundling" in international trade (Baldwin 2006) – i.e., the international fragmentation of production.[1] Most analyses of this phenomenon are based on a specific idea of the production process, according to which individual tasks can be ordered with respect to the cost advantage of performing them abroad (including additional monitoring and communication costs resulting from foreign production), and the decision to offshore a certain task depends on the production and offshoring costs for that particular task. Some recent contributions, however, explicitly consider the fact that in many cases the production process is *sequential* – i.e., individual steps follow a predetermined sequence that cannot be modified at will.[2]

Relative costs of performing individual steps not necessarily increase or decline monotonically along the production path. Instead, a foreign country may have a cost advantage for some particular segment of the production process whereas preceding and following segments can be performed at lower costs in the domestic country. If such a situation is combined with transport costs of shipping intermediate goods across borders, firms may find it disadvantageous to offshore certain steps even if they could be performed at much lower costs abroad. The reason is that the domestic country may have a cost advantage with respect to adjacent steps, and the costs of shifting back and forth intermediate goods may more than eat up potential cost savings from fragmenting the production process. Such a constellation may have important implications for observed offshoring patterns. For example, it may explain why certain production processes are less fragmented internationally as one might expect – given the large international discrepancies in factor prices.

Baldwin and Venables (2013) as well as Harms et al. (2012) have shown how such

[1] A non-exhaustive list of important contributions to this literature includes Jones and Kierzkowski (1990), Feenstra and Hanson (1996), Kohler (2004), and Grossman and Rossi-Hansberg (2008).

[2] See, e.g., Antras and Chor (2013), Baldwin and Venables (2013), Costinot et al. (2012, 2013), Harms et al. (2012), or Kim and Shin (2012). Important earlier approaches are Dixit and Grossman (1982), Sanyal (1983), Yi (2003), and Barba Navaretti and Venables (2004). Fally (2012), Antras and Chor (2013), and Antras et al. (2012) provide empirical measures to characterize sequential production processes.

insights regarding the offshoring decision of individual firms can be obtained already from simple partial equilibrium models in which factor prices are exogenous. However, to arrive at conclusions about the entire economy, we need to account for the influence of firms' offshoring decisions on factor prices at home and abroad, and we have to consider the repercussions of induced factor price changes on firms' optimal behavior – i.e., we need to model offshoring in a general equilibrium framework. This is what the current paper does.

Baldwin and Venables (2013) model firms' offshoring decisions for different configurations of production processes – "snakes" and "spiders" – with spiders reflecting a situation in which different intermediate inputs may be simultaneously produced in different countries to be eventually assembled at a central location, and snakes capturing the notion of sequentiality outlined in the introduction. Using these alternative frameworks, they analyze the consequences of exogenous variations in production costs and offshoring costs, including costs of shipping intermediate goods across borders. Highlighting the "tension between comparative costs creating the incentive to unbundle, and co-location or agglomeration forces binding stages of production together"(Baldwin and Venables 2013: 246), which is also characteristic for our setup, they show that a decrease in shipping costs may result in an *overshooting* of the overall offshoring pattern: Production stages may first be shifted abroad to take advantage of co-location and then return to the domestic country as shipping costs further decrease. In Harms et al. (2012), relative costs fluctuate symmetrically along the production process, allowing a deeper analysis of the technological factors that influence the offshoring decision. In addition to shipping costs, Harms et al. (2012) consider communication and supervision costs caused by offshoring activities as well as the variability of these costs between tasks, or the total length of the production chain.

The current paper starts from a similar symmetric specification of a production-"snake"and places this in a general equilibrium context. We present a two-country (North-South) model in which firms whose production is entirely domestic may coexist with multinational firms who decide on the international distribution of production. Production is based on a rigid sequence of individual steps, and the foreign cost advantage evolves in a non-monotonic fashion along the production chain: some steps are cheaper to perform in the South, some are cheaper to perform in the North, and so on. Finally, every step requires the presence of the unfinished intermediate good, and shifting intermediate goods across borders is associated with transport costs. Wages and prices in both economies are endogenous, and the increasing demand for labor that is generated by accelerating North-South offshoring may eventually result in wage increases that make offshoring less attractive. Using this framework and employing numerical simulations, we explore how changes in transport costs, relative productivities and properties of the production process affect the number of firms whose entire production takes place in the domestic economy, the number of firms that decide to fragment their production process between North and South, and the number of firms that relocate (almost) the entire production process to the foreign country.

In another related approach, Costinot et al. (2012, 2013) use a general-equilibrium model with multiple countries to analyze how a countries' productivity – as reflected by its propensity to commit mistakes – determines the stages of production it attracts. These authors also emphasize the concept of *sequentiality*, i.e., the idea that the order in which tasks have to be performed is exogenously determined. Moreover, the transport costs in our model play a role that is somewhat similar to the "coordination costs" in Costinot et al. (2013: 111), with a decrease in these costs inducing a non-monontonic adjustment of the overall volume of offshoring. However, we deviate from the monotonicity assumption in Costinot et al. (2012, 2013), which stipulates that countries can be ordered by their relative productivities. Conversely, we model a two-country world in which the relative cost advantage of the North fluctuates non-monotonically along the production chain. This enables us to analyze how specific properties of industry-specific production processes affect the volume of offshoring and relative factor prices across countries.

2 The Model

2.1 Preferences

There are two countries (or regions), North and South, with an asterisk denoting South-specific variables. Consumers in both countries have Cobb-Douglas preferences over two consumption goods, X and Y. The X sector produces a continuum of differentiated varieties, whereas goods from the Y sector are homogeneous. Household preferences are

$$Con = X^{\beta} Y^{1-\beta}, \qquad 0 < \beta < 1 , \quad \text{and} \tag{1}$$

$$X = \left[\int_{i \in N} x(i)^{\rho_x} di \right]^{\frac{1}{\rho_x}} , \qquad 0 < \rho_x < 1 . \tag{2}$$

The index i denotes individual varieties, and $\sigma_x = 1/(1-\rho_x)$ is the elasticity of substitution between these varieties. Maximizing utility for a given income level (Inc) yields the following demand system:

$$x(i) = \left(\frac{P_X}{p(i)} \right)^{\sigma_x} X , \tag{3}$$

$$P_X = \left[\int_{i \in N} p(i)^{1-\sigma_x} di \right]^{\frac{1}{1-\sigma_x}} , \quad P_X X = \beta Inc , \quad \text{and} \quad p_Y Y = (1-\beta) Inc . \tag{4}$$

2.2 Technologies and Production Modes

Each country is endowed with fixed quantities of labor $\bar{L}$ (in efficiency units) and a fixed composite factor $\bar{R}$. As Markusen and Venables (1998), we assume that labor can be employed in both sectors, whereas the fixed composite factor, which may be land or a natural resource, is specific to industry Y (see also Markusen, 2002). Good Y is produced in both countries by a competitive industry and can be freely traded. Production of Y employs L and R according to a CES-technology

$$Q_Y = \left[\alpha_R R_Y^{\rho_Y} + \alpha_L L_Y^{\rho_Y} \right]^{\frac{1}{\rho_Y}} , \qquad 0 < \rho_Y < 1 , \tag{5}$$

from which profit maximization yields the following demand for the two factors of production:

$$R_Y = \alpha_R^{\sigma_Y} \left(\frac{p_Y}{r} \right)^{\sigma_Y} Q_Y \quad \text{and} \quad L_Y = \alpha_L^{\sigma_Y} \left(\frac{p_Y}{w} \right)^{\sigma_Y} Q_Y . \tag{6}$$

The term $\sigma_Y = 1/(1-\rho_Y)$ stands for the elasticity of substitution between the two factors R_Y and L_Y, and r and w denote factor prices, respectively. If good Y is produced, perfect competition implies that

$$p_Y = \left[\alpha_R^{\sigma_Y} r^{1-\sigma_Y} + \alpha_L^{\sigma_Y} w^{1-\sigma_Y}\right]^{\frac{1}{1-\sigma_Y}} \ . \tag{7}$$

We choose good Y as our numeraire throughout the paper, and from free trade in Y and product homogeneity we have $p_Y = p_Y^* = 1$.

Varieties of good X can only be produced by firms whose headquarters are located in the North. While we do not explicitly model research and development, this assumption can be rationalized by arguing that only Northern firms are able to develop and use the blueprints necessary for production. Firms in sector X act under monopolistic competition, and each firm has to spend fixed costs in addition to the variable production costs.

The production process of any variety $x(i)$ consists of a continuum of tasks, indexed by t, ranging from 0 to 1. These tasks have to be performed following a strict sequence, i.e., they cannot be re-arranged at will. This property of a sequential production process is important since some of these tasks may be offshored to exploit international cost differences, and these cost differences may vary along the production process in a non-monotonic fashion. A firm producing a given variety of good X may choose between two available *production modes*: domestic (D) and multinational (M). While domestic firms perform the entire production process in the North, multinational firms can offshore some of the tasks to the South. The goal of our analysis is to determine the share of good-X producers that decides to take advantage of the possibility to produce internationally and to derive the amount of offshoring chosen by these firms.

Each task t involves a given quantity of labor. The labor coefficients $a_D(t)$ or $a_M(t)$ denote the efficiency units of labor that are necessary for a domestic (D) or multinational (M) firm to perform task t in the North.[3] For simplicity, we assume that $a_D(t) = a_D$ and $a_M(t) = a_M$ for all t, i.e., input coefficients in the North do not differ across tasks. Being a

[3] By contrast to this assumption of fixed labor coefficients, Jung and Mercenier (2008), who analyze skill/technology upgrading effects of globalization, employ a heterogeneous workers framework in which a worker's productivity is determined endogenously by his own talent and the technology he uses.

multinational firm may come with a higher labor productivity in the North, i.e., $a_M \leq a_D$. Input coefficients $a^*_M(t)$ of performing a task in the South possibly differ from a_M, and – more importantly – these coefficients vary across tasks. For example, some tasks benefit more strongly from a better educated workforce or a better production infrastructure in the North, implying a lower labor coefficient in the North than in the South. Other tasks may be less sophisticated such that the effective labor input for these tasks may be lower in the South than in the North. In addition, offshoring firms possibly have to employ additional labor to monitor tasks performed in the South or to communicate with the headquarter in the North, and these monitoring and communication requirements may also vary across tasks. Summing up, there may be tasks which – given wages w and w^* in the North and the South respectively – are cheaper to perform domestically and tasks which are cheaper to perform abroad. This is represented by Figure 1, which juxtaposes the (constant) costs per task wa_D if these tasks are performed by a Northern domestic firm, the (constant) costs wa_M if they are performed domestically by a multinational firm, and the varying costs $w^*a^*_M(t)$ if these tasks are offshored to the South.

Multinational firms have to adjust to the fact that, at given wages, some tasks that are cheaper to perform in the South may be "surrounded" by tasks for which the North has a cost advantage (and vice versa). In Figure 1, the tasks $t \in [t_1, t_2]$ and $t \in [t_3, t_4]$ would be performed at lower cost if they were offshored to the South by Northern multinational firms. Conversely, all tasks $t \in [0, t_1]$, $t \in [t_2, t_3]$ and $t \in [t_4, 1]$ would be cheaper to perform domestically. To simplify the analysis, we assume that the first and the last task are tied to being performed in the North.

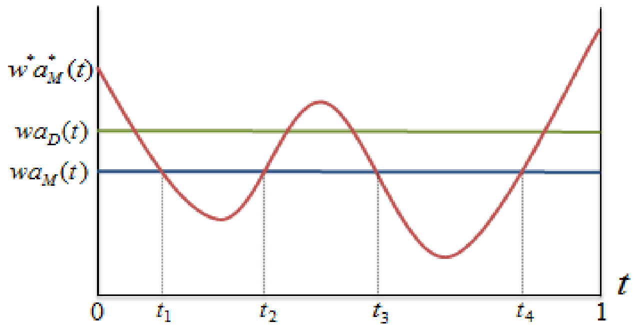

Figure 1: Costs along the Production Chain

As in Yi (2003), Barba Navaretti and Venables (2004), or Harms et al. (2012), we furthermore assume that production tasks require the presence of the unfinished intermediate good, and that moving (intermediate) goods between countries is associated with constant transportation costs per unit. More specifically, any crossing of national borders requires T units of labor in the sending country. It is for this reason that Northern multinational firms may find it profitable to stay with the entire production at home or to adopt a strategy of agglomerating (almost) the entire production process $t \in [t_1, t_4]$ abroad, rather than paying trade costs for each time the unfinished good is crossing borders.

In the rest of this paper, we restrict our attention to a symmetric specification of the $a_M^*(t)$ curve:[4]

$$a_M^*(t) = A \cos(2n\pi t) + B \, . \tag{8}$$

The symmetry engrained in this specification offers a flexible way to capture the non-monotonic evolution of relative costs along the value-added chain while dramatically simplifying the analysis: first and foremost, instead of determining separate cut-off values $t_1, t_2, ..., t_{2n}$, we can exploit the fact that $t_2 = \frac{1}{n} - t_1, t_3 = \frac{1}{n} + t_1$ etc. (see Figure 4 for an illustration). The first cut-off t_1 thus determines all critical values of t_i. Moreover, the individual parameters characterizing the cosine-function have a straightforward economic interpretation: While the shift parameter B reflects the average labor coefficient in the South, the parameter A, which determines the function's amplitude, captures the *het-*

[4] This cosine specification is similar to the working paper version of Harms et al. (2012), which, however, did not incorporate general equilibrium effects.

erogeneity of task-specific input requirements across countries. The variable n ($n \in \mathbb{N}^+$) measures the number of "cycles" that $a_M^*(t)$ completes between $t = 0$ and $t = 1$. We argue that production processes that are characterized by a higher number of cycles – i.e., a larger n – are more *sophisticated*, exhibiting more variability in terms of cost differences. To keep the analysis interesting, we assume that foreign production costs fluctuate around domestic costs more than once ($n \geq 2$).

Offshoring with positive production volumes in both countries can only occur if each country has a cost advantage for some tasks. Technically, this requires that the $w^* a_M^*(t)$ curve in Figure 1 intersects the wa_M line at least once. Therefore, a necessary condition for international production sharing in our paper is

$$\frac{B - A}{a_M} < \frac{w}{w^*} < \frac{B + A}{a_M} \; . \tag{9}$$

Due to the symmetric specification of the $a_M^*(t)$ function, we may distinguish three firm-types $j \in \{D, M^f, M^a\}$: domestic firms (*domestic production,* D), multinational firms that fragment their production chain (*fragmentation,* M^f), or multinational firms that perform most tasks in the South (*production abroad,* M^a).[5] Fragmented multinationals offshore all segments that are cheaper to perform in the South, i.e., all tasks between t_i and t_{i+1} ($i = 1, 3, \ldots, 2n-2$), and produce all other segments at home. Production abroad firms offshore the entire segment between t_1 and t_{2n}, and produce only the first segment between 0 and t_1 and the last segment between t_{2n} and 1 at home.

2.3 Costs and Prices

Defining for future use the dummy term γ ($\gamma = 0$ if $j = D$, and $\gamma = 1$ if $j = M^f, M^a$), marginal costs of firm type j are given by the following expression:

$$C_j = wL_j + \gamma w^* L_j^* \; . \tag{10}$$

[5] The fact that there are only three firm types in equilibrium is due to our symmetric parametrization of $a_M^*(t)$.

The variables L_j and L_j^* stand for the labor input at home and abroad *per unit of output* of a representative type-j firm, given by

$$L_D = a_D \ , \tag{11}$$

$$L_{M^f} = 2na_M t_1 + nT \ , \tag{12}$$

$$L_{M^a} = 2a_M t_1 + T \ , \tag{13}$$

$$\begin{aligned} L^*_{M^f} &= n \int_{t_1}^{\frac{1}{n}-t_1} \left(A\cos(2\pi nt) + B\right) dt + nT \\ &= B\left[1 - 2nt_1\right] - \frac{A}{\pi}\sin\left(2\pi nt_1\right) + nT \ , \quad \text{and} \end{aligned} \tag{14}$$

$$\begin{aligned} L^*_{M^a} &= \int_0^1 \left(A\cos(2\pi nt) + B\right) dt - 2\int_0^{t_1} \left(A\cos(2\pi nt) + B\right) dt + T \\ &= B\left[1 - 2t_1\right] - \frac{A}{n\pi}\sin(2\pi nt_1) + T \ . \end{aligned} \tag{15}$$

Note that with n cycles, fragmented firms (M^f) have $2n+1$ sequential production stages, and transport costs of n times T in each country, while production abroad firms (M^a) have 3 sequential production stages and transport costs of T in each country.

We assume that exporting final X-goods to the South is associated with iceberg trade costs $\tau > 1$ per unit. Using this information as well as equations (3) and (4), we can write the demand faced by a representative firm of type j as

$$x_j^d = \left(\frac{P_X}{p_j}\right)^{\sigma_x} X \quad \text{and} \quad x_j^{d*} = \tau\left(\frac{P_X^*}{p_j \tau}\right)^{\sigma_x} X^* \ . \tag{16}$$

The price indices for the X sector in the North and the South are given by

$$P_X = \left[\sum_j N_j p_j^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}} \quad \text{and} \quad P_X^* = \left[\sum_j N_j \left(p_j \tau\right)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}} , \tag{17}$$

where N_j stands for the mass of firms of each firm-type.

If type-j firms in sector X produce a strictly positive quantity ($x_j > 0$), they charge a constant mark-up rate over their marginal costs:

$$p_j = \frac{\sigma_x}{\sigma_x - 1} C_j \ . \tag{18}$$

We assume that all active firms have to incur fixed costs that take the form of unsold final goods, i.e., they amount to a multiple of marginal costs $F_j C_j$, with $F_j > 0$. Consistent with evidence we assume $F_D < F_{M^f} < F_{M^a}$.[6] Free entry ensures zero profits, such that revenues cover total costs:

$$\frac{1}{\sigma_x} p_j x_j \leq C_j F_j \ , \tag{19}$$

where $N_j > 0$ if the respective condition holds with equality, and $N_j = 0$ otherwise. With positive production, we can combine (18) with (19) to derive production of a firm of type j in equilibrium:

$$x_j = (\sigma_x - 1) F_j \ . \tag{20}$$

3 Equilibrium

3.1 Definition

An equilibrium is defined by an optimal cutoff value $\tilde{t}_1$, a vector of wages as well as prices and quantities in the X and Y sectors, and an industrial structure (N_D, N_{M^a}, N_{M^f}) such that (i) firms of a given type in the X-sector set profit-maximizing prices,
(ii) no firm has an incentive to change its production mode,
(iii) multinational firms of a given type (M^a, M^f) choose the optimal pattern of offshoring, i.e., the cutoff value $\tilde{t}_1$,
(iv) free entry results in zero-profits of all active firms in equilibrium,
(v) in the Y-sector, factor prices reflect their marginal products,
(vi) goods and factor markets are cleared.

3.2 Cutoff Task

The optimal cutoff task $\tilde{t}_1$ is implicitly determined by the following condition:

$$w a_M = w^* a_M^*(\tilde{t}_1) \ . \tag{21}$$

Using (8), we can obtain $\tilde{t}_1$ as

$$\tilde{t}_1 = \frac{1}{2\pi n} \arccos\left(\frac{a_M w / w^* - B}{A}\right) \ . \tag{22}$$

[6] As is common in the literature, we assume that the fixed organizational and set-up costs are higher abroad than domestically. Consequently, they increase in the range of activities performed abroad, implying $F_{M^a} > F_{M^f}$.

That is, the range of offshored tasks depends on the domestic labor coefficient a_M, the average foreign coefficient B, and on the wage w/w^* in the North relative to the South. Not surprisingly, an increase in w/w^* *ceteris paribus* lowers $\tilde{t}_1$, i.e.,

$$\frac{\partial \tilde{t}_1}{\partial \left(w/w^*\right)} = -\frac{a_M}{2\pi n A \sqrt{1-\left(\frac{a_M w/w^* - B}{A}\right)^2}} < 0 \; .$$

The economic intuition behind this result is straightforward: as the domestic wage relative to the foreign wage increases, firms have an incentive to perform a smaller share of the production process at home and a larger share abroad. In a similar fashion, we may determine the influence of the variables determining average costs, a_M and B, as well as the heterogeneity parameters A and n.

3.3 Market Clearing

Labor market equilibrium requires

$$\begin{aligned}\bar{L} =& L_Y + L_D\left(x_D + F_D\right) N_D + \\ & L_{M^f}\left(x_{M^f} + F_{M^f}\right) N_{M^f} + L_{M^a}\left(x_{M^a} + F_{M^a}\right) N_{M^a} \quad \text{and} \qquad (23)\\ \bar{L^*} =& L_Y^* + L_{M^f}^*(x_{M^f} + F_{M^f}) N_{M^f} + L_{M^a}^*(x_{M^a} + F_{M^a}) N_{M^a} \; , \qquad (24)\end{aligned}$$

where $L_D(x_D + F_D)$ is the fixed and variable labor input required by domestic firms to produce x_D units of output, $L_{M^f}(x_{M^f} + F_{M^f})$ the fixed and variable labor input used by fragmenting multinationals in the North etc. For the market of the fixed composite factor to be in equilibrium we need

$$\bar{R} = R_Y \quad \text{and} \quad \bar{R^*} = R_Y^* \; . \qquad (25)$$

The equilibrium conditions on the factor markets determine w, r, w^* and r^*, respectively, and final-goods' market equilibrium imposes that

$$\begin{aligned} x_j = x_j^d + x_j^{d*} \quad & \text{as well as} \qquad (26)\\ Q_Y + Q_Y^* = Y + Y^* \; . & \qquad (27)\end{aligned}$$

Finally, incomes depend on factor prices and (fixed) factor supplies:

$$Inc = r\overline{R} + w\overline{L} \quad \text{and} \quad Inc^* = r^*\overline{R^*} + w^*\overline{L^*} \; . \qquad (28)$$

3.4 Production Regimes

In what follows, we distinguish between equilibria in which all X-sector firms choose the same production mode and equilibria in which firms with different production modes coexist. We will refer to these constellations as *production regimes*: a *domestic* production regime is characterized by $N_{M^a} = N_{M^f} = 0$ and $N_D > 0$, a *mixed domestic/fragmented* regime is characterized by $N_{M^a} = 0$ and $N_D > 0, N_{M^f} > 0$, etc.

In a regime in which different production modes j and k coexist, equation (20) implies $x_j/x_k = F_j/F_k$. Since it follows from (16) and (18) that $x_j/x_k = (p_j/p_k)^{-\sigma_x}$ and $p_j/p_k = C_j/C_k$, we obtain

$$C_k = \left(\frac{F_k}{F_j}\right)^{-1/\sigma_x} C_j \ . \tag{29}$$

Unit costs C_k and C_j are given by (10) combined with (11) – (15). For further reference we define

$$\varphi_f = \left(\frac{F_{M^f}}{F_D}\right)^{-1/\sigma_x} \qquad \text{and} \qquad \varphi_a = \left(\frac{F_{M^a}}{F_D}\right)^{-1/\sigma_x} \tag{30}$$

as the fixed cost advantage of producing domestically compared to fragmented production or production abroad, respectively. Since $F_D < F_{M^f} < F_{M^a}$, we have $\varphi_a < \varphi_f < 1$. With these definitions, we can characterize conditions for the different production regimes. For example, if $C_{M^f}/\varphi_f = C_D$, domestic and fragmented production coexist. If $C_{M^f}/\varphi_f = C_a/\varphi_a$, fragmented production and production abroad coexist. If, however, $C_D < C_{M^f}/\varphi_f$ and $C_D < C_{M^a}/\varphi_a$, all firms choose domestic production. From (10) to (15), and combining (8) and (21) we have

$$\frac{C_D}{w} = a_D \ , \tag{31}$$

$$\frac{C_{M^f}}{w} = 2na_M\tilde{t}_1 + nT + \frac{a_M}{A\cos(2\pi n\tilde{t}_1) + B}\left[B\left(1 - 2n\tilde{t}_1\right) - \frac{A}{\pi}\sin\left(2\pi n\tilde{t}_1\right) + nT\right] \ , \tag{32}$$

$$\frac{C_{M^a}}{w} = 2a_M\tilde{t}_1 + T + \frac{a_M}{A\cos(2\pi n\tilde{t}_1) + B}\left[B\left(1 - 2\tilde{t}_1\right) - \frac{A}{n\pi}\sin\left(2\pi n\tilde{t}_1\right) + T\right] \ . \tag{33}$$

Figure 2 depicts marginal costs (relative to φ and w) as a function of the optimal cutoff value $\tilde{t}_1$.[7]

[7] From taking the derivatives of (32) and (33) and from $1/\varphi_f < 1/\varphi_a$, it can be shown that $\partial\left(C_{M^f}/w\right)/\partial\tilde{t}_1 < \partial\left(C_{M^a}/w\right)/\partial\tilde{t}_1$, as depicted in Figure 2.

From the intersection points $C_{M^f}(\tilde{t}_1)/\varphi_f = C_D$ and $C_{M^a}(\tilde{t}_1)/\varphi_a = C_{M^f}(\tilde{t}_1)/\varphi_f$, we may determine critical values $\tilde{t}_1^f$ and $\tilde{t}_1^a$, respectively. These values define different possible firm regimes:[8]

(i) If $\tilde{t}_1 > \tilde{t}_1^f$, i.e., $C_D < C_{M^f}/\varphi_f$, we have $N_D > 0$ and $N_{M^f} = N_{M^a} = 0$

(ii) At $\tilde{t}_1^f$, we have $N_D > 0$, $N_{M^f} > 0$ and $N_{M^a} = 0$

(iii) If $\tilde{t}_1^a < \tilde{t}_1 < \tilde{t}_1^f$, we have $N_{M^f} > 0$ and $N_D = N_{M^a} = 0$

(iv) At $\tilde{t}_1^a$, we have $N_{M^f} > 0$, $N_{M^a} > 0$ and $N_D = 0$

(v) If $\tilde{t}_1 < \tilde{t}_1^a$, we have $N_{M^a} > 0$ and $N_D = N_{M^f} = 0$.

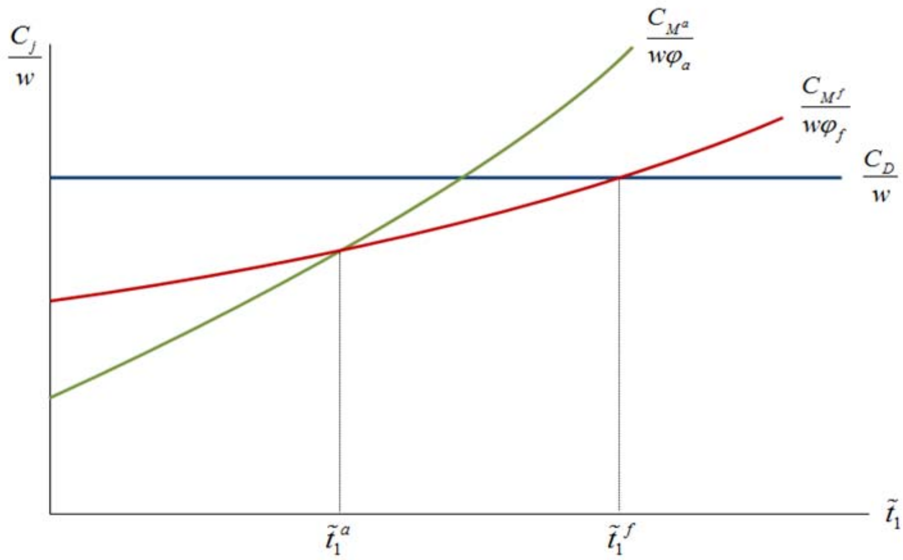

Figure 2: Equilibrium Regimes and Cutoff

Note that the ordering of regimes may also differ from that depicted in Figure 2. For example, if $\tilde{t}_1^f < \tilde{t}_1^a$, we only have firms producing abroad (for low $\tilde{t}_1$) and/or domestic firms (for higher $\tilde{t}_1$). Furthermore, if $C_{M^f}/\varphi_f < C_{M^a}/\varphi_a$ at $\tilde{t}_1 = 0$, there is no intersection point $\tilde{t}_1^a$, such that only fragmented firms and/or domestic firms exist in equilibrium. Such an outcome is likely for low transport costs T. Finally, if it happens to be $\tilde{t}_1^f = \tilde{t}_1^a$, all three firm types coexist at this point.

[8] Since (22) determines a monotonic relationship between the optimal cut-off $\tilde{t}_1$ and the relative wage w/w^*, we could also define critical values for w/w^* instead.

3.5 Model Solution

This section illustrates the necessary steps to determine the equilibrium values of the model variables in a *given* production regime. As a starting point, we consider the most straightforward case, the domestic production regime in which there is no offshoring to the South. From equations (5) and (6) we can derive

$$\frac{w}{w^*} = \left(\frac{\alpha_R \left(\frac{\bar{R}}{L_Y}\right)^{\rho_Y} + \alpha_L}{\alpha_R \left(\frac{\bar{R}^*}{L_Y^*}\right)^{\rho_Y} + \alpha_L} \right)^{\frac{1-\rho_Y}{\rho_Y}} . \tag{34}$$

This shows that the relative wage decreases in domestic employment in the Y-sector (L_Y) and increases in foreign employment in that sector (L_Y^*). Given the diminishing marginal product of labor in that sector, this result is not surprising. In a production regime in which all firms of the X-sector produce only domestically, we have $N_{M^f} = N_{M^a} = 0$. Using this property, the firm-level output (20) as well as the domestic labor-market clearing condition (23) yields

$$L_Y = \bar{L} - a_D \sigma_x F_D N_D . \tag{35}$$

Purely domestic production also implies $L_Y^* = \bar{L}^*$. To compute the number of firms in equilibrium (N_D), we substitute the supply equation (20) as well as the demand and pricing equations (16) and (17) into the goods market equilibrium condition (26) . This results in

$$N_D = \left(\frac{X + \tau X^*}{(\sigma_x - 1) F_D} \right)^{\frac{\sigma_x - 1}{\sigma_x}} . \tag{36}$$

Combining (4), (6), (10), (11), (17), (18), and (28) yields

$$X = N_D^{\frac{1}{\sigma_x - 1}} \frac{\beta(\sigma_x - 1)}{a_D \sigma_x} \left(\bar{L} + \frac{\alpha_R}{\alpha_L} \left(\frac{L_Y}{\bar{R}} \right)^{\frac{1}{\sigma_Y}} \bar{R} \right) \quad \text{and} \tag{37}$$

$$X^* = N_D^{\frac{1}{\sigma_x - 1}} \frac{\beta(\sigma_x - 1)}{a_D \sigma_x \tau} \left(\frac{w^*}{w} \right) \left(\bar{L}^* + \frac{\alpha_R}{\alpha_L} \left(\frac{\bar{L}^*}{\bar{R}^*} \right)^{\frac{1}{\sigma_Y}} \bar{R}^* \right) . \tag{38}$$

Equations (36), (37) and (38) imply

$$N_D = \Lambda \left[\bar{L} + \frac{\alpha_R}{\alpha_L} \left(\frac{L_Y}{\bar{R}} \right)^{\frac{1}{\sigma_Y}} \bar{R} + \left(\frac{w^*}{w} \right) \left(\bar{L}^* + \frac{\alpha_R}{\alpha_L} \left(\frac{\bar{L}^*}{\bar{R}^*} \right)^{\frac{1}{\sigma_Y}} \bar{R}^* \right) \right] , \tag{39}$$

where $\Lambda \equiv \frac{\beta}{a_D \sigma_x F_D}$. Combining (35) and (39) yields L_Y as a function increasing in the relative wage w/w^*. This relationship is driven by the fact that an increasing domestic wage raises costs in the X-sector and reduces the number of firms N_D. This, in turn, raises the supply of labor in the Y-sector. Together with (34), which establishes a negative relationship between L_Y and w/w^*, this uniquely determines the relative wage in equilibrium. Once we have identified the equilibrium value of w/w^* for a given parameter constellation, we have to check whether it is indeed the case that $C_D < C_{M^f}/\varphi_f$ and $C_D < C_{M^a}/\varphi_a$.

The analysis of the two other regimes that are characterized by a single firm type proceeds in a similar fashion. Compared to the domestic-production regime, the only complication that arises is that we cannot set $L_Y^* = \bar{L}^*$ and that the cost function of domestic firms in the X sector involves the optimal threshold value $\tilde{t}_1$ (see (12) – (15)). However, equation (22) characterizes $\tilde{t}_1$ as a function of the relative wage w/w^*, which closes the model.

In a production regime with *two* firm types we have to determine an additional endogenous variable (e.g., N_{M^f}, if domestic and fragmented firms coexist). This can be achieved by using the free-entry condition (29) as an additional equation. In the mixed *domestic/fragmented regime*, for example, we have to determine both N_D and N_{M^f}, and we can use the additional condition $C_{M^f}/\varphi_f = C_D$, combined with (10), (11), (12), (13) and (22) to uniquely determine the relative wage w/w^*. Again, once we have computed prices and quantities for such a regime, we have to make sure that no firm has an incentive to deviate from its choice of production mode. The analysis of other mixed production regimes proceeds in the same fashion.

3.6 Comparative Static Analysis

Given the complexity of our model, it is hard to derive general qualitative comparative-static results. However, for the case of a mixed production regime, we can use the free-entry condition (29) and the formula for the optimal cutoff value (22) to analyze how exogenous variations of parameter values affect the relative wage and the intensive margin of offshoring. We specify this for the case of a regime in which domestic and fragmented production coexist. As mentioned in the previous section, this requires $C_{M^f}/\varphi_f = C_D$.

Inserting for C_{Mf} and C_D yields $L_{Mf}w + L^*_{Mf}w^* = \varphi_f L_D w$, or

$$\frac{w}{w^*} = \frac{L^*_{Mf}}{\varphi_f a_D - L_{Mf}}$$
$$= \frac{B\left[1 - 2n\tilde{t}_1\right] - \frac{A}{\pi}\sin(2\pi n\tilde{t}_1) + nT}{\varphi_f a_D - 2na_M\tilde{t}_1 - nT} \ . \tag{40}$$

This condition establishes a relationship between the optimal cutoff value $\tilde{t}_1$ and the relative wage w/w^*. Taking the derivative of (40) with respect to $\tilde{t}_1$ yields

$$\frac{\partial(w/w^*)}{\partial\tilde{t}_1} = \frac{-2na^*_M(\tilde{t}_1)\left(\varphi_f L_D - L_{Mf}\right) + 2na_M L^*_{Mf}}{\left(\varphi_f L_D - L^*_{Mf}\right)^2}$$
$$= \frac{-2na_M w/w^*\left(\varphi_f L_D - L_{Mf}\right) + 2na_M L^*_{Mf}}{\left(\varphi_f L_D - L^*_{Mf}\right)^2}$$
$$= \frac{-2na_M L^*_{Mf} + 2na_M L^*_{Mf}}{\left(\varphi_f L_D - L^*_{Mf}\right)^2}$$
$$= 0 \ .$$

A marginal change in the optimal cutoff does not influence the relative wage. This result is an application of the envelope theorem: a change in w/w^* would be necessary if a variation of $\tilde{t}_1$ affected firms' marginal costs at the given relative wage. However, since $\tilde{t}_1$ is chosen to minimize the marginal costs of fragmented firms, this effect disappears.

Figure 3 depicts the "free-entry" (FE) relationship (40) and the "optimal cutoff" (OC) condition (22). As we have seen in section 3.2, the optimal cutoff $\tilde{t}_1$ decreases in the relative wage w/w^*. The point of intersection defines the unique cut-off value $\tilde{t}_1$ and the relative wage w/w^* which prevail in a regime in which domestic and fragmented production coexist. For a mixed regime with domestic firms and firms *producing abroad*, we can set up a similar figure as the free entry condition is also horizontal in this case.

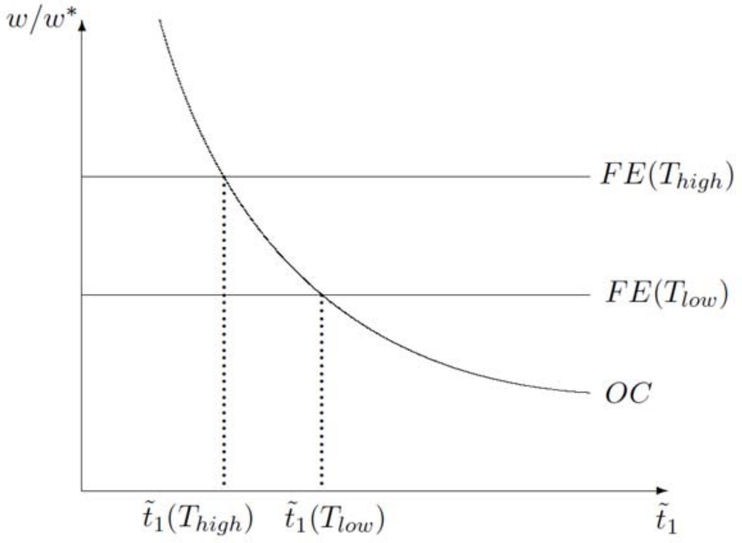

Figure 3: Equilibrium Wages and Cutoff Task in the D/M^f Regime

We are particularly interested in the effect of varying the transport costs T on the model equilibrium. From (40) we see that a decline in T shifts down the horizontal FE-curve in Figure 3. Conversely, the OC-curve is unaffected.

The resulting shift of the point of intersection shows that lowering T raises $\tilde{t}_1$, i.e., it reduces the share of tasks that multinational firms with fragmented production delegate to the South. The reason is that lower transport costs make offshoring more attractive. The resulting increase in labor demand in the South relative to the North reduces w/w^* and thus provides an incentive to extend the part of the production process that is performed at home. Similarly, it is straightforward to show that $d\tilde{t}_1/da_D > 0$, $d\tilde{t}_1/da_M < 0$, $d\tilde{t}_1/dA > 0$, and $d\tilde{t}_1/dB > 0$, $d\tilde{t}_1/dn < 0$ from the symmetric cosine specification.

This, however, does not necessarily mean that globalization – as represented by a decrease of transport costs – lowers the total volume of offshoring. While firms reduce offshoring at the *intensive margin*, lowering the share of tasks performed abroad, offshoring may increase at the *extensive margin*, with more firms choosing fragmented production. To analyze as to whether this conjecture is correct, we turn to a numerical simulation in the next section.

4 A Numerical Appraisal

In this section, we perform a numerical exercise to further understand the forces that determine the extent of offshoring. As outlined above, our model allows for two dimensions along which the extent of offshoring changes as exogenous parameters vary: first, the share of the production process that is performed abroad for a given firm-type may increase or decrease (the *intensive margin*). Second, the number of firms of a certain type may vary (the *extensive margin*). We analyze how offshoring reacts at the extensive and at the intensive margin to a decline in transport costs, changes in relative productivities and other properties of the production process.

4.1 Calibration

The two countries are scaled so that initially about half of the domestic consumption of good Y is imported from the South, while X goods are produced only by Northern firms and exported to the South. With preference parameter $\beta = 0.5$, the North is endowed with one third of the world's R and two thirds of the world's L, while the South is endowed with the rest.[9] In industry X, we set $\sigma_x = 4$ and $n = 2$ as benchmark values for the substitution elasticity and the number of cycles of $a^*_M(t)$. We choose somewhat arbitrarily – but within the ranges consistent with our theoretical constraints – the fixed costs and the trade cost of each border crossing of intermediate inputs: $F_D = 1.0$; $F_{M^f} = 1.3$; $F_{M^a} = 1.5$; $T = 0.2$. With this functional form and parameter values, we calibrate the key technological parameters – A, B, a_D and a_M –, so that initially about half of the total X is produced by D-type firms and the other half by M^f-type firms. Figure 4 displays the three calibrated technologies and the resulting cutoff tasks.[10]

[9] Note that our framework adopts elements from both the Ricardian and the Heckscher-Ohlin framework, i.e., trade is driven by differences in factor endowments and by technological differences. While our assumption that the North is relatively abundant in labor may seem unjustified at first glance, note that L reflects *effective* labor supply, which is determined by both demographic and technological factors.

[10] Appendix A reports the benchmark parameter and equilibrium variable values.

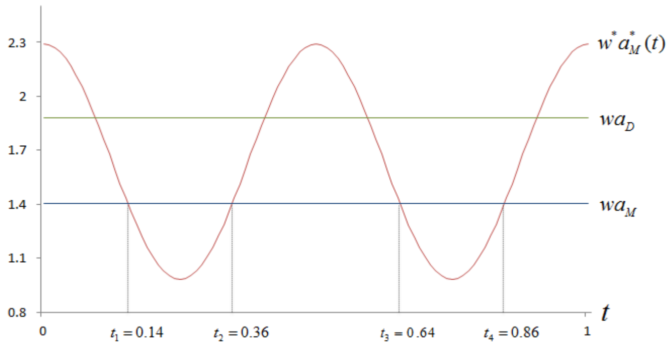

Figure 4: Technologies and Cutoff Tasks

4.2 Factor Endowments and Production Regimes in Equilibrium

Given our benchmark parameter values, Figure 5 presents the equilibrium production regimes for different allocations of production factors between the two countries. The vertical axis is the total world endowment of effective labor, and the horizontal axis is the total world endowment of the composite factor, with the North measured from the southwest (SW) and the South from the northeast (NE).

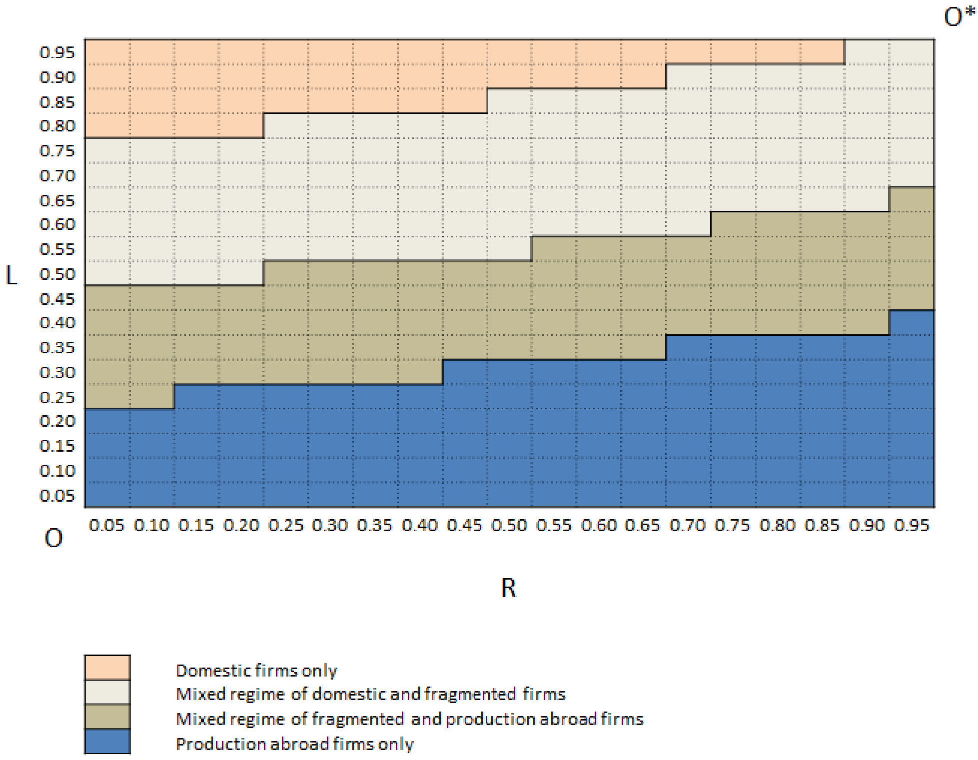

Figure 5: Equilibrium Regimes

We see that the equilibrium regimes are associated with differences in relative factor endowments. Intuitively, if the North is highly abundant in effective labor – such that this factor is relatively cheap – no offshoring occurs. Conversely, if the South is highly abundant in labor, we may expect that most firms produce abroad. For intermediate allocations of factors of production, fragmented firms should dominate.[11]

Figure 6 displays the equilibrium number of each firm-type along the NW-SE diagonal – the two countries differ in relative factor endowments – and along the SW-NE diagonal – the two countries have identical relative factor endowments but differ in size.

[11] Indeed, altering the distribution of the world endowment in much finer steps, we also have regimes in which only fragmented firms exist between the two regimes of $\{D, M^f\}$ and $\{M^f, M^a\}$.

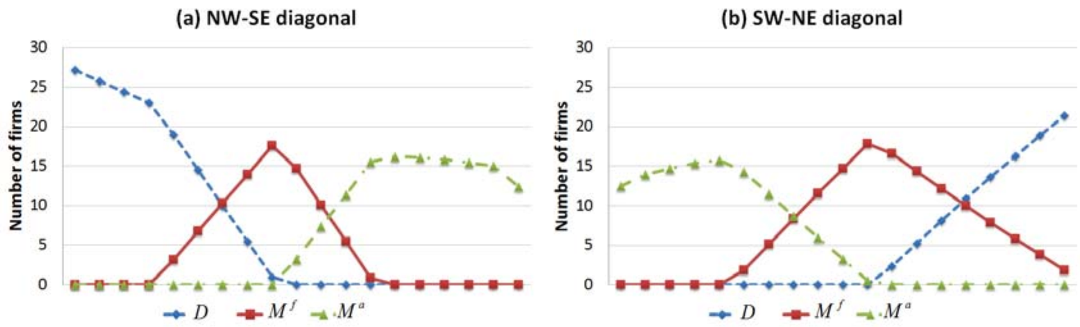

Figure 6: Endogenous Market Structure along the NW-SE Diagonal (North becomes less labor-abundant) and SW-NE Diagonal (North becomes bigger)

As we move from left to right in panel (a) of Figure 6, the relative share of the North in global labor supply decreases, and its share in the composite factor increases . This raises the Northern wage rate, making offshoring more attractive and thus reducing the number of domestic firms. First, we observe the emergence of fragmented firms that allocate their production process to different countries. Eventually, as the bulk of global labor is located in the South, firms decide to offshore the biggest possible part of the value-added chain, leaving only M^a firms in business.

The effect of increasing the size of the North – leaving relative factor endowments constant – is depicted in panel (b) of Figure 6. As the North is growing bigger, it hosts an increasing share of the global labor supply. Since the labor endowment is decisive for the attractiveness of offshoring, the picture is first dominated by multinationals producing in the South, and eventually by domestic firms. At intermediate stages – i.e., for constellations at which both countries are of roughly equal size – most of global production is performed by fragmenting multinationals that intensively exploit international cost differences.

4.3 The Effects of Globalization

We now investigate the effects of globalization, which we interpret as a decline in T.[12] Figure 7 reports the effects of a decline in T on unit production costs and entry of each

[12] Given that all final X-varieties are produced in the North and exported to the South, a decline in τ – the iceberg trade costs in final X-goods – only affects welfare of the South (positively).

firm-type. Note that the horizontal axis is inverted, moving from higher to lower values – i.e., the extent of globalization increases (with transport costs decreasing) from left to right.

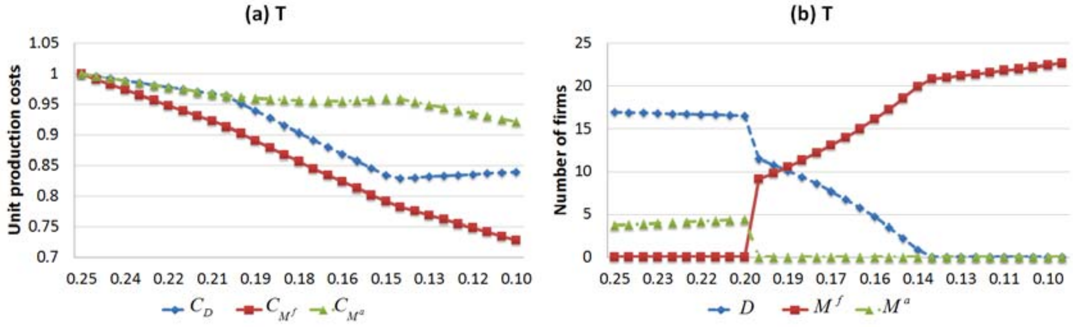

Figure 7: Globalization and C_j and N_j

We see that in general a fall in T reduces costs for all three types of firms, with fragmented firms benefitting most strongly. Conversely, the costs of firms producing abroad decrease much less: First, a decline in transport costs has a lower impact for firms that produce abroad compared with fragmented firms. Second, the reduced transport costs are partly offset by increasing wages in the South. As for the adjustment of globalization at the extensive margin, domestic and production abroad firms prevail if transport costs are high. As T decreases, fragmentation becomes more attractive, and eventually, this becomes the dominant mode of production.

Figure 8 presents the induced variations in cutoff tasks $\tilde{t}_1$ and $\tilde{t}_2$ (with symmetric effects on $\tilde{t}_3$ and $\tilde{t}_4$ due to our cosine specification).

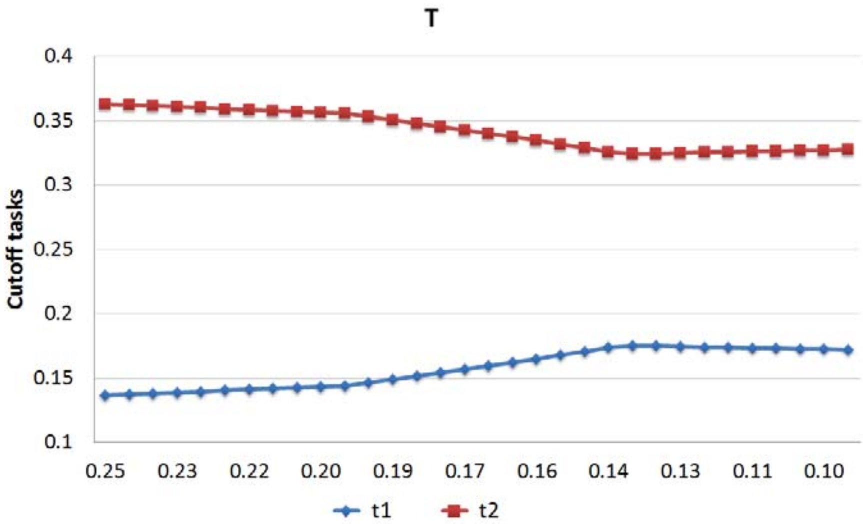

Figure 8: Globalization and Cutoff Tasks

As already shown in the comparative static analysis for the mixed production regime, a decline in trade costs first lowers the range of tasks offshored. This is due to a general equilibrium effect: a higher demand for labor in the South, resulting from an increasing number of offshoring firms lowers the wage ratio (w/w^*), as shown by Figure 9. The indifference conditions on production costs (21) indicate that this makes it less profitable to offshore a large range of tasks. However, for very low levels of T, as only fragmented firms exists, the effect of a decline in T on the cutoff values is reversed. This is due to the fact that lowering the transport costs and thus releasing labor in both countries, has a bigger impact on the South given that the North is relatively L-abundant in our benchmark. As shown by Figure 9, this raises w/w^* so that it is profitable to offshore a higher range of tasks to the South.

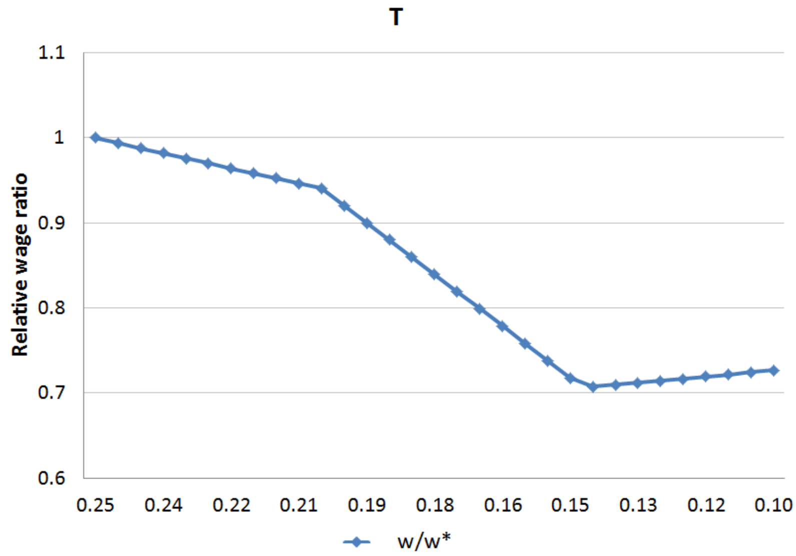

Figure 9: Globalization and Relative Wage (w/w^*)

4.4 Technological Change

In this subsection, we explore how technological changes affect the relative importance of alternative production modes. Again, the horizontal axis is inverted, moving from higher to lower values – i.e., productivity increases are reflected by decreasing input coefficients from left to right.

Figure 10 shows the effects of domestic technological change on a_D and a_M. Intuitively, technological progress in domestic firms – a fall in a_D – makes these firms more competitive compared to multinationals. Of the two types of offshoring fragmented firms are affected most negatively. More entry by domestic firms raises the domestic wage, which is more detrimental to fragmented firms given that they perform more tasks in the North than firms producing almost everything abroad. In contrast, productivity improvements of multinational firms – a fall in a_M – reduces the number of domestic firms. Between the two types of multinational firms, fragmented firms benefit more by the same logic as above.

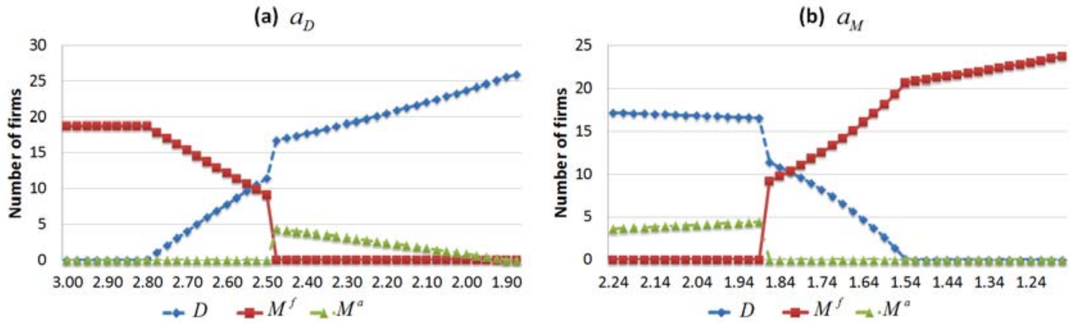

Figure 10: Technological Change on a_D and a_M

Figure 11 focuses on the South and presents the effects of varying the parameters A and B. Recall that raising A increases the amplitude of the $a_M^*(t)$ function, representing greater cost differences between different tasks in the production chain. Conversely, reducing A implies a decline in the cost advantage of producing in the South for tasks $t \in [\tilde{t}_1, \tilde{t}_2]$ and $[\tilde{t}_3, \tilde{t}_4]$. This makes fragmentation less attractive, and eventually, only domestic firms and production abroad firms prevail. A similar pattern emerges if B – i.e., the average costs associated with producing abroad – increases: in this case, the number of firms choosing any type of offshoring declines.

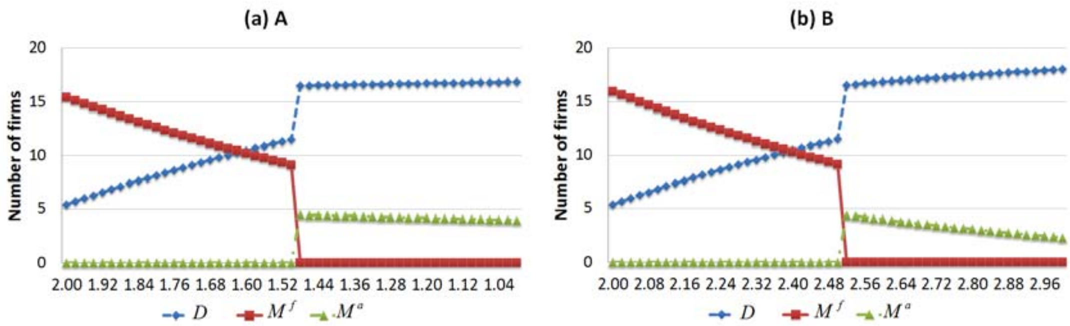

Figure 11: Technological Change on A and B

Finally, Figure 12 displays the effect of an increase in the number of cycles (n) on the relative importance of alternative production modes.

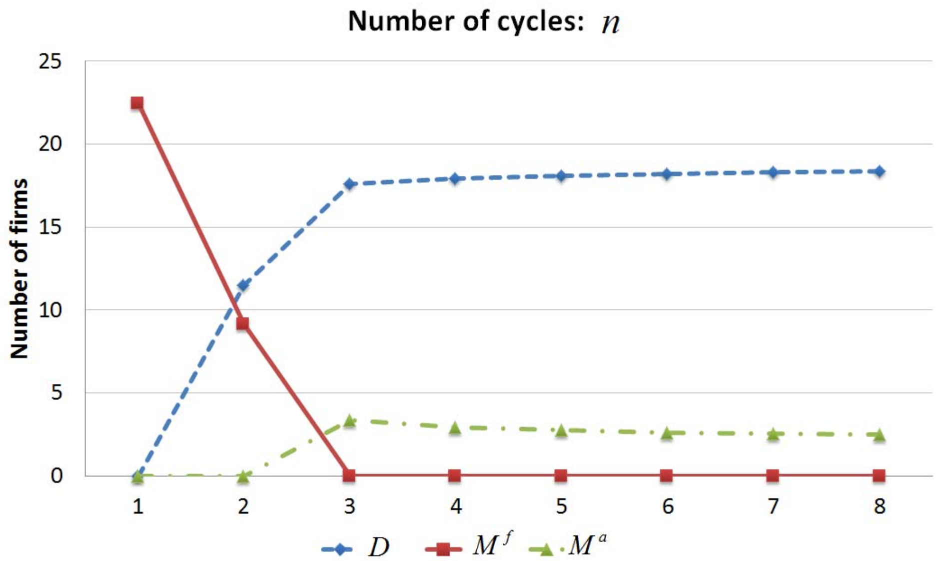

Figure 12: Increase in Sophistication of the Sequential Production Chain

Increasing the *sophistication* of the production process – as reflected by a more complex structure of cost differences along the value-added chain – has similar effects as an increase in T. Given that fragmented offshoring requires transportation costs of n-times T in each country, raising n reduces N_{M_f} while the number of domestic firms increases.

5 Summary and Conclusion

In this paper, we have analyzed the extent of offshoring in a two-country general equilibrium model that is based on three crucial assumptions: First, firms' production process follows a rigid structure that defines the sequence of production steps. Second, domestic and foreign relative productivities vary in a non-monotonic fashion along the production chain. Third, each task requires the presence of an unfinished intermediate good whose transportation across borders is costly. We believe that these assumptions are quite plausible, e.g., characterizing production processes in the automotive industry. As a consequence, some firms may be reluctant to offshore individual production steps, even if performing them abroad would be associated with cost advantages: the reason is that adjacent tasks may be cheaper to perform in the domestic economy and that high transport costs do not justify shifting the unfinished good abroad and back home.

Using this basic structure and setting up a simple general equilibrium model along these lines, we have analyzed the influence of technological progress and globalization – interpreted as a variation in border crossing costs – on the volume of offshoring at the extensive and the intensive margin. As relative production costs vary, firms adjust the share of tasks they perform abroad (the intensive margin), and the number of firms that fragments its production process or produces entirely abroad changes (the extensive margin). Both adjustments may affect relative wages at home and abroad, which can reinforce or dampen the initial impulse. We have shown that globalization in the form of declining transport costs may have different effects on offshoring at the extensive and intensive margin. Our analysis suggests a *decrease* of offshoring at the intensive margin – i.e., firms offshore a smaller part of the entire production process – but an *increase* in the number of firms that perform at least some tasks abroad.

We believe that the simplicity of our model – in particular, the symmetry of the a_M^* function – has allowed us to derive some novel results, which are likely to carry over into a more general environment. The challenge ahead is, of course, to expand the framework to accommodate additional features of reality. The second challenge is to gauge the relative importance of sequential production processes for the economy as a whole. Our contribution rested on the assumption that *all* firms have to cope with a rigid sequence of production steps. This may be as unrealistic as the notion that production processes can be re-arranged freely by every firm. We believe that characterizing real-world production processes in terms of "sequentality" holds ample promise for future research.

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Appendix A: Benchmark Parameter and Variable Values

$\bar{R}$	$\bar{L}$	$\bar{R}^*$	$\bar{L}^*$	β	σ_x	a_D	a_M	n	A
100.00	200.00	200.00	100.00	0.50	4.00	2.50	1.86	2.00	1.50
B	t_{min}	t_{max}	τ	T	F_D	F_{M^f}	F_{M^a}	σ_Y	α_R
2.50	0.00	1.00	1.30	0.20	1.00	1.30	1.50	3.00	0.50
α_L	Inc	Con	P_{Con}	X	Y	x_D^d	$x_{M^f}^d$	$x_{M^a}^d$	P_X
0.50	190.48	101.14	1.88	107.41	95.24	1.68	2.19	2.52	0.89
Inc^*	Con^*	P_{Con}^*	X^*	Y^*	x_D^{d*}	$x_{M^f}^{d*}$	$x_{M^a}^{d*}$	P_X^*	x_D
149.49	69.62	2.15	64.84	74.75	1.32	1.71	1.98	1.15	3.00
x_{M^f}	x_{M^a}	C_D	C_{M^f}	C_{M^a}	p_D	p_{M^f}	p_{M^a}	N_D	N_{M^f}
3.90	4.50	1.88	1.76	1.70	2.51	2.35	2.27	11.48	9.13
N_{M^a}	t_1	t_2	t_3	t_4	Q_Y	Q_Y^*	p_Y	p_Y^*	L_Y
0.00	0.14	0.36	0.64	0.86	51.42	118.57	1.00	1.00	15.11
R_Y	L_Y^*	R_Y^*	w	r	w^*	r^*			
100.00	52.78	200.00	0.75	0.40	0.65	0.42			