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Simon Beyeler and Sylvia Kaufmann

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Simon Beyeler* and Sylvia Kaufmann†‡

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Abstract

We combine the factor augmented VAR framework with recently developed estimation and identification procedures for sparse dynamic factor models. Working with a sparse hierarchical prior distribution allows us to discriminate between zero and non-zero factor loadings. The non-zero loadings identify the unobserved factors and provide a meaningful economic interpretation for them. Applying our methodology to US macroeconomic data reveals indeed a high degree of sparsity in the data. We use the estimated FAVAR to study the effect of a monetary policy shock and a shock to the term premium. Factors and specific variables show sensible responses to the identified shocks.

JEL classification: C32, C55, E32, E43, E52
Key words: Bayesian FAVAR; sparsity; factor identification

*Study Center Gerzensee and University of Bern, Dorfstrasse 2, CH-3115 Gerzensee, simon.beyeler@szgerzensee.ch

†Study Center Gerzensee, Dorfstrasse 2, CH-3115 Gerzensee, sylvia.kaufmann@szgerzensee.ch

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1 Introduction

The use of factor models has become a common tool in macroeconomic analysis, it facilitates the work when a wide range of data is used to study the properties of an economy. In recent times with the extraordinary developments in information technology the ability to handle large amounts of data has become crucial in the field of economics. Extracting relevant information from many different time series measuring different aspects of an economy and compressing it using factor analysis is a neat way to circumvent the curse of dimensionality without ignoring possibly important features.

Bernanke et al. (2005) (henceforth BBE05) suggested augmenting standard small scale vector autoregression (VAR) models by adding unobserved latent factors estimated from a large macro dataset to include additional information in the analysis. They motivated the framework by observing that some economic concepts like output gap, the business cycle stance or inflation sometimes may not be observed without error by the econometrician (and maybe neither by the policy maker). Extracting the relevant information from a large dataset covering the main areas of the economy into factors addresses the issue. On the other hand, variables observable in a timely manner and without large errors, could be defined as *observed* factors and included without transformation into the factor augmented VAR (FAVAR) system.

To estimate the unobserved factors, BBE05 apply two frameworks. In the first and their preferred one, factors are extracted by principal components (Stock and Watson 2002). To ensure that unobserved factors are purged from the information content of observed factors for all other variables, factors are estimated in a two-step procedure. In the first step, factors are extracted from all variables including the observed factors. These first-stage estimates are then purged from the information content of the observed factors conditioning on factors extracted from so-called slow moving variables by a regression-based approach. Later studies purged the initial estimates of common factors by a regression-based approach and iterated the procedure up to convergence (Boivin and Giannoni 2007). In the parametric framework (Stock and Watson 1989, Geweke and Zhou 1996), factor estimation conditions on observed factors and additional restrictions ensure that estimated unobserved factors are contemporaneously uncorrelated to observed factors. BBE05 achieve this by imposing restrictions on the leading square matrix of factor loadings, and estimate unobserved factors by Bayesian Markov chain Monte Carlo (MCMC) methods. However, parametric factor estimation extracts factors which usually lack a proper interpretation. This is inherently also the case for factors estimated by principal components. Bai and Ng (2013) propose to obtain an interpretation of factors by ex-post rotation and re-ordering of series. In the Bayesian approach, BBE05 suggest to assign some of the variables exclusively to one of the factors (in addition to the identification restrictions) to obtain an interpretation of factors iin terms of economic concepts.

In the present paper, we propose to estimate a sparse dynamic factor model for the FA part in the FAVAR approach. Sparse factor models have been traditionally applied in gene expression analysis (West 2003, Carvalho et al. 2008). They are based on the idea that a single factor is not necessarily related to all variables in the underlying data set. Rather, it may only account for the co-movement in a subset of variables. We proceed along the lines of Kaufmann and Schumacher (2018) (henceforth KS18), who propose to estimate the factors independently of variable ordering and without pre-setting identifying restrictions with respect to factor position and sign. We estimate factors in a parametric way by Bayesian MCMC methods and propose an alternative identification scheme. First, to exclude that unobserved factors contain linear combinations of variables involving observed factors, we assume that observed and unobserved factors are contemporaneously uncorrelated. This is implemented by restricting the error covariance between unobserved and observed factors to be block-diagonal, see also Bai et al. (2016). The space of unobserved factors is identified while estimating the sparse factor model, i.e. the sparse factor loading matrix will define the factor basis. The estimated factors obtain their interpretation by the non-zero loadings on specific or a group of series related to an economic concept like production, financial risk, prices etc.. Hence, factor interpretation is obtained by model estimation rather than by imposing additional restrictions on variable ordering and timing restrictions of variables' responses to shocks in factors (Boivin et al. 2016). For scaling purposes the unobserved factors' innovation variance is set to unity. An additional advantage of estimating a sparse factor loading matrix is that the off-diagonal elements of the innovations' covariance matrix can be left unrestricted (Conti et al. 2014). Empirical results in KS18, Kaufmann and Schumacher (2017) and in the present paper document that there is a lot of sparsity in large economic datasets. After estimation and identification of factor position and sign, we can assess identification by comparing the factor loading structure to identification schemes commonly used in the literature (Geweke and Zhou 1996; Aguilar and West 2010; Frühwirth-Schnatter and Lopes 2010).

The estimated sparse FAVAR model provides a basis for further structural analysis. The identified factors allow for a richer, factor-specific interpretation of results from structural FAVAR models. In studies analyzing monetary policy transmission and related issues like price stickiness (Boivin et al. 2009; Boivin et al. 2011;

Baumeister et al. 2013), the results usually focus on the response in the common component of specific (groups of) variables to the identified monetary policy shock. Factor identification allows us to discriminate between factor-specific response. Estimating a sparse factor loading matrix may also help identifying structural shocks in FAVAR models with time-varying parameters (Korobilis 2013) and combining series of mixed-frequency (Marcellino and Sivec 2016).

In the next section, we describe model specification and discuss the identification strategy. Section 3 presents the Bayesian MCMC sampling scheme and in particular the estimation of the factors. The section also describes the post-processing procedure to identify factor position and factor sign. To illustrate the method, in Section 4 we work with a large panel of series for the US macroeconomy, for which we estimate and identify seven unobserved factors next to the federal funds rate (FFR) included as observed factor. We find evidence for a substantial amount of sparsity in the dataset, and the structure of non-zero factor loadings yields an economic interpretation for all unobserved factors. Despite the amount of sparsity and the small number of factors, the common component explains a large fraction of the sample variance. We proceed with a structural VAR analysis to study the effects of a FFR and a term-premium factor shock. The estimated factors and specific variables all show sensible responses to the FFR shock. In line with the findings in Kurmann and Otrok (2013), the term premium shock generates very similar impulse responses to a news shock. We also show how the identification strategy described in Uhlig (2003) can be adapted to the FAVAR environment. Section 5 concludes. The interested reader finds details on prior specifications and the derivation of posterior distributions in Apendices A and B, respectively. Derivations for applying Uhlig (2003) to the FAVAR framework are described in Appendix C. Appendix E displays additional results of the application, while Appendix F lists the data set including labels.

2 Model specification and identification

2.1 The model

The framework proposed in BBE05 collects N non-trending observed variables in a $N \times 1$ vector X_t, $t = 1, \ldots, T$. These variables are assumed to contain information on some pervasive k, $k << N$, economic factors f_t^* which are not directly observable to the econometrician but are relevant determinants of some m observed series Y_t.

The FAVAR representation for $(f_t^{*\prime}\ Y_t^{\prime})$ writes

$$\begin{aligned}
\begin{bmatrix} X_t \\ Y_t \end{bmatrix} &= \begin{bmatrix} \lambda^{*f} & \lambda^{*Y} \\ 0 & I_m \end{bmatrix} \begin{bmatrix} f_t^* \\ Y_t \end{bmatrix} + \begin{bmatrix} \xi_t \\ 0 \end{bmatrix} \\
\Phi^*(L) \begin{bmatrix} f_t^* \\ Y_t \end{bmatrix} &= \begin{bmatrix} \eta_t^{*f} \\ \eta_t^{Y} \end{bmatrix} \quad \eta_t^* \sim N(0, \Sigma^*) \\
\Psi(L)\xi_t &= \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega)
\end{aligned} \tag{1}$$

where λ^{*f} and λ^{*Y} are factor loading matrices of dimension $N \times k$ and $N \times m$, respectively, and I_m represents the identity matrix of dimension m. An autoregressive process of order p characterizes the FAVAR process$(f_t^{*\prime}\ Y_t^{\prime})$, $\Phi^*(L) = I_{k+m} - \Phi_1^* L - \cdots - \Phi_p^* L^p$. We assume that the common co-movement in X_t is fully explained by f_t^* and Y_t. Therefore, common and idiosyncratic shocks are uncorrelated, i.e. $E(\eta_t^* \varepsilon_t^{\prime}) = 0$, and idiosyncratic components ξ_t follow series-specific independent VAR processes, i.e. $\Psi(L)$ and Ω are, respectively, diagonal processes and diagonal with elements $\{\Psi(L), \Omega\} = \{\psi_i(L), \omega_i^2 | \psi_i(L) = 1 - \psi_{i1}L - \cdots - \psi_{iq}L^q, i = 1, \ldots, N\}$.

The * in model (1) indicates that we work with a sparse factor model and estimate sparse factor loading matrices λ^{*f} and λ^{*Y}, i.e. matrices that potentially contain zero loadings. This extends the framework of BBE05 in the sense that the non-zero loadings in a column potentially yield an explicit interpretation of the corresponding factor in f_t^*. For example, a factor only loading on price variables may reflect nominal conditions of an economy while a factor loading mostly on real variables may reflect business cycle conditions. On the other hand, rows of zero loadings in λ^{*f} indicate variables that are irrelevant for the estimation of the factors. As shown in KS18, such variables do not contain relevant information for estimating the factors and deteriorate estimation efficiency if included for estimation. Sparsity in λ^{*Y} captures the idea that the observed variables Y_t also reflect (observable) information common to specific groups of variables. For example, changes in the policy interest rate, if included in Y_t, may affect other interest rates included in X_t, while not contemporaneously affecting real variables like consumption or investment.

In this paper, we estimate model (1) in a Bayesian parametric framework based on Gibbs sampling. BBE05 prefer the non-parametric two-step estimation based on principal components analysis over parametric estimation. They achieve structural identification of shocks by imposing a recursive scheme on Σ^*. They argue that structural identification in the parametric framework is more difficult to establish, in particular because structural identification is ultimately linked to factor identification in the sense of factor interpretation. They suggest to obtain factor identification by restricting additionally the factor loading matrix λ^{*f}. For example, those series perceived to be related to business cycle conditions would be restricted to load onto the factor defined to reflect business cycle conditions, etc. This procedure would come close to confirmatory factor analysis or to a dedicated factor model, see e.g. Lawley and Maxwell (1971) or more recently Conti et al. (2014).

We show that using a sparse parametric approach eventually yields factor identification, not by imposing variable-factor association a priori but by letting the data tell us the variable-specific factor association. After estimation, structural identification of shocks is ultimately obtained by factor interpretation. In other words, inducing sparsity into the factor loading matrix identifies the factor space. Obviously, this implies the existence of a sparse factor representation of data. So far, our experience with economic data is very promising in that respect. The estimated sparse factor models all yield economically interpretable factors.

2.2 Implementing sparsity

To induce sparsity, we work with a hierarchical point mass-normal mixture prior distribution on the factor loadings λ^*_{ij}, $i = 1, \ldots, N$, $j = 1, \ldots, k+m$ (see e.g. West 2003, Carvalho et al. 2008)

$$\pi(\lambda^*_{ij}|\beta_{ij}, \tau_j) = (1-\beta_{ij})\delta_0(\lambda^*_{ij}) + \beta_{ij} N(0, \tau_j) \tag{2}$$

$$\pi(\beta_{ij}|\rho_j) = (1-\rho_j)\delta_0(\beta_{ij}) + \rho_j B(ab, a(1-b)) \tag{3}$$

$$\pi(\rho_j) = B(r_0 s_0, r_0(1-s_0)) \tag{4}$$

where δ_0 is a Dirac delta function that assigns all probability mass to zero and $B(uv, u(1-v))$ denotes a beta distribution with mean v and parameter u determining precision. For τ_j, we assume an inverse Gamma prior distribution $IG(g_0, G_0)$. The factor-independent parametrization of the hyperparameters renders the prior distribution invariant with respect to factor ordering an sign. This is useful to apply random permutation sampling to draw from the unconstrained multimodal posterior distribution. Posterior mode identification, i.e. identifying factor position and sign, is obtained by processing the posterior output, see section 3.4.

Setting up the prior in this way implies a common probability across series of a non-zero loading on factor j equal to $\rho_j b$. With appropriate parametrization of layer (3), we can implement the viewpoint that for many variables the probability of association with anyone factor is zero, while for a few it will be high.

The point mass-normal mixture prior (2)-(4) explicitly discriminates between zero and non-zero loadings. This allows us to perform variable selection simultaneously while estimating the model, see e.g. George and McCulloch (1997). In this way, we can avoid proceeding in a two-step manner to identify the relevant variables (Forni et al. 2001, Bai and Ng 2008).

2.3 Identification

As well known in factor analysis, model (1) has an infinite number of observationally equivalent decompositions (Lawley and Maxwell 1971). For any non-singular matrix $Q = \begin{bmatrix} Q^f & Q^{fY} \\ Q^{Yf} & Q^Y \end{bmatrix}$,[1] we can rotate representation (1) into:

$$\begin{aligned} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} &= \begin{bmatrix} \lambda^{*f} & \lambda^{*Y} \\ 0 & I_m \end{bmatrix} Q^{-1} Q \begin{bmatrix} f_t^* \\ Y_t \end{bmatrix} + \begin{bmatrix} \xi_t \\ 0 \end{bmatrix} \\ Q\Phi^*(L)Q^{-1}Q \begin{bmatrix} f_t^* \\ Y_t \end{bmatrix} &= Q \begin{bmatrix} \eta_t^{*f} \\ \eta_t^Y \end{bmatrix} \quad Q\eta_t^* \sim N\left(0, Q\Sigma^* Q'\right) \end{aligned} \tag{5}$$

which yields

$$\begin{aligned} \begin{bmatrix} \hat{f}_t \\ \hat{Y}_t \end{bmatrix} &= Q \begin{bmatrix} f_t^* \\ Y_t \end{bmatrix} \\ \hat{f}_t &= Q^f f_t^* + Q^{fY} Y_t \quad (6) \\ \hat{Y}_t &= Q^{Yf} f_t^* + Q^Y Y_t \quad (7) \end{aligned}$$

It seems obvious to require that observed variables remain observed after rotation. This is ensured by restricting $Q^{Yf} = 0$ and $Q^Y = I_m$. Imposing all restrictions prior to estimation, we would need $k^2 + km$ additional restrictions (Bai et al. 2016).

We proceed as follows. We rule out the possibility that unobserved factors involve linear combinations of observed variables as in (6), which implies $\hat{\lambda}^f = \lambda^{*f}(Q^f)^{-1}$ and $\hat{\lambda}^Y = \lambda^{*Y} - \lambda^{*f}(Q^f)^{-1}Q^{fY}$ for the loadings. For any nonsingular $k \times k$ matrix Q^f, the requirement can be achieved by restricting the $k \times m$ matrix $Q^{fY} = 0$. For $Q^f = I_k$, this implies $E(f_t^* Y_t' | \mathcal{I}_{t-1}) = 0$, with $\mathcal{I}_{t-1}$ denoting information up to period $t-1$. So, $Q^{fY} = 0$ implies that conditional on past information, unobserved factors f_t^* are contemporaneously uncorrelated to observed variables Y_t, and we implement this notion by setting Σ^* block-diagonal, $\Sigma^* = \begin{bmatrix} \Sigma_f^* & 0 \\ 0 & \Sigma_Y \end{bmatrix}$. This provides us with km restrictions.[2]

[1] We save on notation by using single superscripts for the diagonal submatrices.

[2] This restriction emulates the idea of the restriction imposed by BBE05 on the factor loading matrix. The $k \times k$ leading identity matrix defining the leading k series in the panel as the factors together with the zero loadings of observed factors on these k leading variables in fact imply that unobserved factors are contemporaneously uncorrelated with observed factors.

With remaining k^2 restrictions, we have to identify the space or basis of unobservable factors and its orientation.[3] First, we scale factors by assuming the diagonal elements of Σ_f^* equal to 1, $\sigma_{fj}^* = 1$, $j = 1, \ldots, k$, and leave off-diagonal elements unrestricted (k restrictions). Hence, Σ_f^* is interpretable as a correlation matrix. Usually, the remaining $k(k-1)$ identification restrictions are then imposed on the leading $k \times k$ matrix λ_k^{*f} in the factor loading matrix λ^{*f}. Setting $\lambda_k^{*f} = D$, D diagonal with positive elements ($k(k-1)$ zero restrictions), and requiring a specific ordering identifies the factor space together with factor position and factor sign.

Obviously, this procedure needs careful choice of the k leading variables in the panel, because these in fact are the factors. However, variable ordering is usually not perceived as an issue in factor estimation. Few papers address the issue and present ways of determining relevant leading variables, the so-called *factor founders*, while estimating the model (Carvalho et al. 2008; Frühwirth-Schnatter and Lopes 2010). Recently, Aßmann et al. (2016) and Chan et al. (2018) propose order-invariant estimation of factor models and ex-post rotational factor identification. In particular, Chan et al. (2018) show that imposing restrictions prior to estimation can seriously bias inference about the number of factors.

We proceed in a different way. We estimate the factor model independently of variable ordering and do not impose the remaining $k(k-1)$ restrictions prior to estimation. We exploit the fact that λ^{*f} is sparse and that the non-zero loadings will identify the factors and imply a factor basis. Model estimation then identifies the factors and all factor-specific parameters up to factor position and sign. The unconstrained posterior distribution will display $2^k k!$ modes, and we determine factor position and sign ex-post by processing the posterior Gibbs output, see section 3.4 and KS18 for details.

Having estimated the model, we may evaluate the structure of λ^{*f} to assess model identification against criteria derived in the literature (Anderson and Rubin 1956, Geweke and Zhou 1996, Bai and Wang 2014). For example, an appropriate

Our experience with economic data suggests that sparsity is pervasive enough to relax the assumption of a block-diagonal Σ^*. The estimation of the model for US data yields factor estimates which are pretty uncorrelated contemporaneously and quite distinct from the FFR. Therefore, in a future version we plan to re-estimate the model with a full correlation matrix Σ^*.

[3]This interpretation relies on a geometrical interpretation of the factor decomposition:

$$Cov\left([X_t' \; Y_t']'\right) = \lambda^* \Sigma^* \lambda^{*\prime} + Cov\left(\xi_t\right)$$

For example, $\Sigma^* = I_{k+m}$ would provide an orthonormal basis for the factor space, with series-specific factor loadings $(\lambda_{i1}^*, \ldots \lambda_{i,k+m}^*)$ representing the numerical coordinates of a point in a $k+m$ Cartesian system. So, a general Σ^* reflects a correlated factor basis. Conditional on the factor basis Σ^*, rotational indeterminacy remains with respect to the orientation of the basis, including factor order and sign. This means that conditional on Σ^*, Q^f restricts to the set of orthogonal matrices.

variable re-ordering B, would rank first all variables loading only on the first factor, then rank those variables loading only on the second factor, and so on, $\hat{X}_t = BX_t$. Our experience with empirical economic datasets is that such an ordering can usually be defined ex-post, implying a re-ordered factor loading matrix $\hat{\lambda}^{*f} = B\lambda^{*f}$ with a generalized lower-diagonal structure.

3 Bayesian estimation

To outline model estimation, we introduce additional notation. We stack factor observations and initial values f^{*p} into the vector $F^* = (f^{*p'}, f_1^{*'}, ..., f_T^{*'})'$. While X_t denotes observations in period t, X^t indicates observations up to period t, and similarly for other variables. All parameters and hyperparameters are included in $\theta = \{\lambda^{*f}, \lambda^{*Y}, \mathbf{\Phi}^*, \mathbf{\Psi}, \Omega, \Sigma_f^*, \Sigma_Y^*, \vartheta\}$, where $\mathbf{\Phi}^* = \{\phi_{ij,l}^*|i,j = 1,\dots,k+m,\ l = 1,\dots,p\}$, $\mathbf{\Psi} = \{\psi_{il}|i = 1,\dots,N,\ l = 1,\dots,q\}$, and $\vartheta = \{\beta,\rho,\tau\}$ with $\beta = \{\beta_{ij}|i = 1,\dots,N,\ j = 1,\dots,k+m\}$, $\{\rho,\tau\} = \{\rho_j,\tau_j|j = 1,\dots,k+m\}$.

3.1 Likelihood and prior specification

Conditional on factors, the likelihood takes the form

$$L(X^T, Y^T|F^*, \theta) = \prod_{t=1}^{T} \pi(X_t|Y_t, f_t^*, \theta)\pi(Y_t|Y^{t-1}, f^{*t-1}, \theta) \tag{8}$$

with multivariate normal observation densities

$$\pi(X_t|Y_t, f_t^*, \theta) = \frac{1}{(2\pi)^{N/2}|\Omega|^{1/2}} \exp\left(-\frac{1}{2}\varepsilon_t'\Omega^{-1}\varepsilon_t\right) \tag{9}$$

$$\pi(Y_t|Y^{t-1}, f^{*t-1}, \theta) = \frac{1}{(2\pi)^{m/2}|\Sigma_Y|^{1/2}} \exp\left(-\frac{1}{2}\eta_t^{Y\prime}\Sigma_Y^{-1}\eta_t^Y\right) \tag{10}$$

The prior density of unobserved factors is formulated conditional on observed factors Y_t

$$\pi(F^*|Y^T, \theta) = N\left(0, \mathbf{F}_0\right) \tag{11}$$

where $\mathbf{F}_0^{-1} = \mathbf{\Phi}^{f\prime}\mathbf{\Sigma}_f^{-1}\mathbf{\Phi}^f$, with $\mathbf{\Phi}^f$ and $\mathbf{\Sigma}_f$ appropriately banded matrices, see section 3.3.

For the model parameters, we assume independent priors

$$\pi(\theta) = \pi(\lambda^*|\vartheta)\pi(\vartheta)\pi(\mathbf{\Phi}^*)\pi(\mathbf{\Psi})\pi(\Omega)\pi(\Sigma_f^*)\pi(\Sigma_Y) \tag{12}$$

The hierarchical sparse prior distribution $\pi(\lambda^*|\vartheta)\pi(\vartheta)$ is given in (2)-(4). Except for Σ_f^*, all remaining parameters have standard prior distributions, see appendix A. As discussed in section 2.3, our identification scheme treats Σ_f^* as correlation matrix, i.e. with 1s on the diagonal and unrestricted otherwise. Instead of defining a prior distribution for the correlation matrix, which is not trivial, we use parameter extension as proposed in Conti et al. (2014). Defining the working parameter V, a $k \times k$ non-singular diagonal matrix, we expand the correlation matrix to a regular covariance matrix $\hat{\Sigma}_f = V^{\frac{1}{2}} \Sigma_f^* V^{\frac{1}{2}}$, which allows us to formulate a conjugate inverse Wishart prior distribution $\pi\left(\hat{\Sigma}_f | S_f\right) \sim IW(\nu_f, S_f)$.

3.2 Posterior sampler

To obtain a sample from the posterior distribution

$$\pi\left(F^*, \theta | X^T, Y^T\right) \propto L\left(X^T, Y^T | F^*, \theta\right) \pi\left(F^* | Y^T, \theta\right) \pi(\theta) \tag{13}$$

we repeatedly draw from:

(i) the sparse posterior of factor loadings $\pi\left(\lambda^{f*}, \lambda^{*Y} | X^T, Y^T, F^*, \mathbf{\Psi}, \Omega\right)$, and update the hyperparameters $\pi(\vartheta | \lambda^{*f}, \lambda^{*Y})$,

(ii) the posterior of factors: $\pi\left(F^* | X^T, Y^T, \theta\right)$

(iii) the posterior distribution of model parameters $\pi\left(\mathbf{\Phi^*}, \mathbf{\Psi}, \Omega, \Sigma_f^*, \Sigma_Y | X^T, Y^T, F^*, \lambda^{*f}, \lambda^{*Y}\right)$,

and

(iv) permute randomly factor position and sign.

Most of the posterior distributions for model parameters are standard and derived in detail in appendix B. Given the unconventional approach of estimating factors parametrically in the FAVAR framework, we briefly expose the sampler in the following section.

3.3 Sampling the factors

To draw from the posterior of factors, $\pi(F^* | X^T, Y^T, \theta)$ we first condition on observed variables Y_t:

$$\bar{X}_t = X_t - \lambda^{*Y} Y_t - \lambda^{*f} \mu_{f^* | Y^{t-1}}$$
$$\bar{f}_t = f_t^* - \mu_{f_t^* | Y^{t-1}}$$

where $\mu_{f_t^*|Y^{t-1}} = \Phi_1^{*fY} Y_{t-1} + ... + \Phi_p^{*fY} Y_{t-p}$, with Φ_j^{*fY} the right-upper submatrix of $\Phi_j^* = \begin{bmatrix} \Phi_j^{*f} & \Phi_j^{*fY} \\ \Phi_j^{*Yf} & \Phi_j^{*Y} \end{bmatrix}$. Then, we condense the conditional system:

$$\begin{aligned}
\Psi(L)\bar{X}_t &= \\
\tilde{X}_t &= \lambda^{*f}\bar{f}_t - \lambda^{*f} \odot (\psi_{\cdot 1} \otimes \mathbf{1}_{1\times k})\,\bar{f}_{t-1} - \cdots - \lambda^{*f} \odot (\psi_{\cdot q} \otimes \mathbf{1}_{1\times k})\,\bar{f}_{t-q} + \varepsilon_t \\
\bar{f}_t &= \Phi_1^{*f}\bar{f}_{t-1} + \cdots + \Phi_p^{*f}\bar{f}_{t-p} + \eta_t^{*f}, \quad \eta_t^{*f} \sim N\left(0, \Sigma_f^*\right)
\end{aligned}$$

where $\odot$ and $\otimes$ represent the Hadamar and the Kronecker product, respectively. $\mathbf{1}_{1\times k}$ is a row vector containing k ones as elements. Stack all observations to obtain the matrix representation

$$\begin{aligned}
\tilde{\mathbf{X}} &= \mathbf{\Lambda}^f \bar{F} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim N\left(0, I_{T-q} \otimes \Omega\right) && (14) \\
\mathbf{\Phi}^f \bar{F} &= \boldsymbol{\eta}^f \quad \boldsymbol{\eta}^f \sim N\left(0, \mathbf{\Sigma}_f\right) && (15)
\end{aligned}$$

where $\tilde{\mathbf{X}} = \left(\tilde{X}'_{q+1}, \ldots, \tilde{X}'_T\right)'$ contains all data, $\bar{F} = \left(\bar{f}'_{q+1-\max(p,q)}, \ldots, \bar{f}'_{q+1}, \ldots, \bar{f}'_T\right)'$ stacks all unobserved factors, including initial states. The matrices $\mathbf{\Lambda}^f$ and $\mathbf{\Phi}^f$ are respectively of dimension $(T-q)N \times (T+d)\,k$ and square $(T+d)k$, with $d = (p-q)I\,\{p>q\}$. Typically, these matrices are sparse and banded around the main diagonal (Chan and Jeliazkov 2009)

$$\mathbf{\Lambda}^f = \left[\begin{array}{c|ccccccc}
 & -\lambda^{*f} \odot (\psi_{\cdot q} \otimes \mathbf{1}_{1\times k}) & \ldots & \lambda^{*f} & 0\ldots & & & 0 \\
\mathbf{0}_{(T-q)N\times dk} & & \ddots & \ddots & & \ddots & & \vdots \\
 & 0\ldots & 0 & & -\lambda^{*f} \odot (\psi_{\cdot q} \otimes \mathbf{1}_{1\times k}) & & \ldots & \lambda^{*f}
\end{array}\right]$$

$$\mathbf{\Phi}^f = \left[\begin{array}{cccccccc}
 & I_p \otimes I_k & & 0 & \ldots & & & \\
\hline
-\Phi_p^{*f} & \ldots & -\Phi_1^{*f} & I_k & 0 & \ldots & & \\
 & & & & \ddots & & & \\
 & \ldots & 0 & -\Phi_p^{*f} & \ldots & -\Phi_1^{*f} & I_k &
\end{array}\right],$$

$$\mathbf{\Sigma}_f = \begin{bmatrix} I_p \otimes \Sigma_{f0} & 0 & \ldots \\ 0 & & \\ \vdots & & I_{T+d-p} \otimes \Sigma_f^* \end{bmatrix}$$

where Σ_{f0} represents the variance of the initial states of the unobserved factors (see appendix B.2).

Combining the prior (11) with the likelihood $\pi(\tilde{\boldsymbol{X}}|\bar{F}, \theta) \sim N(\mathbf{\Lambda}^f \bar{F}, I_{T-q} \otimes \Omega)$ we obtain the posterior distribution

$$\bar{F}|\tilde{\boldsymbol{X}}, \theta \sim N(\boldsymbol{\mu}_{\bar{f}}, \mathbf{F}) \tag{16}$$

$$\mathbf{F}^{-1} = \mathbf{F}_0^{-1} + \boldsymbol{\Lambda}^{f\prime}(I_{T-q} \otimes \Omega^{-1})\boldsymbol{\Lambda}^f \tag{17}$$

$$\boldsymbol{\mu}_{\bar{f}} = \mathbf{F}\boldsymbol{\Lambda}^{f\prime}(I_{T-q} \otimes \Omega^{-1})\tilde{\boldsymbol{X}} \tag{18}$$

In order to avoid the full inversion of $\mathbf{F}$ we take the Cholesky decomposition, $\mathbf{F}^{-1} = L'L$, then $\mathbf{F} = L^{-1}L^{-1\prime}$. We obtain a draw $\bar{F}$ by setting $\bar{F} = \boldsymbol{\mu}_{\bar{f}} + L^{-1}\boldsymbol{\nu}$, where $\boldsymbol{\nu}$ is a $(T+d)k$ vector of independent draws from the standard normal distribution. We retrieve a draw F^* by adding back the conditional mean, $f_t^* = \bar{f}_t + \mu_{f_t^*|Y^{t-1}}$.

Model estimation does not identify factor position and factor sign. Given that we formulate a factor-invariant prior distribution on the loadings and on the factor-specific parameters, the prior is invariant with respect to factor ordering and sign. Therefore, the posterior (13) will also be invariant with respect to factor and sign permutations $\rho(\cdot)$, $\pi\left(F^*, \theta|X^T, Y^T\right) = \pi\left(\rho(F^*, \theta)|X^T, Y^T\right)$. To explore the full unconditional distribution, we apply random permutation of factor order and factor sign at the end of each iteration (Frühwirth-Schnatter 2001). The posterior output will have $2^k k!$ modes. We identify factor order and sign ex-post by sorting out the multimodal posterior output, see the next section.

3.4 Ex post mode identification

Model estimation yields G draws out of the multimodal posterior distribution. We post-process the draws to define factor position and sign. We proceed as in Kaufmann and Schumacher (2018, section 3.3) who suggest to identify factor position based on the posterior draws of the *factors* rather than using the *loadings* as usually done in the literature.

In brief, we first identify κ relevant *factor representatives*, f^{*c}, $c = 1, \dots, \kappa$, which form the basis to identify factor positions. To determine factor representatives, we form clusters of highly correlated (in absolute terms) factor draws. From those clusters which contain a significant number of draws, say e.g. $0.9G$ draws, we estimate a factor representative by the mean of the (sign-adjusted) clustered draws.

The intuition behind the procedure is the following. Assume that all k factors in the estimated model are relevant, i.e. model estimation is not overfitting the number of factors. Then, the posterior output should contain G posterior draws for each of the k factors, whereby the respective G draws should be relatively highly correlated. Therefore, we should be able to identify $\kappa = k$ factor representatives. On the other hand, if an estimated model is overfitting the number of factors, $k > k^{true}$, then

$G(k-k^{true})$ factor draws will be sampled out of the prior, given that the data are uninformative for the $k-k^{true}$ redundant factors. These $G(k-k^{true})$ factor draws would be loosely correlated, if at all. The clustering procedure will then identify $\kappa = k^{true} < k$ factor representatives.

After determining the factor representatives, we then re-order each posterior draw according to maximum correlation with the κ factor representatives. Concretely, we determine the permutation $\varrho^{(g)} = (\varrho_1^{(g)}, \dots, \varrho_k^{(g)})$ of $\{1,\dots,k\}$ for draw $g = 1,\dots,G$:

$$\varrho^{(g)} = \Big\{ \varrho_c^{(g)} = j \Big| \left|\operatorname{corr}\left(f_j^{*(g)}, f^{*c}\right)\right| = \max_{l=1,\dots,k} \left|\operatorname{corr}\left(f_l^{*(g)}, f^{*c}\right)\right|, \; j = 1,\dots,k, c = 1,\dots,\kappa \Big\} \tag{19}$$

where $f_j^{*(g)} = (f_{j1}^{*(g)}, \dots, f_{jT}^{*(g)})'$ represents the gth draw of the jth factor. If $\varrho^{(g)}$ is a unique permutation of $\{1,\dots,k\}$, we retain draw g for posterior inference. The permutation is applied as detailed in Kaufmann and Schumacher (2018, equation (10)) to factors, factor loadings λ^{*f} and factor-specific parameters and hyperparameters. The permutation step is completed by sign-permuting each factor draw negatively correlated to the factor representative. Appropriate sign-adjustment also applies to factor loadings λ^{*f} and dynamic parameters $\mathbf{\Phi}^*$.

The permutation step is slightly adjusted in case we identify fewer factor representatives than estimated factors, i.e. $\kappa < k$. This is an indication that the model may be re-estimated conditional on a lower number of factors. Nevertheless, we may perform posterior inference on the κ relevant factors. In this case, permutation (19) is re-defined. After determining $\varrho^{(g)}$ as in (19), the factor draws correlated the lowest with factor representatives are ranked last, in no specific order, $\varrho^{(g)} := \left(\varrho^{(g)}, \{1,\dots,k\} \setminus \varrho^{(g)}\right)$.

4 Application to the US economy

In this section, we apply our methodology to a large panel of time series for the US economy to illustrate estimation and identification of the sparse FAVAR. We find evidence for a high degree of sparsity which can be exploited for model identification. Besides including the FFR as one observed factor, we estimate seven unobserved factors. The variance share explained by the common component amounts to slightly more than 50 percent. Further, we perform a structural analysis to study how structural shocks like a monetary policy shock or a productivity shock affect the economy. Against the background of estimating an unrestricted factor error covariance matrix, this exercise illustrates how to apply traditional structural identification schemes to the sparse FAVAR model. The FAVAR offers an advantage over small scale VAR models in that it allows us to include pervasive information and extend the analysis to a broad set of variables.

4.1 Data and prior specification

We work with the FRED-QD database available for download from the website of the Federal Reserve Bank of St. Louis. The data is a quarterly companion to the Monthly Database for Macroeconomic Research (FRED-MD) assembled by McCracken and Ng (2015). It consists of 253 macroeconomic time series for the US economy which are regularly updated and reported at a quarterly frequency starting in 1959Q1. The FRED-QD database has been constructed along the lines of the data set used in Stock and Watson (2012). In addition, we include utilization adjusted total factor productivity (TFP) from Fernald (2012). In our analysis, we focus on the period 1965Q1 - 2015Q2 and drop the series with missing observations, which leaves us with 224 variables in total.[4] Where necessary, series are transformed to non-trending series by applying first differences either to logs or to levels. For an easier understanding of the results and given our Bayesian estimation setup, we depart from the transformations suggested in FRED-QD and avoid second differences. A complete list of included series and their transformations is available in appendix F.

Following BBE05 we treat the FFR as the only observed factor, given its role as a policy instrument and the fact that it is observed without error. The preferred specification includes $k = 7$ unobserved factors, which capture well the covariance structure of the underlying data. The choice of k is justified in various ways. First, $k = 7$ mirrors well the number of groups into which the series may be classified, like e.g. economic activity variables, prices, interest rates and so on. Second, the average variance share explained by the common component lies above 50 percent and does not increase any more substantially when estimating additional factors, see Figure E.1 in Appendix E. As a last device, we apply the eigenvalue-ratio based criterion proposed by Lam and Yao (2012). The right panel of Figure E.1 in Appendix E shows that the minimum ratio is achieved at $(k+1) = 2$. However, there are further local minima at 5, 8 and 11, which indicates that next to two strong factors there is evidence for additional weaker factors. Overall, the evidence leads us to prefer $k = 7$.

[4] The series with missing observations stem from various larger groups of series. After removing them, each group keeps a representative number of series. Therefore, we expect no significant data information loss by removing series with missing observations.

The parametrization for the prior distributions is listed in Table D.1. For the two layer sparse prior we set the mean $s_0 = 0.35$, the precision $r_0 = 200$.[5] We allow for two lags in the dynamics of the factors as well as the idiosyncratic components, $p = q = 2$.[6] The sampler converges quickly, we draw 8000 times from the posterior, discard the first 3000, and retain every second one. We are left with $G = 2500$ draws to perform posterior inference.

4.2 Results

Figure 1 shows a heatmap of the mean posterior probabilities of non-zero factor loadings.[7] It nicely reveals the sparsity in the data. While for some variables the probability of non-zero loadings is high (red entries), there are also a lot of loadings with very low non-zero probabilities (white entries). To create the figure, the factors are first ranked in decreasing order of the number of non-zero factor loadings.[8] In addition, the variables have been sorted such that variables loading only on the first factor are ordered first, followed by those that load only on Factor 2 and on Factors 1 and 2 and so on. In doing so, we get a generalized lower triangular structure, which reveals that the structure of the estimated factor loading matrix yields an identified model.

The estimated sparse factor loading matrix yields a clear economic interpretation for all seven unobserved factors. Figure 2 plots the posterior mean estimates of the seven unobserved factors along with the 68 percent highest posterior density interval[9] (HPDI) and the FFR as well. The first three latent factors are all related to the real part of the economy. The first factor represents production as it loads on production and output series like real GDP, real investment, industrial production measures, manufacturing sales as well as new orders for durable manufacturing goods. It further loads on real consumption expenditures as well as various employment and unemployment variables. The second unobserved factor is positively correlated to the first factor. We interpret it as employment factor given that it mostly loads on employment and unemployment data, including employees in different sectors, the unemployment rate and hours worked. Further, it positively loads on some credit variables such as consumer loans as well as commercial and industrial

[5] Changing the prior degree of sparsity has almost no influence on the results. As expected, a higher sparsity degree slightly increases the number of estimated zero elements in λ^*, but leaves the results qualitatively unchanged.

[6] Again, increasing the number of lags does not alter the results as the coefficient estimates for $p, q > 2$ are close to zero.

[7] We compute the average number of nonzero draws for λ^*_{ij}, $P(\lambda^*_{ij} \neq 0|\cdot) = \frac{1}{G}\sum_{g=1}^{G} I\{\lambda^{(g)}_{ij} \neq 0\}$.

[8] We define a loading as non-zero for which $P(\lambda^*_{ij} \neq 0|\cdot) > 0.5$

[9] This also applies to all plots of impulse responses if not stated otherwise.

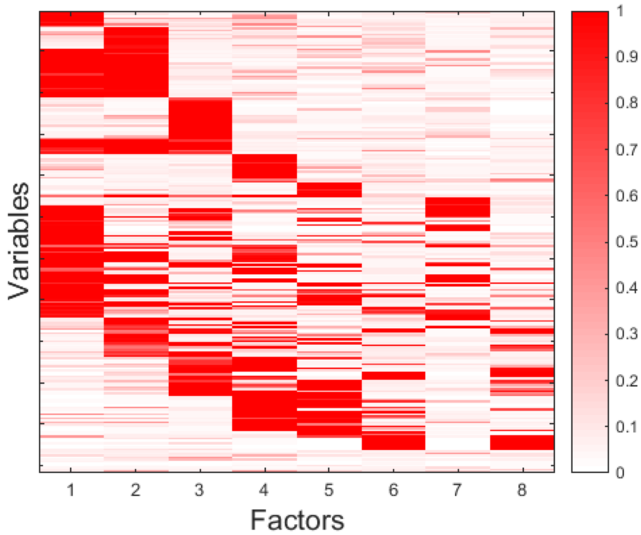

Figure 1: Posterior probabilities of a non-zero factor loading. The loadings are sorted such that variables loading only on the first factor are ordered first, followed by those that load only on Factor 2 and on Factors 1 and 2 and so on.

loans, indicating that credit is co-moving with employment. The third latent factor represents the housing market. It mostly loads on variables like building permits and housing starts. In addition, it is informative for stock market variables. The fourth and the fifth factor capture nominal features of the US economy. While the fourth factor loads mostly on consumer price inflation series, the fifth factor takes up producer price inflation series as well as energy price inflation such as the changes in the oil price. The sixth factor loads on interest rates and partly explains spreads between long and short term interest rates. We interpret it as a term premium factor, as it happens to be highly correlated with measures of the term premium for government bonds as computed in Adrian et al. (2013), see Figure 3. The figure plots the median of the term premium factor against different measures of the term premium for government bonds available on the website of the Federal Reserve Bank of New York. For expositional convenience, all series including the factor estimate have been standardized. Excluding the period 1965Q1 - 1969Q4, the correlations between the median estimate of the factor and the five different measures are between 0.7 and 0.8. The last unobserved factor is taking up productivity, as it loads positively on TFP and on real output per hour as well. It also loads negatively on unit labor costs and positively on several measures of output. Finally, the FFR explains a large fraction of co-movement between interest rates. Table D.3 lists those series most correlated with each factor. According to these, we obtain essentially the same interpretation for factors as just given.

Despite the small number of factors, the common component explains on aver-

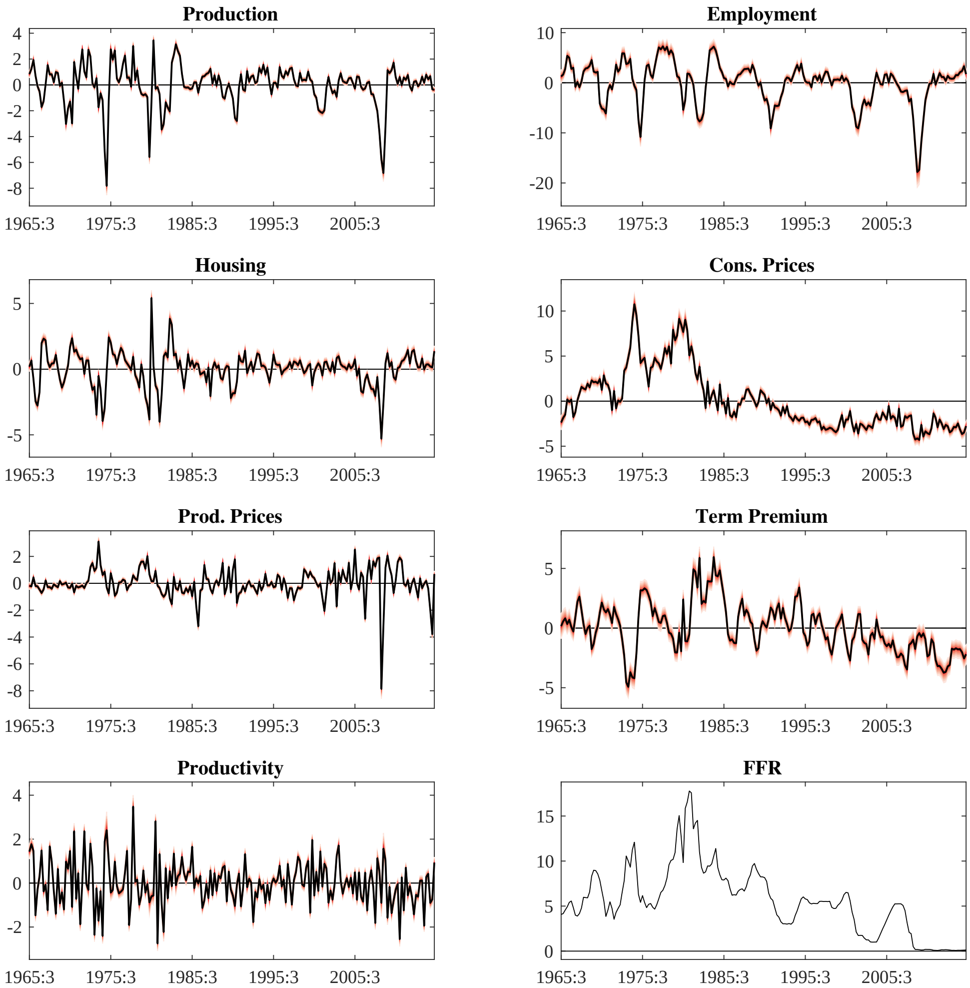

Figure 2: Estimated unobserved factors median along with 68% HPDI and FFR (observed factor) over the whole sample period.

age more than 50 percent of data variation. Table D.2 shows the variance share explained by the common component for some selected variables from seven different groups, the number shown is the median over all MCMC draws. The model does a good job in explaining real GDP and industrial production growth. The common component accounts for 99 and 95 percent of, respectively, GDP growth and industrial production growth variation. The common component further explains 56 percent of the variance in real consumption expenditures and 37 percent of TFP variation. However, the factors do a poor job in explaining capacity utilization in the manufacturing sector (CUMFNS), for which more than 90 percent of variance remains unexplained. The common component also accounts for a large

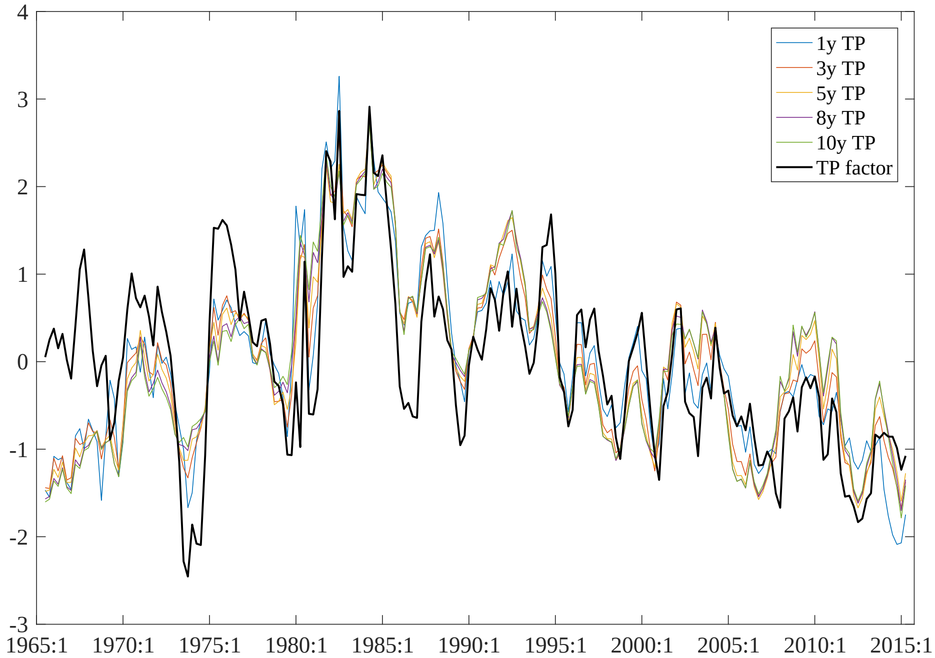

Figure 3: Factor 6 (median, standardized) and one to ten-years government bond term premia (standardized).

variance share in employment variables but government employees. However, this is not surprising, as the number of government employees is not expected to highly correlate with the economic situation. Overall, the common component accounts for a large share of variance in variables linked to the housing market, to sales, prices as well as interest rates. On the other hand, the common component explains only a minor share of variance in variables of the financial sector such as loans or stock market prices, which indicates that additional driving forces are captured by the idiosyncratic component.

4.3 Monetary policy

One of the main reasons why BBE05 proposed to combine the VAR methodology with factor analysis was the probable lack of important information in a small scale VAR to obtain structural identification of e.g. monetary policy shocks. A well known example is the price puzzle, i.e. the positive reaction of inflation in response to an unexpected interest rate hike. According to Sims (1992), a rationale for the price puzzle may be that the policy maker's information set includes more variables of high forecasting power than the econometrician's small VAR does. Another rationale is given by Giordani (2004), who thinks that biased measures of the output gap may lead to a price puzzle.

Since the FFR is the only included observed factor and given that we assume independence between innovations in unobserved and observed factors, η_t^Y can be interpreted as a monetary policy shock. Unanticipated changes in the FFR do not affect any of the unobserved factors on impact, at the same time the FFR does not respond contemporaneously to innovations in the unobserved factors. In that sense the identification scheme here is somewhat more restrictive than the often used recursive Cholesky restrictions, as they allow for a contemporaneous response of the FFR to other innovations. However, we do not expect that this artefact stemming from the factor identification will have a dramatic impact on the results. We exploit our data rich model to study how these monetary policy shocks affect the rest of the economy. Figure 4 plots the impulse response functions of the estimated factors to a monetary policy shock. First, we note that the shock to the FFR (Factor 8) dies out gradually over time. Factors 1 to 3, i.e. the production, employment and housing factors, all show an inverse humped shaped pattern. As expected, an increase in the FFR leads to a transitory slowdown in economic activity. The effect on the two price factors (Factors 4 and 5) is positive on impact. So, even though our model includes a broad range of information, the price puzzle still remains. It takes three to five quarters until the effect turns quite persistently negative. Compared to other factors, the uncertainty surrounding the impulse response function of the consumer price factor turns out to be much higher. We further observe that the response of producer and energy prices falls more rapidly into negative territory and dies out more quickly than the response of consumer prices. This indicates that consumer prices are somewhat stickier than producer and energy prices. The productivity factor does not show a strong reaction in response to the monetary policy shock.

Figure 5 plots the impulse responses to a FFR shock for some selected variables along with the 68 percent HPDI. Clearly, an interest rate hike has an adverse effect on economic activity and leads to a temporary decrease in industrial production. The effect dies out after about 15 quarters. For consumer as well as producer prices the short term effect is positive, the median response (black line) falls below zero only after several quarters. However the negative effect is only significant for producer prices. The response of the five year government bond yield indicates that a hike in the FFR also translates into a persistent increase in longer term interest rates. The negative effect on both the monetary base and M2 reflects a liquidity effect. Figure E.4 in the appendix contains the shares of forecast error variance (FEV) explained by the monetary policy shock for the same eight variables. The shares are highest for the three interest rates, while the shock explains relatively small portions of the variance of the remaining series.

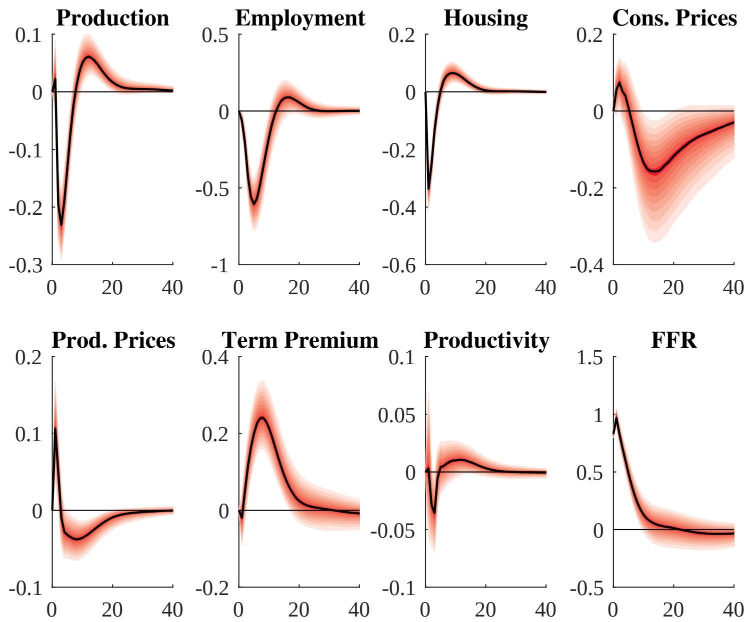

Figure 4: Impulse responses of the factors to an unanticipated change in the FFR.

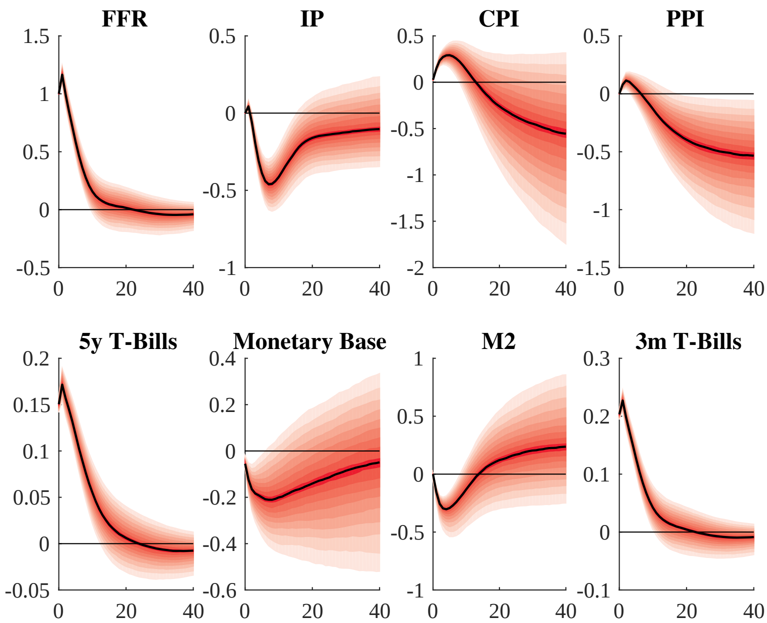

Figure 5: Impulse responses of selected variables to an unanticipated change in the FFR.

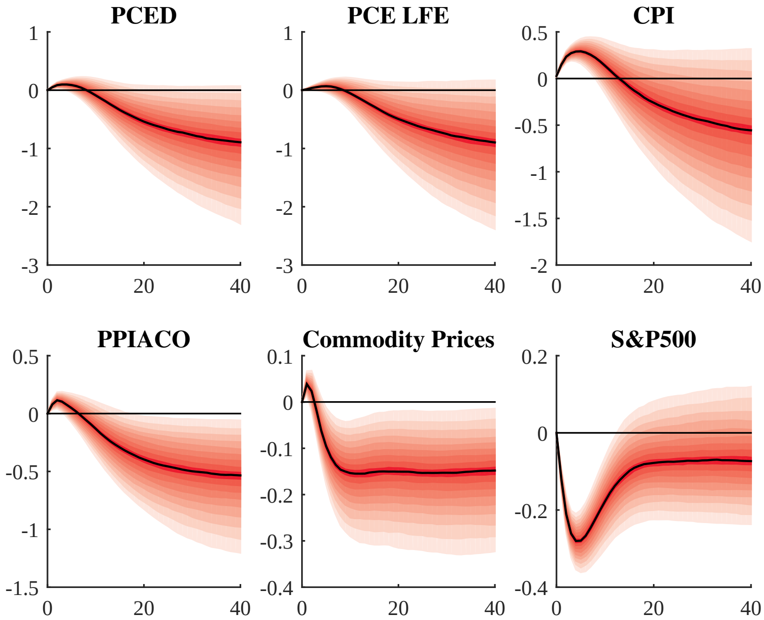

Figure 6: Impulse responses of selected price variables to an unanticipated change in the FFR.

Considering that the price puzzle is still present in the estimates, we may evaluate the responses of other price inflation series to a FFR shock. Figure 6 plots the impulse responses for six selected price indices. The upper panel contains the responses of three different measures of consumer prices, namely the personal consumption expenditures (PCED), PCED excluding food and energy (PCE LFE) and, for comparison, the consumer price index reproduced from Figure 5. While the effect of a FFR increase on the CPI is strongly positive in the short run, this is less the case for the other two series. Nevertheless, they have in common that it takes a while (again several quarters) until the median response reaches negative territory. The lower panel of Figure 6 plots the impulse responses of producer prices (PPI-ACO), commodity prices and the S&P 500. While the reaction of producer prices is similar to those of consumer prices, commodity prices take a shorter time to contract. The response of stock prices is typical for a FFR shock. They considerably fall on impact and follow an inverted hump shaped pattern. Our results stand in contrast to those of Baumeister et al. (2013), who do not report a price puzzle for aggregate price level measures. Their model allows for time varying parameters and is estimated over a shorter sample period which excludes recent years. Figure E.2 in appendix E reveals that the positive response of prices is partly linked to the great recession, during which interest rates were lowered to the zero lower bound. We plot the same impulse responses obtained when the model is estimated with data ending in 2007Q2. We observe that prices still take a while to decrease after a FFR hike, but only CPI still shows a slight positive reaction in the short run. This may reflect some degree of time variation in price responses to FFR shocks. Given the recent introduction of unconventional monetary policy measures, this instability does not come as a big surprise. It is further interesting to note that the response of the S&P 500 is also quite different when the model is estimated without the great recession. In this case the negative effect of an interest rate hike is much weaker compared to the full sample estimation, and at a longer horizon the median response stays persistently on a positive level. This points towards a much stronger reaction of stock markets to monetary policy shocks during and after the financial crisis.

An alternative explanation for the observed price puzzle would be that our identified monetary policy shocks are in fact no monetary policy shocks. To check this we compare our identified shock series to the monetary shock measure of Romer and Romer (2004), and find a considerable similarity between the two series for the available time period[10]. Figure E.3 in the Appendix plots the two measures against each other over the available time span of the original Romer and Romer data, the

[10]We took the original series that covers the time span from March 1969 to December 1996 and converted it to quarterly data.

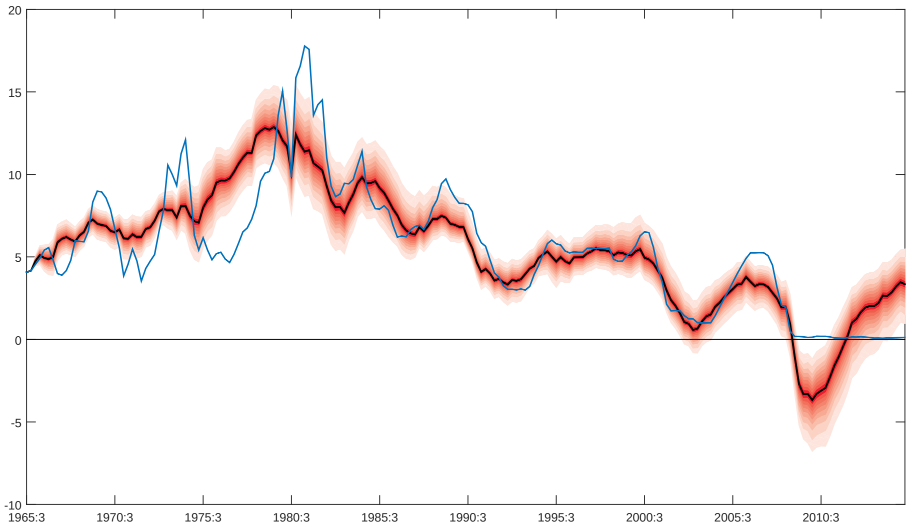

Figure 7: Predicted FFR (median and 68% HPDI) along with the realized values of the FFR (in blue).

estimated correlation coefficient between the series is 0.63. Our identified shock series seems to be consistent with their findings.

Given that the FFR is the monetary policy instrument and the observed factor in the FAVAR, we are implicitly estimating a reaction function for monetary policy and can compute a prediction of the FFR conditional on the observed data. The prediction can be seen as a sort of Taylor interest rate implied by the model[11]. Figure 7 plots the prediction for the FFR along with the actual values (in blue). The plot reveals some interesting insights. First, during the late 1970s, the actual FFR lies clearly below our estimate, indicating that monetary policy has been relatively loose during that period. At the beginning of the 1980s during the Volcker era, the opposite is true. During this period, relative to the model based interest rate monetary policy was tight in order to fight against high inflation rates. Otherwise, the differences between the predicted and the actual value remain small, although there is a tendency for the actual value to exceed the former during boom phases. This is particularly the case in 2005 before the beginning of the financial crisis. We also see how monetary policy has been trapped at the lower bound after the outbreak of the great recession. The model based prediction of the FFR dives deep into negative territory in response to the financial crisis, but also shows a much earlier and faster tapering thereafter.

[11] In analogy to the interest rate feedback rule proposed in Taylor (1993).

4.4 Term premium shock

We suggested that one of the identified unobserved factors seems to be special as it mainly loads on interest rate spreads and highly correlates with measures of the term premium, see Figure 3. To get a better understanding of its role, we identify a structural shock that only affects the term premium factor on impact, leaving the other factors unaffected.

A shock to the term premium factor leads to pro-cyclical responses in GDP, consumption, investment, housing, employment and hours worked, see Figures 8 and 9. Consumer as well as producer prices fall in response to this shock, while stock prices increase. Consumer confidence measured by the University of Michigan's consumer sentiment index increases, while the VXO volatility index falls. The spread of the 10 year government bond over the 3 month treasury bills increases as the bond return increases more strongly. In Figure 9, we observe that the spread between 1 year and 3 month treasuries also transitorily increases. The spread between Moody's seasoned BAA corporate bond yield and the return on 10 year treasuries falls, which indicates a higher risk appetite of investors. We further observe an increase in the amount of total outstanding consumer credit, the same is true for commercial and industrial loans. TFP is unaffected on impact, which is by construction so. Then, it starts increasing as well.

The impulse responses of consumption, TFP, CPI and the spread between long and short term interest rates to the term premium shock closely mirror those of a slope shock in Kurmann and Otrok (2013) (KO13 henceforth). In KO13 the slope shock is identified as the shock that maximizes the FEV of the slope of the term structure. Both their and our shock lead to similar impulse responses of key macroeconomic variables, which seems quite natural as the slope of the term structure and our term premium factor capture the same economic concept linked to expectations about the future state of the economy. KO13 point out the strong similarities between the impulse responses of a TFP news shock[12] and a slope shock, which leads them to conclude that the main driver of movements in the slope of the term structure are in fact TFP news shocks. Further, for this observation they assign a key role to the endogenous response of monetary policy *"...the news shock seems to be a major determinant of movements in the slope through its influence on monetary policy at the short end of the term structure."* (KO13 p. 2623). In their impulse responses, the increase in the spread results from a decrease in short term interest rates while long term interest rates barely move. In contrast, our impulse responses document an increase in both short term and long term interest rates, whereby the latter re-

[12]KO13 identify the TFP news shock along the lines of Barsky and Sims (2011) in a VAR framework.

act more strongly. Hence, our findings show that an unanticipated shock that also leads to a change at the longer end of the term structure can produce the same responses of key macroeconomic variables as in KO13. Economically, an increase of interest rates in response to a news shock is also plausible. In anticipation of a future technology shock consumption starts to increase, which might put savings under pressure and lead to an increase in interest rates.

4.5 Identification of interpretable shocks

Sparsity in the factor loading matrix helps identifying and interpreting factors and factor innovations in FAVAR models. However, we may want to identify a shock in the factor VAR that acts as the main driver of a certain variable in the underlying data set. For this, we can rely on the method proposed by Uhlig (2004) which identifies an orthogonal shock based on the explained fraction of the FEV of a given variable. Concretely, the method determines the impact effects of a shock that maximizes the FEV of the variable of interest over a given forecast horizon. This identification strategy can easily be adapted to the FAVAR framework, in which the observed variables do not enter the VAR directly, see appendix C for computational details. To illustrate the method, we identify a technology shock as the main driver of TFP, i.e. the shock which accounts for the highest fraction of the FEV of TFP at a horizon up to four quarters. The identified shock permanently raises TFP, see Figure 10. The shock leads to a permanent increase in GDP and a permanent decrease in the CPI. Interestingly, total hours worked fall on impact as higher productivity seems to lower the demand for labor. Consumption increases quite persistently, the effect dying out only slowly. Interest rates fall gradually, while the spread between long and short term interest rates increases slightly in response to a technology shock. The impulse response of hours worked are in line with findings in Gali (1999). They find that the conditional correlations of hours worked and productivity are negative for technology shocks and that hours worked show a persistent decline in response to a positive technology shock. The technology shock explains almost all the FEV of TFP up to a horizon of 40 quarters and nearly 30 percent of the FEV of GDP (see Figure E.5 in the Appendix).

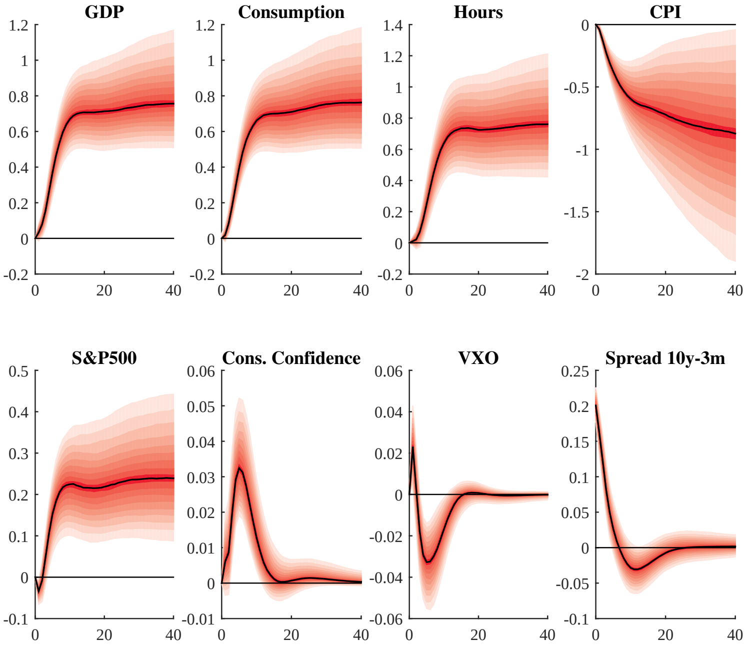

Figure 8: Impulse Responses of selected variables to an unanticipated change in the term premium.

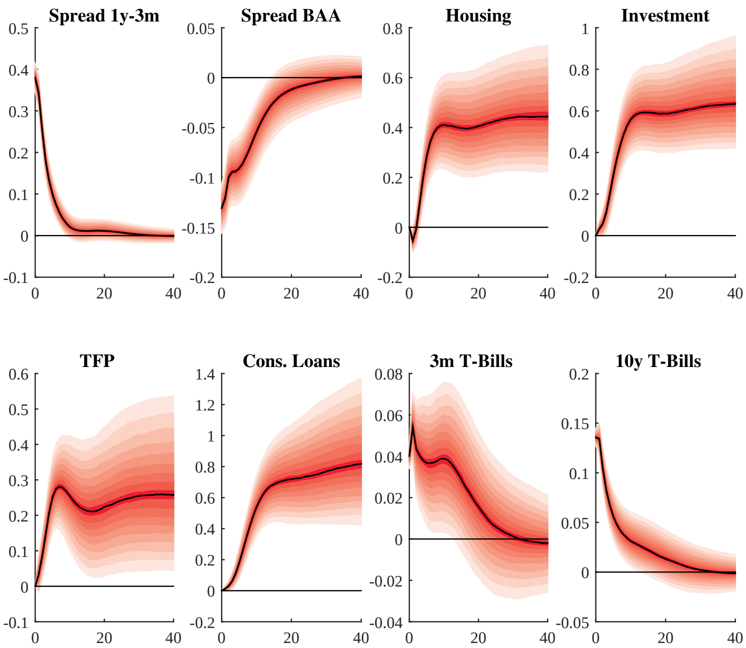

Figure 9: Impulse Responses of selected variables to an unanticipated change in the term premium.

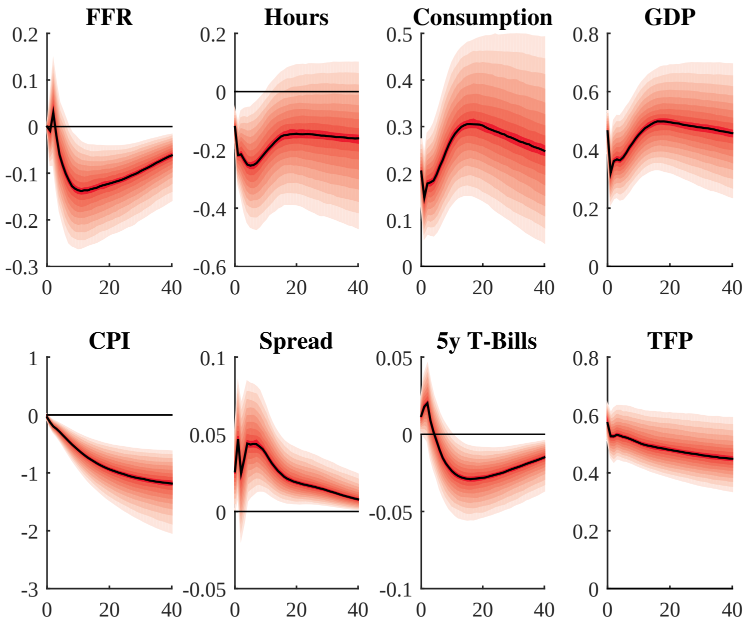

Figure 10: Impulse responses of selected variables to a technology shock.

5 Conclusion

In the present paper we combine the FAVAR framework with the estimation and identification procedures for sparse dynamic factor models. Sparse factor models are widely used in other fields and we think they are very valuable to analyze economic data. Introducing sparsity in the context of FAVAR provides one solution to the identification problem common to all factor models. It further allows us to assign a meaningful economic interpretation to the identified factors due to the sparse structure in the factor loading matrix. An additional distinction to traditional factor models is that we depart from the strong assumption of orthogonal common shocks and work with correlated factor shocks instead. This allows us to identify structural shocks using different strategies that have been proposed in the structural VAR literature. We apply our methodology to an empirical data set for the US macro economy (FRED QD) and find that there is indeed a high degree of sparsity present in the data. The proposed estimation and identification procedure is successful in identifying seven unobserved factors representing production, employment, the housing market, consumer and producer prices, productivity and term premia. Together, they account for about 52 percent of variation in the data. We utilize the role of the FFR, the monetary policy instrument, as observed factor to study the effects of monetary policy on the economy. The estimated factors as well as specific variables all show reasonable responses to an unanticipated interest rate hike. However, we find that the monetary policy shock exhibits a mild price puzzle which seems to be linked to the great recession, as it nearly vanishes when the period after 2007Q3 is excluded from the sample. One of the estimated unobserved factors is partly explaining the term premia in government bond yields. The impulse responses to an innovation in the term premium factor closely mirror those to the slope shock in KO13, and are in fact very similar to those of the news shock identified in Barsky and Sims (2011). However, the main difference to KO13 is that in response to the term premium shock short and long term interest rates increase, whereas KO13 report a decrease in short term interest rates and no effect at the longer end of the yield curve. Finally, we identify the technology shock as the one which maximizes the explained fraction of FEV in TFP by adapting the methodology of Uhlig (2004) to the FAVAR environment. In line with the findings in Gali (1999), the impulse response of hours to a technology shock decline and hence, show a negatively correlated reaction to TFP.

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A Prior distributions

The idiosyncratic components are independent. Therefore we formulate variable-specific prior distributions for $\psi_i = (\psi_{i1}, \ldots, \psi_{iq})'$ and ω_i^2,

$$\begin{aligned}
\pi(\psi_i) &= N(q_0, Q_0) I_{\{Z(\psi_i)>1\}} \\
\pi(\omega_i^2) &= IG(u_0, U_0), \quad i = 1, \ldots, N
\end{aligned}$$

where $I_{\{\cdot\}}$ is an indicator function that takes on the value one if the roots of the characteristic polynomial of the underlying process lie outside the unit circle.

For the factor autoregressive parameters $vec(\Phi^{*\prime})$, where $\Phi^* = [\Phi_1^*, \ldots, \Phi_p^*]$, we assume multivariate normal priors truncated to the stationary region

$$\pi(vec(\Phi^{*\prime})) = N(p_0, P_0) I_{\{Z(\Phi^*)>1\}}$$

We formulate an inverse Wishart prior on the error covariance matrix of observed variables Y_t, $\Sigma_Y \sim IW\left(\nu_Y, S_Y\right)$.

B Posterior distributions

B.1 The factor loadings λ^*

To simplify notation let $\lambda^* = [\lambda^{*f} \quad \lambda^{*Y}]$ and $\mathcal{F}_t^* = [f_t^{*\prime} \quad Y_t^{\prime}]'$. The first step to get the posterior for the factor loadings $\pi(\lambda_{ij}^* | \mathcal{F}^{*T}, X^T, Y^T, \Psi(L), \Omega)$ is to integrate out the variable specific prior probability of zero loading for each factor j. The prior described above implies a common base rate of non-zero factor loading of $E(\beta_{ij}) = \rho_j b$ across variables. The marginal distribution then becomes

$$\pi(\lambda_{ij}^* | \rho_j, \tau_j) \sim (1 - \rho_j b)\delta_0(\lambda_{ij}^*) + \rho_j b N(0, \tau_j) \tag{B.1}$$

To isolate the effect of factor j on variable i we transform the variables to

$$x_{it}^* = \psi_i(L) x_{it} - \sum_{l=1, l \neq j}^{k+m} \lambda_{il}^* \psi_i(L) \mathcal{F}_{jt}^* + \varepsilon_{it} \tag{B.2}$$

Now we combine the marginal prior with data to sample independently across i from

$$\pi(\lambda_{ij}^*|\cdot) = \prod_{t=q+1}^{T} \pi(x_{it}^*|\cdot)\{(1-\rho_j b)\delta_0(\lambda_{ij}^*) + \rho_j b N(0,\tau_j) \tag{B.3}$$

$$= P(\lambda_{ij}^* = 0|\cdot)\delta_0(\lambda_{ij}^*) + P(\lambda_{ij}^* \neq 0|\cdot))N(m_{ij}, M_{ij}) \tag{B.4}$$

with observation density $\pi(x_{it}^*|\cdot) = N(\lambda_{ij}^*\psi_i(L)\mathcal{F}_{jt}^*, \omega_i^2)$ and where

$$M_{ij} = \left(\frac{1}{\omega_i^2}\sum_{t=q+1}^{T}(\psi_i(L)f_{jt}^*)^2 + \frac{1}{\tau_j}\right)^{-1} \tag{B.5}$$

$$m_{ij} = M_{ij}\left(\frac{1}{\omega_i^2}\sum_{t=q+1}^{T}(\psi_i(L)f_{jt}^*)x_{it}^*\right) \tag{B.6}$$

To obtain the posterior odds $P(\lambda_{ij}^* \neq 0|\cdot)/P(\lambda_{ij}^* = 0|\cdot)$ the prior odds of the non-zero factor loading are updated:

$$\frac{P(\lambda_{ij}^* \neq 0|\cdot)}{P(\lambda_{ij}^* = 0|\cdot)} = \frac{\pi(\lambda_{ij}^*)|_{\lambda_{ij}^*=0}}{\pi(\lambda_{ij}^*|\cdot)|_{\lambda_{ij}^*=0}}\frac{\rho_j b}{1-\rho_j b} = \frac{N(0;0,\tau_j)}{N(0;m_{ij},M_{ij})}\frac{\rho_j b}{1-\rho_j b} \tag{B.7}$$

Conditional on λ_{ij}^* the variable specific probabilities β_{ij} are updated and sampled from $\pi(\beta_{ij}|\lambda_{ij}^*,\cdot)$. When $\lambda_{ij}^* = 0$

$$\pi(\beta_{ij}|\lambda_{ij}^* = 0,\cdot) \propto (1-\beta_{ij})[(1-\rho_j)\delta_0(\beta_{ij}) + \rho_j B(ab, a(1-b))] \tag{B.8}$$

$$P(\beta_{ij} = 0|\lambda_{ij}^* = 0,\cdot) \propto (1-\rho_j), \quad P(\beta_{ij} \neq 0|\lambda_{ij}^* = 0,\cdot) \propto (1-\beta_j)\rho_j \tag{B.9}$$

That is, with posterior odds $(1-b)\rho_j/(1-\rho_j)$ we sample from $B(ab, a(1-b)+1)$ and set β_{ij} equal to zero otherwise. Conditional on $\lambda_{ij}^* \neq 0$ we obtain

$$\pi(\beta_{ij}|\lambda_{ij}^* \neq 0,\cdot) \propto \beta_{ij}N(\lambda_{ij}^*;0,\tau_j)[(1-\rho_j)\delta_0(\beta_{ij}) + \rho_j B(ab, a(1-b))] \tag{B.10}$$

$$P(\beta_{ij} = 0|\lambda_{ij}^* \neq 0,\cdot) = 0, \quad P(\beta_{ij} \neq 0|\lambda_{ij}^* \neq 0,\cdot) = 1 \tag{B.11}$$

In this case we sample β_{ij} from $B(ab+1, a(1-b))$.
The posterior update of the hyperparameters τ_j and ρ_j is sampled from an inverse Gamma, $\pi(\tau_j)|\cdot) \sim IG(g_j, G_j)$ and a Beta distribution $\pi(\rho_j|\cdot) \sim B(r_{1j}, r_{2j})$, respec-

tively, with

$$g_j = g_0 + \frac{1}{2}\sum_{i=1}^{N} I_{\{\lambda^*_{ij}\neq 0\}}, \quad G_j = G_0 + \frac{1}{2}\sum_{i=1}^{N} \lambda^*_{ij} \tag{B.12}$$

$$r_{1j} = r_0 s_0 + S_j, \quad r_{2j} = r_0(1-s_0) + N - S_j \tag{B.13}$$

where $S_j = \sum_{i=1}^{N} I_{\{\beta_{ij}\neq 0\}}$

B.2 Sampling the factors: Covariance of initial states

If Σ_{f0} in $\mathbf{\Sigma}_f$ is not chosen to be diffuse, we may set it equal to the stationary variance. From the companion form of a VAR(p) process, $\bar{F}_t = \tilde{\mathbf{\Phi}}^f \bar{F}_{t-1} + \boldsymbol{\eta}^f_t$, $\boldsymbol{\eta}^f_t \sim N\left(0, \begin{bmatrix} \Sigma^*_f & 0_{k\times k(p-1)} \\ \multicolumn{2}{c}{0_{k(p-1)\times kp}} \end{bmatrix}\right)$, with

$$\tilde{\mathbf{\Phi}}^f = \begin{bmatrix} \tilde{\mathbf{\Phi}}^f_1 \\ \tilde{\mathbf{\Phi}}^f_2 \end{bmatrix}, \; \tilde{\mathbf{\Phi}}^f_1 = \begin{bmatrix} \Phi^{*f}_1 & \dots & \Phi^{*f}_p \end{bmatrix}, \; \tilde{\mathbf{\Phi}}^f_2 = \begin{bmatrix} I_{k(p-1)} & \mathbf{0}_{k(p-1)\times k} \end{bmatrix}$$

we obtain $E(\bar{F}_t\bar{F}'_t) = \tilde{\mathbf{\Phi}}^f E(\bar{F}_{t-1}\bar{F}'_{t-1})\tilde{\mathbf{\Phi}}^{f\prime} + \Sigma_{\boldsymbol{\eta}^f}$ and $\Sigma_{\bar{F}} = \tilde{\mathbf{\Phi}}^f \Sigma_{\bar{F}} \tilde{\mathbf{\Phi}}^{f\prime} + \Sigma_{\boldsymbol{\eta}^f}$. The vec operator yields

$$\mathrm{vec}(\Sigma_{\bar{F}}) = \left[\mathbf{I}_{(pk)^2} - (\tilde{\mathbf{\Phi}}^f \otimes \tilde{\mathbf{\Phi}}^f)\right]^{-1} \times \mathrm{vec}\left(\Sigma_{\boldsymbol{\eta}^f}\right)$$

from which we can retrieve the corresponding values for Σ_{f0}.

B.3 The idiosyncratic components

The posterior simulation of the parameters is divided in two blocks. The dynamics of the idiosyncratic components $\psi_i = (\psi_{i1}, ..., \psi_{iq})'$ are sampled individually.

$$\pi(\psi_i | X_i, \mathcal{F}^*, \theta_{-\mathbf{\Psi}}) = N(q_i, Q_i), \quad i = 1, ..., N \tag{B.14}$$

where

$$Q_i = \left(\omega_i^{-2}\tilde{X}_i^{-\prime}\tilde{X}_i^{-} + Q_0^{-1}\right)^{-1} \tag{B.15}$$

$$q_i = Q_i\left(\sigma_i^{-2}\tilde{X}_i^{-\prime}\tilde{X} + Q_0^{-1}q_0\right) \tag{B.16}$$

$$\tilde{X}_i = \begin{bmatrix} X_{iq+1} - \lambda_i^*\mathcal{F}_{q+1}^* \\ \vdots \\ X_{iT} - \lambda_i^*\mathcal{F}_T^* \end{bmatrix} \tag{B.17}$$

$$\tilde{X}_i^- = \begin{bmatrix} X_{iq} - \lambda_i^*\mathcal{F}_q^* & \cdots & X_{i1} - \lambda_i^*\mathcal{F}_1^* \\ \vdots & & \vdots \\ X_{iT-1} - \lambda_i^*\mathcal{F}_{T-1}^* & \cdots & X_{iT-q} - \lambda_i^*\mathcal{F}_{T-q}^* \end{bmatrix} \tag{B.18}$$

The variance of the idiosyncratic component, ω_i^2, is simulated from independent inverse Gamma distributions $IG(u_i, U_i)$, $i = 1, ..., N$ with $u_i = u_0 + 0.5(T - p)$ and $U_i = U_0 + 0.5(\tilde{X}_i - \tilde{X}_i^-\psi_i)'(\tilde{X}_i - \tilde{X}_i^-\psi_i)$.

B.4 The parameters for the factor dynamics

The dynamics of the unobserved factors f_t^* and observed variables Y_t are jointly sampled from

$$\pi(vec(\Phi^{*\prime})|X, \mathcal{F}^*, \Sigma^*) = N(p, P)I_{\{Z(\Phi^*)>1\}} \tag{B.19}$$

where

$$P = \left([I_{k+m} \otimes f^{*-}]'[I_{k+m} \otimes f^{*-}] + P_0^{-1}\right)^{-1} \tag{B.20}$$

$$p = P\left([I_{k+m} \otimes f^{*-}]'vec(f^*) + P_0^{-1}p_0\right) \tag{B.21}$$

where $f^* = \left[\mathcal{F}_{p+1}^*, \ldots, \mathcal{F}_T^*\right]'$ and

$$f^{*-} = \begin{bmatrix} \mathcal{F}_p^{*\prime} & \cdots & \mathcal{F}_1^{*\prime} \\ \vdots & & \vdots \\ \mathcal{F}_{T-1}^{*\prime} & \cdots & \mathcal{F}_{T-p}^{*\prime} \end{bmatrix}$$

B.5 The error covariance matrix of factors Σ^*

We depart from the assumption of independent factor innovations and require only that the innovations of the unobserved factors be orthogonal to those of the observed ones. The two blocks Σ_f^* and Σ_Y are thus full matrices. While the elements of the latter are unrestricted, we set the diagonal elements of Σ_f^* to one in order to normalize factor scale. Sampling Σ_f^* is thus equivalent to sample a correlation matrix for the unobserved factors, for which we lack a standard distribution. Following Conti et al. (2014) we rely on marginal data augmentation techniques and temporarily expand the parameter space of the model with the variances of the unobserved latent factors as working parameters when it comes to sampling Σ^*_f. Using the decomposition $\hat{\Sigma}_f = V^{\frac{1}{2}}\Sigma^*_f V^{\frac{1}{2}}$, any covariance matrix can be decomposed into two parts, a correlation matrix Σ^*_f and a matrix V that contains the variances on its diagonal. Assuming a hierarchical inverse Wishart prior distribution $\hat{\Sigma}|S_f \sim IW(\nu_f, S_f)$, the joint distribution of V and S_f can be factored as $p(V, S_f|\Sigma^*_f) = p(V|S_f, \Sigma^*_f)p(S_f)$, and it can be shown that each diagonal element of V, v_j, follows an inverse Gamma distribution

$$v_j|\Sigma^*_f, s_j \sim IG\left(\frac{\nu}{2}, \frac{s_j\sigma^{*-}_{fj}}{2}\right), \; j = 1, \ldots, k \tag{B.22}$$

where s_j and σ^{*-}_{fj} are the jth diagonal elements of, respectively, S_f and Σ^{*-1}_f. For S_f we impose the Huang and Wand (2013) prior as in Conti et al. (2014), hence S_f is a nonsingular diagonal matrix with its non-zero elements following a Gamma distribution[13]

$$s_j \sim G\left(\frac{1}{2}, \frac{1}{2\nu^* C_j^2}\right), \; j = 1, \ldots, k \tag{B.23}$$

At iteration (m), we proceed as follows:

(i) Sample V_{prior} from (B.22) and (B.23).

(ii) Expand the model

$$\begin{aligned} \hat{f}_t^{*(m)} &= V_{prior}^{\frac{1}{2}} f_t^{*(m)}, \; \hat{\lambda}^{*f(m)} = \lambda^{*f(m)} V_{prior}^{-\frac{1}{2}} \\ \hat{\Phi}_l^{*f(m)} &= V_{prior}^{\frac{1}{2}} \Phi_l^{*f(m)} V_{prior}^{-\frac{1}{2}} \text{ for } l = 1, ..., p \end{aligned}$$

In this expanded model the residuals are distributed as $\hat{\eta}_t^{f(m)} \sim N\left(0, \hat{\Sigma}_f^{*(m)}\right)$ with

$$\hat{\Sigma}_f^{*(m)} = V_{prior}^{\frac{1}{2}} \Sigma_f^{*(m-1)} V_{prior}^{\frac{1}{2}}$$

(iii) Update the covariance matrix

$$\hat{\Sigma}_f^{*(m)}|S \sim IW\left(\nu_f + (T-p), S_f + \sum_{t=p+1}^{T} \hat{\eta}_t^{*f(m)} \hat{\eta}_t^{*f(m)\prime}\right)$$

and update the working parameter V_{post} by setting it to the diagonal elements of $\hat{\Sigma}_f^{*(m)}$.

[13] It is parametrized such that $\nu^* = \nu - k + 1$ and $E(s_j) = \nu^* C_j^2$

(iv) Transform back to the identified model

$$
\begin{aligned}
f_t^{*(m)} &\leftarrow V_{post}^{-\frac{1}{2}} \hat{f}_t^{*(m)}, \quad \lambda^{*f(m)} \leftarrow \hat{\lambda}^{*f(m)} V_{post}^{\frac{1}{2}} \\
\Phi_l^{*f(m)} &\leftarrow V_{post}^{-\frac{1}{2}} \hat{\Phi}_l^{*f(m)} V_{post}^{\frac{1}{2}}, \quad l = 1, ..., p \\
\Sigma_f^{*(m)} &= V_{post}^{-\frac{1}{2}} \hat{\Sigma}_f^{*(m)} V_{post}^{-\frac{1}{2}}
\end{aligned}
$$

We then proceed with the second block of the covariance matrix, which is left unrestricted and can be drawn from an inverse Wishart distribution.

$$
\Sigma_Y^* \sim IW\left(\nu_Y + (T-p), S_Y + \sum_{t=p+1}^{T} \eta_t^Y \eta_t^{Y\prime}\right)
$$

C Identification of structural shocks by maximizing the explained share of the forecast error variance

The approach was originally proposed by Uhlig (2003,2004). The idea is to identify $s \leq k$ orthogonal shocks that explain the maximum fraction of the forecast error variance (FEV) over a given prediction horizon $t + \underline{h}$ to $t + \bar{h}$ for one variable included in the VAR. In the present paper, we adapt the approach to the FAVAR framework. The target will not be to explain a maximum share in the FEV of a factor. Rather, we maximize the explained share in the FEV of a selected variable in X_t, for example TFP. In this section, we use notation similar to Uhlig (2003) for a better understanding. The VAR for factors writes

$$
\Phi^*(L)\mathcal{F}_t^* = \eta_t^*
$$

where η_t^* are the one step ahead prediction errors with variance-covariance matrix Σ^*. If the VAR is stationary, we can invert to the moving average representation:

$$
\mathcal{F}_t^* = C(L)\eta_t^*
$$

where

$$
C(L) = \sum_{l=0}^{\infty} C_l L^l
$$

To identify the structural shocks, we need to find a matrix A which fulfills $\eta_t^* = A\upsilon_t$ and $E[\upsilon_t \upsilon_t'] = I_{k+m}$. Note that in our setup the last element in η_t^*, the monetary policy shock, is orthogonal to the other elements by construction (all off-diagonal elements in the last row and column of Σ^* are set to zero). To identify additional structural shocks, we are interested in finding a $k \times k$ submatrix, A_1, of A, such that $A_1\eta_t^{*f} = \upsilon_t^1$, $E[\upsilon_t^1\upsilon_t^{1'}] = I_k$ and

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & 1 \end{bmatrix}$$

The impulse responses to the structural shocks are then computed as

$$R(L) = C(L)A$$

An obvious candidate for A_1 is the Cholesky decomposition of the leading $k \times k$ submatrix of Σ^*. But using any orhtogonal matrix Q_1 satisfying $Q_1 Q_1' = I_k$, yields another valid candidate $\tilde{A}_1 = A_1 Q_1$ with impulse responses

$$\left[\tilde{R}(L) = R(L)Q\right], \; Q = \begin{bmatrix} Q_1 & 0 \\ 0 & 1 \end{bmatrix}$$

Call $e_{t+h|t-1}$ the h-step ahead prediction error of $\mathcal{F}_{t+h}$ given all the data up to $t-1$,

$$e_{t+h|t-1} = \sum_{l=0}^{h} R_l Q \nu_{t+h-l}$$

with covariance matrix

$$\Sigma(h) = \sum_{l=0}^{h} R_l R_l' = \sum_{j=1}^{k+m} \sum_{l=0}^{h} (R_l q_j)(R_l q_j)'$$

were q_j is the jth vector of the matrix Q. The last term represents the covariance matrix as the sum of each (orthogonal) shock's covariance component.

In Uhlig (2003) the goal is to find the vector q_1 that explains the maximum share in the FEV over a pre-defined horizon of a variable i included in the VAR

$$\Sigma(\underline{h}, \bar{h}, i) = \sum_{h=\underline{h}}^{\bar{h}} \Sigma(h)_{ii}$$

This vector is given by the eigenvector associated with the largest eigenvalue of the matrix

$$\tilde{S} = \sum_{h=\underline{h}}^{\bar{h}} \sum_{l=0}^{h} (\iota_i R_l)'(\iota_i R_l)$$

where ι_i is the selection vector with a 1 at position of variable i.

Our focus lies on the object

$$\Sigma(\underline{h}, \bar{h}, i) = \sum_{h=\underline{h}}^{\bar{h}} \lambda_i^* \Sigma(h) \lambda_i^{*'}$$

which is the forecast error variance of variable i in X_t. Therefore, the vector q_1 will be the eigenvector corresponding to the largest eigenvalue of the matrix

$$\tilde{S} = \sum_{h=\underline{h}}^{\bar{h}} \sum_{l=0}^{h} (\lambda_i^* R_l)'(\lambda_i^* R_l)$$

D Tables

Factor loadings	$r_0 = 200$, $s_0 = 0.35$, $\tau_j \sim IG(2, 0.125)$, $a = 0.01$, $b = 0.4$
Factor VAR	$vec(\Phi) \sim N(0, P_0)$, P_0: Minnesota with prior diagonal variance 0.25 and shrink factor for off-diagonals 0.025, $\nu = k + m + 1$, $\nu^* = \nu - (k + m) + 1$
Idiosyncratic component	$\psi_i \sim N(0, 0.25)$, $\sigma_i^2 \sim IG(2, 0.25)$

Table D.1: Prior specification

NIPA and Production	
GDPC96	0.99
PCECC96	0.56
GPDIC96	0.74
FPIx	0.77
PRFIx	0.67
INDPRO	0.95
CUMFNS	0.10
TFP	0.37

Employment	
PAYEMS	0.93
USPRIV	0.97
MANEMP	0.91
UNRATE	0.87
USGOVT	0.03
HOABS	0.84

Housing	
HOUST	0.78
PERMIT	0.84

Sales	
CMRMTSPLx	0.81

Prices	
PCECTPI	0.98
DGOERG3Q086SBEA	0.89
CPIAUCSL	0.96
PPIACO	0.79
OILPRICEx	0.58
DNDGRG3Q086SBEA	0.96

Interest Rates	
TB3MS	0.99
GS1	0.99
GS5	0.95
GS10	0.84
AAA	0.62
TB3SMFFM	0.81
GS10TB3Mx	0.64

Credit and Stocks	
BUSLOANSx	0.24
CONSUMERx	0.11
REALLNx	0.16
TOTALSLx	0.46
S0x26P500	0.27

Table D.2: Median variance share explained by the common component.

Factor 1	IPMANSICS(0.98) INDPRO(0.97) IPMAT(0.94) IPDMAT(0.93) IPFINAL(0.91) CMRMTSPLx(0.88) HOANBS(0.85) NAPMPI(0.85)
Factor 2	USPRIV(0.93) PAYEMS(0.92) USWTRADE(0.91) USTPU(0.91) USGOOD(0.90) SRVPRD(0.87) DMANEMP(0.86) MANEMP(0.86)
Factor 3	PERMIT(0.93) HOUST(0.89) PERMITS(0.88) PER-MITW(0.80) HOUSTS(0.78) PERMITMW(0.77) HOUSTW(0.75) PRFIx(0.74)
Factor 4	PCEPILFE(0.99) DSERRG3Q086SBEA(0.97) GDPCTPI(0.96) DHCERG3Q086SBEA(0.96) IPDBS(0.94) CPILFESL(0.92) PCECTPI(0.92) DDURRG3Q086SBEA(0.89)
Factor 5	DGOERG3Q086SBEA(0.94) DNDGRG3Q086SBEA(0.92) CPITRNSL(0.90) CUSR0000SAC(0.87) PPIFCG(0.86) PPIACO(0.85) PPIIDC(0.84) DGDSRG3Q086SBEA(0.81)
Factor 6	GS1TB3Mx(0.79) GS10(0.59) AAA(0.58) BAA(0.57) T5YFFM(0.56) GS5(0.56) TB6M3Mx(0.51) GS10TB3Mx(0.47)
Factor 7	OPHPBS(0.83) OPHNFB(0.80) GDPC96(0.67) OUTBS(0.66) OUTNFB(0.63) TFP(0.58) UNLPNBS(0.54) GCEC96(0.40)

Table D.3: Series most correlated with unobserved factors, correlation coefficient in brackets.

E Additonal figures

E.1 Choosing the number of factors

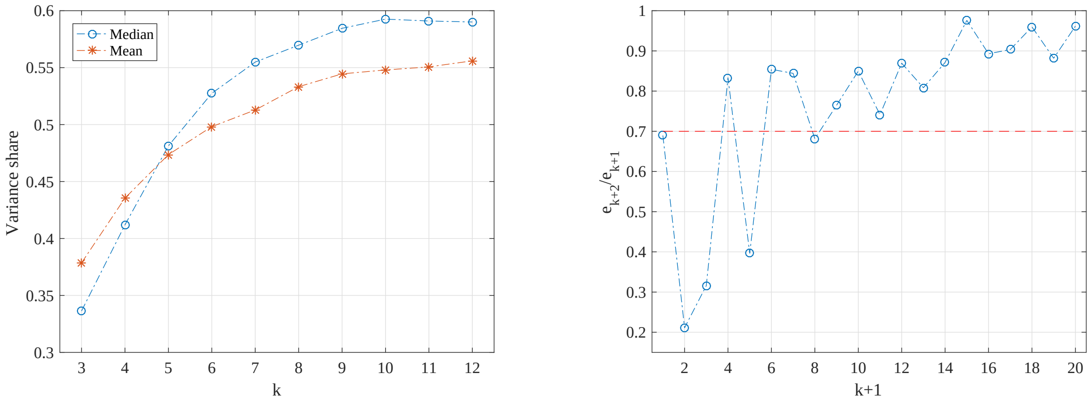

Figure E.1: Left: Variance shares explained by the common component conditional on $k = 3, \ldots, 12$ estimated unobserved factors. Right: Eigenvalue-ratio based criterion for the number of factors. The global maximum indicates 2 strong factors, the local minima at 5, 8, 11 and 13 indicate further so-called weaker factors. We cut off at a ratio of 0.7.

E.2 Additional impulse responses and variance decompositions

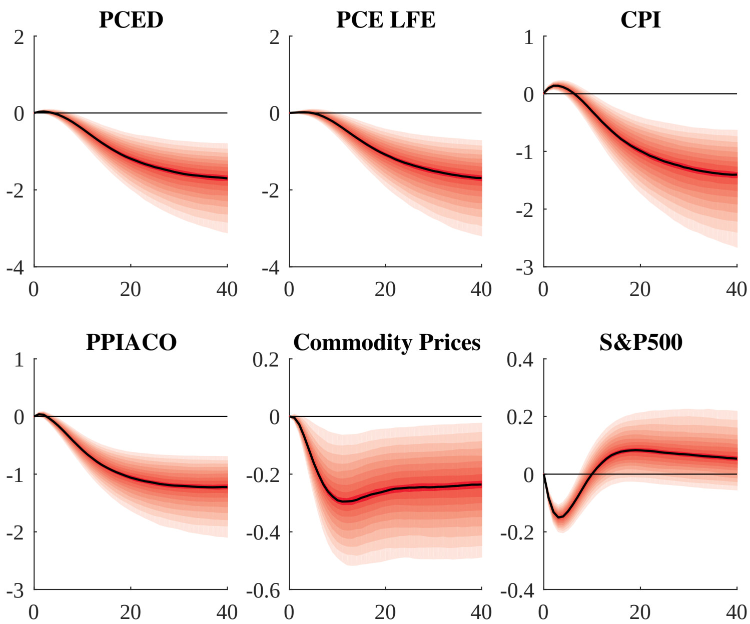

Figure E.2: Impulse responses of selected price indices to a FFR shock when the estimation sample ends in 2007Q2, i.e. when we exclude the great recession.

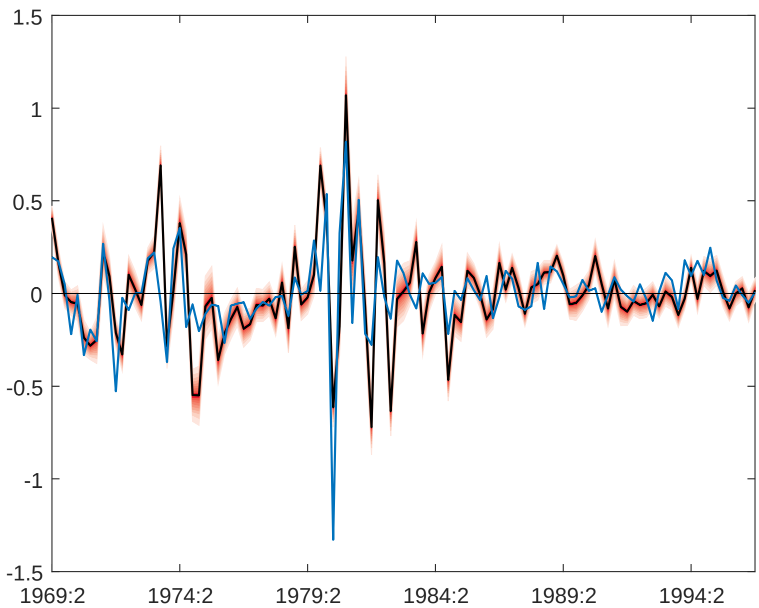

Figure E.3: Identified monetary policy shock vs. Romer and Romer (2004) monetary shock (blue line).

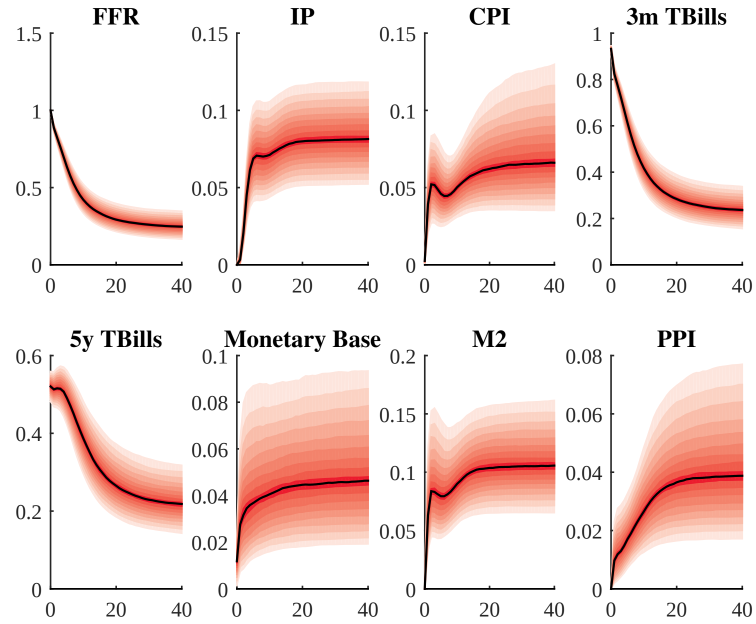

Figure E.4: Share of the forecast error variance in selected variables explained by the FFR shock.

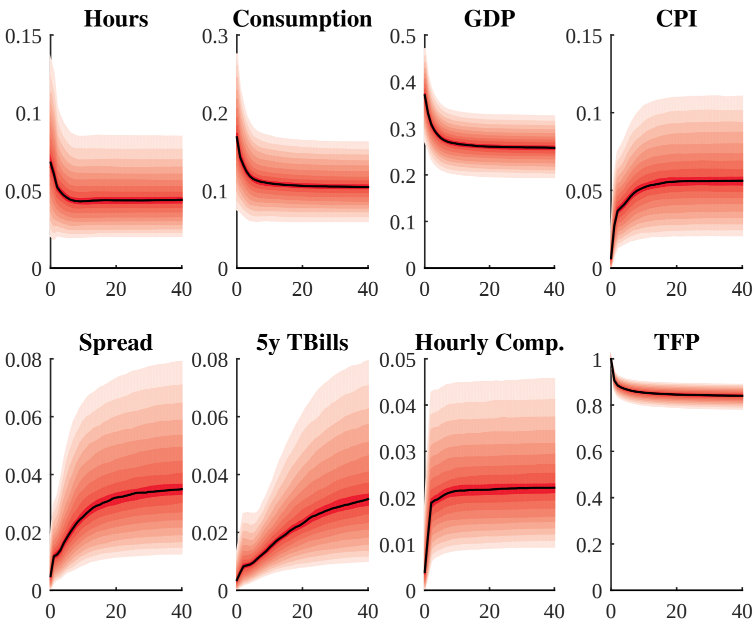

Figure E.5: Share of the forecast error variance in selected variables explained by the technology shock.

E.3 Factor loadings

The following figures contain the factor loadings with a posterior probability of a non-zero entry larger than 0.5 for each factor.

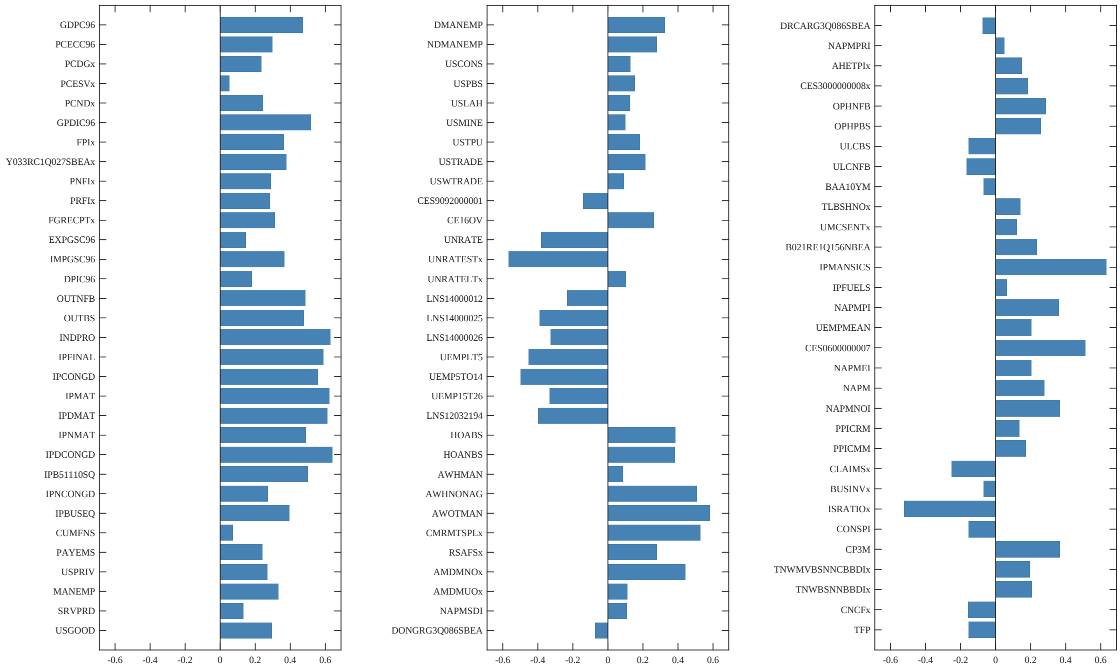

Figure E.6: Non-zero loadings for factor 1.

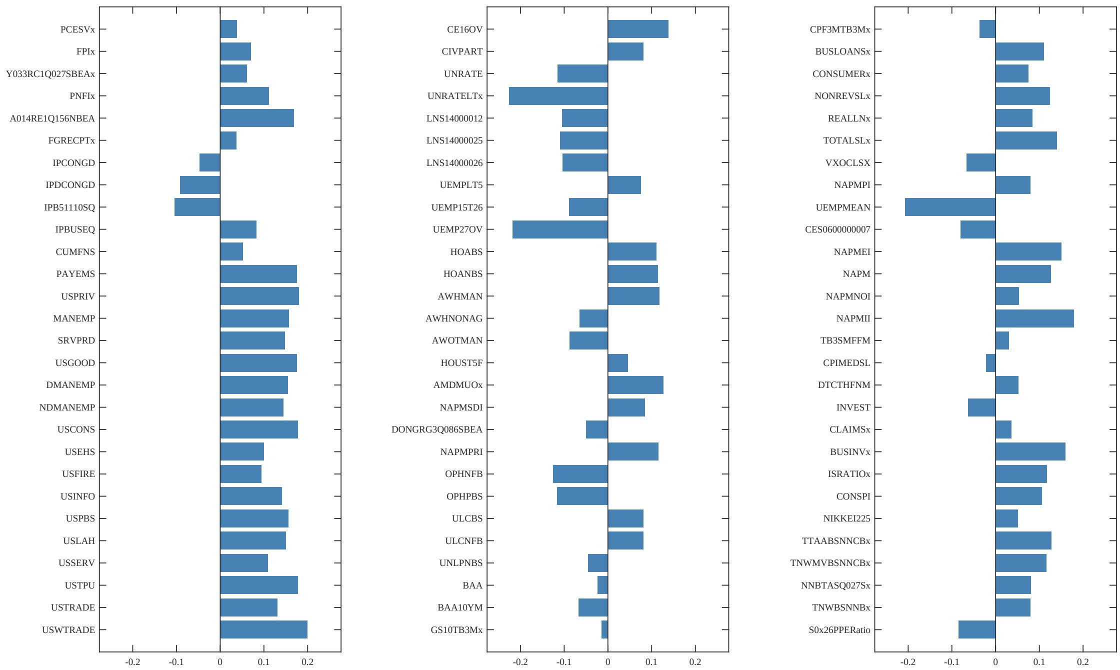

Figure E.7: Non-zero loadings for factor 2.

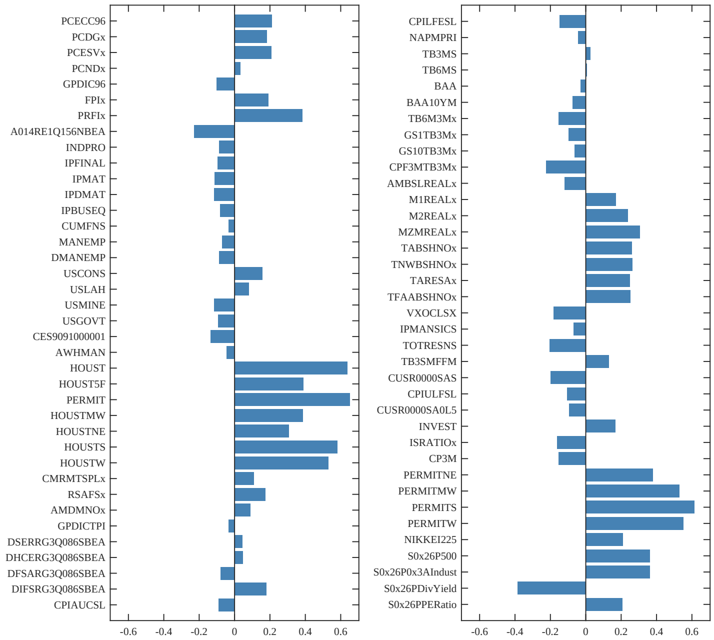

Figure E.8: Non-zero loadings for factor 3.

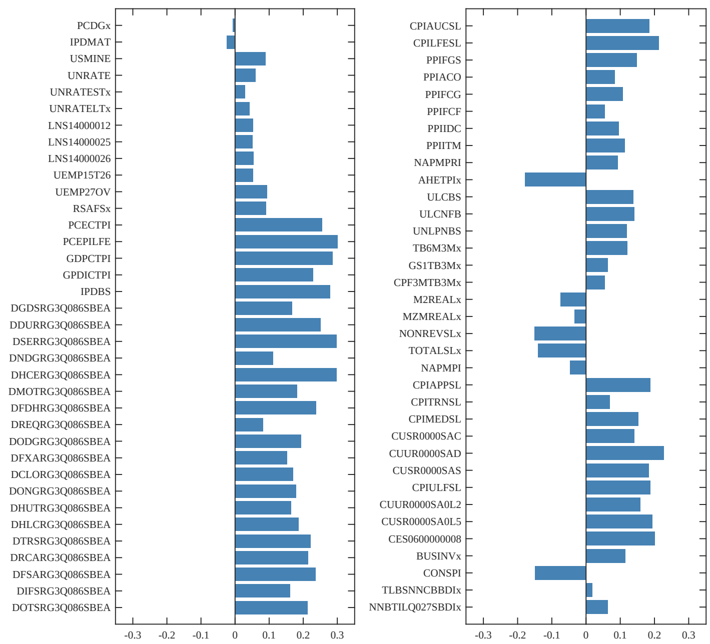

Figure E.9: Non-zero loadings for factor 4.

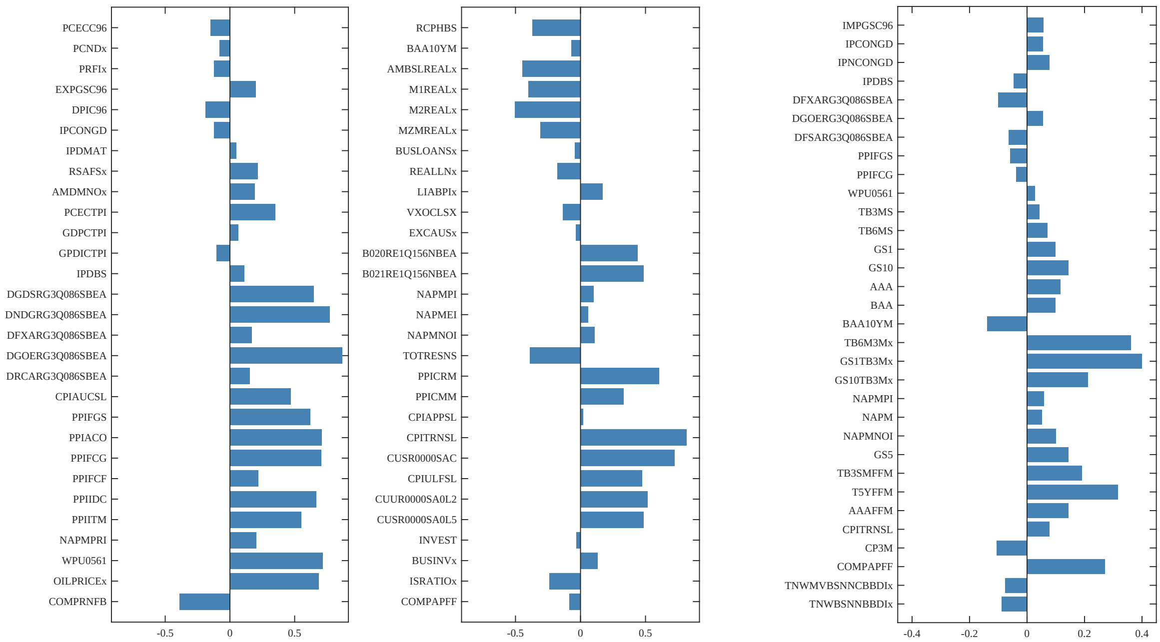

Figure E.10: Non-zero loadings for factor 5 (left and middle) and factor 6 (right).

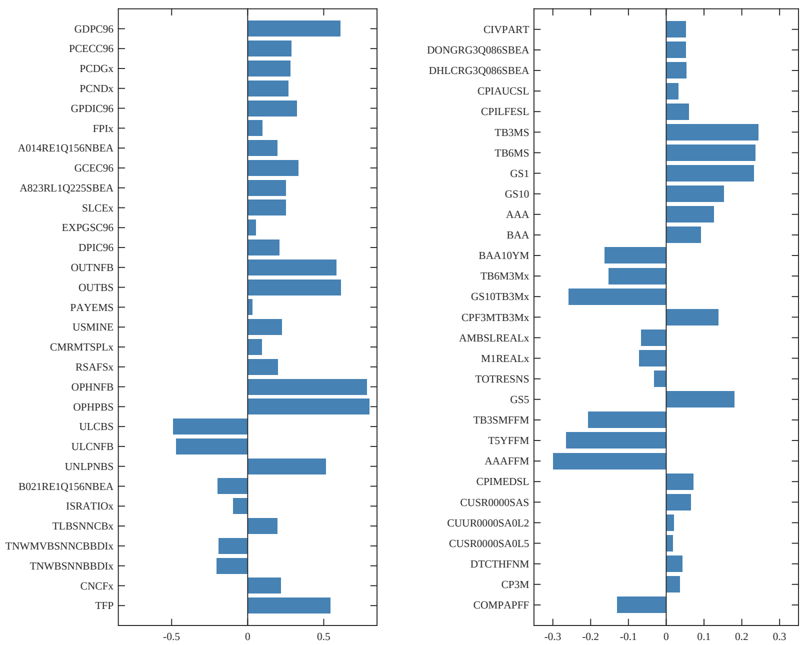

Figure E.11: Non-zero loadings for factor 7 (left) and 8 (right).

F Data

Table F.1: Time series. Transformations: level (lv), first difference (fd), first log difference (fl).

ID	MNEMONIC	Description	TCode
1	GDPC96	Real Gross Domestic Product, 3 Decimal (Billions of Chained 2009 Dollars)	fl
2	PCECC96	Real Personal Consumption Expenditures (Billions of Chained 2009 Dollars)	fl
3	PCDGx	Real personal consumption expenditures: Durable goods (Billions of Chained 2009 Dollars), deflated using PCE	fl
4	PCESVx	Real Personal Consumption Expenditures: Services (Billions of 2009 Dollars), deflated using PCE	fl
5	PCNDx	Real Personal Consumption Expenditures: Nondurable Goods (Billions of 2009 Dollars), deflated using PCE	fl
6	GPDIC96	Real Gross Private Domestic Investment, 3 decimal (Billions of Chained 2009 Dollars)	fl
7	FPIx	Real private fixed investment (Billions of Chained 2009 Dollars), deflated using PCE	fl
8	Y033RC1Q027SBEAx	Real Gross Private Domestic Investment: Fixed Investment: Nonresidential: Equipment (Billions of Chained 2009 Dollars), deflated using PCE	fl
9	PNFIx	Real private fixed investment: Nonresidential (Billions of Chained 2009 Dollars), deflated using PCE	fl
10	PRFIx	Real private fixed investment: Residential (Billions of Chained 2009 Dollars), deflated using PCE	fl
11	A014RE1Q156NBEA	Shares of gross domestic product: Gross private domestic investment: Change in private inventories (Percent)	lv
12	GCEC96	Real Government Consumption Expenditures & Gross Investment (Billions of Chained 2009 Dollars)	fl
13	A823RL1Q225SBEA	Real Government Consumption Expenditures and Gross Investment: Federal (Percent Change from Preceding Period)	lv
14	FGRECPTx	Real Federal Government Current Receipts (Billions of Chained 2009 Dollars), deflated using PCE	fl
15	SLCEx	Real government state and local consumption expenditures (Billions of Chained 2009 Dollars), deflated using PCE	fl
16	EXPGSC96	Real Exports of Goods & Services, 3 Decimal (Billions of Chained 2009 Dollars)	fl
17	IMPGSC96	Real Imports of Goods & Services, 3 Decimal (Billions of Chained 2009 Dollars)	fl
18	DPIC96	Real Disposable Personal Income (Billions of Chained 2009 Dollars)	fl
19	OUTNFB	Nonfarm Business Sector: Real Output (Index 2009=100)	fl
20	OUTBS	Business Sector: Real Output (Index 2009=100)	fl
21	INDPRO	Industrial Production Index (Index 2012=100)	fl
22	IPFINAL	Industrial Production: Final Products (Market Group) (Index 2012=100)	fl
23	IPCONGD	Industrial Production: Consumer Goods (Index 2012=100)	fl
24	IPMAT	Industrial Production: Materials (Index 2012=100)	fl
25	IPDMAT	Industrial Production: Durable Materials (Index 2012=100)	fl
26	IPNMAT	Industrial Production: Nondurable Materials (Index 2012=100)	fl
27	IPDCONGD	Industrial Production: Durable Consumer Goods (Index 2012=100)	fl
28	IPB51110SQ	Industrial Production: Durable Goods: Automotive products (Index 2012=100)	fl
29	IPNCONGD	Industrial Production: Nondurable Consumer Goods (Index 2012=100)	fl
30	IPBUSEQ	Industrial Production: Business Equipment (Index 2012=100)	fl
31	IPB51220SQ	Industrial Production: Consumer energy products (Index 2012=100)	fl
32	CUMFNS	Capacity Utilization: Manufacturing (SIC) (Percent of Capacity)	lv
33	PAYEMS	All Employees: Total nonfarm (Thousands of Persons)	fl
34	USPRIV	All Employees: Total Private Industries (Thousands of Persons)	fl
35	MANEMP	All Employees: Manufacturing (Thousands of Persons)	fl
36	SRVPRD	All Employees: Service-Providing Industries (Thousands of Persons)	fl
37	USGOOD	All Employees: Goods-Producing Industries (Thousands of Persons)	fl
38	DMANEMP	All Employees: Durable goods (Thousands of Persons)	fl
39	NDMANEMP	All Employees: Nondurable goods (Thousands of Persons)	fl
40	USCONS	All Employees: Construction (Thousands of Persons)	fl
41	USEHS	All Employees: Education & Health Services (Thousands of Persons)	fl

Table F.1: Time series, continued. Transformations: level (lv), first difference (fd), first log difference (fl).

ID	MNEMONIC	Description	TCode
42	USFIRE	All Employees: Financial Activities (Thousands of Persons)	fl
43	USINFO	All Employees: Information Services (Thousands of Persons)	fl
44	USPBS	All Employees: Professional & Business Services (Thousands of Persons)	fl
45	USLAH	All Employees: Leisure & Hospitality (Thousands of Persons)	fl
46	USSERV	All Employees: Other Services (Thousands of Persons)	fl
47	USMINE	All Employees: Mining and logging (Thousands of Persons)	fl
48	USTPU	All Employees: Trade, Transportation & Utilities (Thousands of Persons)	fl
49	USGOVT	All Employees: Government (Thousands of Persons)	fl
50	USTRADE	All Employees: Retail Trade (Thousands of Persons)	fl
51	USWTRADE	All Employees: Wholesale Trade (Thousands of Persons)	fl
52	CES9091000001	All Employees: Government: Federal (Thousands of Persons)	fl
53	CES9092000001	All Employees: Government: State Government (Thousands of Persons)	fl
54	CES9093000001	All Employees: Government: Local Government (Thousands of Persons)	fl
55	CE16OV	Civilian Employment (Thousands of Persons)	fl
56	CIVPART	Civilian Labor Force Participation Rate (Percent)	fd
57	UNRATE	Civilian Unemployment Rate (Percent)	fd
58	UNRATESTx	Unemployment Rate less than 27 weeks (Percent)	fd
59	UNRATELTx	Unemployment Rate for more than 27 weeks (Percent)	fd
60	LNS14000012	Unemployment Rate - 16 to 19 years (Percent)	fd
61	LNS14000025	Unemployment Rate - 20 years and over, Men (Percent)	fd
62	LNS14000026	Unemployment Rate - 20 years and over, Women (Percent)	fd
63	UEMPLT5	Number of Civilians Unemployed - Less Than 5 Weeks (Thousands of Persons)	fl
64	UEMP5TO14	Number of Civilians Unemployed for 5 to 14 Weeks (Thousands of Persons)	fl
65	UEMP15T26	Number of Civilians Unemployed for 15 to 26 Weeks (Thousands of Persons)	fl
66	UEMP27OV	Number of Civilians Unemployed for 27 Weeks and Over (Thousands of Persons)	fl
67	LNS12032194	Employment Level - Part-Time for Economic Reasons, All Industries (Thousands of Persons)	fl
68	HOABS	Business Sector: Hours of All Persons (Index 2009=100)	fl
69	HOANBS	Nonfarm Business Sector: Hours of All Persons (Index 2009=100)	fl
70	AWHMAN	Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing (Hours)	lv
71	AWHNONAG	Average Weekly Hours Of Production And Nonsupervisory Employees: Total private (Hours)	fd
72	AWOTMAN	Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing (Hours)	fd
73	HOUST	Housing Starts: Total: New Privately Owned Housing Units Started (Thousands of Units)	fl
74	HOUST5F	Privately Owned Housing Starts: 5-Unit Structures or More (Thousands of Units)	fl
75	PERMIT	New Private Housing Units Authorized by Building Permits (Thousands of Units)	fl
76	HOUSTMW	Housing Starts in Midwest Census Region (Thousands of Units)	fl
77	HOUSTNE	Housing Starts in Northeast Census Region (Thousands of Units)	fl
78	HOUSTS	Housing Starts in South Census Region (Thousands of Units)	fl
79	HOUSTW	Housing Starts in West Census Region (Thousands of Units)	fl
80	CMRMTSPLx	Real Manufacturing and Trade Industries Sales (Millions of Chained 2009 Dollars)	fl
81	RSAFSx	Real Retail and Food Services Sales (Millions of Chained 2009 Dollars), deflated by Core PCE	fl
82	AMDMNOx	Real Manufacturers& New Orders: Durable Goods (Millions of 2009 Dollars), deflated by Core PCE	fl
83	AMDMUOx	Real Value of Manufacturers& Unfilled Orders for Durable Goods Industries (Million of 2009 Dollars), deflated by Core PCE	fl
84	NAPMSDI	ISM Manufacturing: Supplier Deliveries Index (lin)	lv
85	PCECTPI	Personal Consumption Expenditures: Chain-type Price Index (Index 2009=100)	fl
86	PCEPILFE	Personal Consumption Expenditures Excluding Food and Energy (Chain-Type Price Index) (Index 2009=100)	fl

Table F.1: Time series, continued. Transformations: level (lv), first difference (fd), first log difference (fl).

ID	MNEMONIC	Description	TCode
87	GDPCTPI	Gross Domestic Product: Chain-type Price Index (Index 2009=100)	fl
88	GPDICTPI	Gross Private Domestic Investment: Chain-type Price Index (Index 2009=100)	fl
89	IPDBS	Business Sector: Implicit Price Deflator (Index 2009=100)	fl
90	DGDSRG3Q086SBEA	Personal consumption expenditures: Goods (chain-type price index)	fl
91	DDURRG3Q086SBEA	Personal consumption expenditures: Durable goods (chain-type price index)	fl
92	DSERRG3Q086SBEA	Personal consumption expenditures: Services (chain-type price index)	fl
93	DNDGRG3Q086SBEA	Personal consumption expenditures: Nondurable goods (chain-type price index)	fl
94	DHCERG3Q086SBEA	Personal consumption expenditures: Services: Household consumption expenditures (chain-type price index)	fl
95	DMOTRG3Q086SBEA	Personal consumption expenditures: Durable goods: Motor vehicles and parts (chain-type price index)	fl
96	DFDHRG3Q086SBEA	Personal consumption expenditures: Durable goods: Furnishings and durable household equipment (chain-type price index)	fl
97	DREQRG3Q086SBEA	Personal consumption expenditures: Durable goods: Recreational goods and vehicles (chain-type price index)	fl
98	DODGRG3Q086SBEA	Personal consumption expenditures: Durable goods: Other durable goods (chain-type price index)	fl
99	DFXARG3Q086SBEA	Personal consumption expenditures: Nondurable goods: Food and beverages purchased for off-premises consumption (chain-type price index)	fl
100	DCLORG3Q086SBEA	Personal consumption expenditures: Nondurable goods: Clothing and footwear (chain-type price index)	fl
101	DGOERG3Q086SBEA	Personal consumption expenditures: Nondurable goods: Gasoline and other energy goods (chain-type price index)	fl
102	DONGRG3Q086SBEA	Personal consumption expenditures: Nondurable goods: Other nondurable goods (chain-type price index)	fl
103	DHUTRG3Q086SBEA	Personal consumption expenditures: Services: Housing and Utilities (chain-type price index)	fl
104	DHLCRG3Q086SBEA	Personal consumption expenditures: Services: Health care (chain-type price index)	fl
105	DTRSRG3Q086SBEA	Personal consumption expenditures: Transportation Services (chain-type price index)	fl
106	DRCARG3Q086SBEA	Personal consumption expenditures: Recreation Services (chain-type price index)	fl
107	DFSARG3Q086SBEA	Personal consumption expenditures: Services: Food Services and accommodations (chain-type price index)	fl
108	DIFSRG3Q086SBEA	Personal consumption expenditures: Financial Services and insurance (chain-type price index)	fl
109	DOTSRG3Q086SBEA	Personal consumption expenditures: Other Services (chain-type price index)	fl
110	CPIAUCSL	Consumer Price Index for All Urban Consumers: All Items (Index 1982-84=100)	fl
111	CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy (Index 1982-84=100)	fl
112	PPIFGS	Producer Price Index by Commodity for Finished Goods (Index 1982=100)	fl
113	PPIACO	Producer Price Index for All Commodities (Index 1982=100)	fl
114	PPIFCG	Producer Price Index by Commodity for Finished Consumer Goods (Index 1982=100)	fl
115	PPIFCF	Producer Price Index by Commodity for Finished Consumer Foods (Index 1982=100)	fl
116	PPIIDC	Producer Price Index by Commodity Industrial Commodities (Index 1982=100)	fl
117	PPIITM	Producer Price Index by Commodity Intermediate Materials: Supplies & Components (Index 1982=100)	fl
118	NAPMPRI	ISM Manufacturing: Prices Index (Index)	lv
119	WPU0561	Producer Price Index by Commodity for Fuels and Related Products and Power: Crude Petroleum (Domestic Production) (Index 1982=100)	fl
120	OILPRICEx	Real Crude Oil Prices: West Texas Intermediate (WTI) - Cushing, Oklahoma (2009 Dollars per Barrel), deflated by Core PCE	fl
121	AHETPIx	Real Average Hourly Earnings of Production and Nonsupervisory Employees: Total Private (2009 Dollars per Hour), deflated by Core PCE	fl

Table F.1: Time series, continued. Transformations: level (lv), first difference (fd), first log difference (fl).

ID	MNEMONIC	Description	TCode
122	CES2000000008x	Real Average Hourly Earnings of Production and Nonsupervisory Employees: Construction (2009 Dollars per Hour), deflated by Core PCE	fl
123	CES3000000008x	Real Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing (2009 Dollars per Hour), deflated by Core PCE	fl
124	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour (Index 2009=100)	fl
125	RCPHBS	Business Sector: Real Compensation Per Hour (Index 2009=100)	fl
126	OPHNFB	Nonfarm Business Sector: Real Output Per Hour of All Persons (Index 2009=100)	fl
127	OPHPBS	Business Sector: Real Output Per Hour of All Persons (Index 2009=100)	fl
128	ULCBS	Business Sector: Unit Labor Cost (Index 2009=100)	fl
129	ULCNFB	Nonfarm Business Sector: Unit Labor Cost (Index 2009=100)	fl
130	UNLPNBS	Nonfarm Business Sector: Unit Nonlabor Payments (Index 2009=100)	fl
131	FEDFUNDS	Effective Federal Funds Rate (Percent)	lv
132	TB3MS	3-Month Treasury Bill: Secondary Market Rate (Percent)	lv
133	TB6MS	6-Month Treasury Bill: Secondary Market Rate (Percent)	lv
134	GS1	1-Year Treasury Constant Maturity Rate (Percent)	lv
135	GS10	10-Year Treasury Constant Maturity Rate (Percent)	lv
136	AAA	Moodys Seasoned Aaa Corporate Bond Yield (Percent)	lv
137	BAA	Moodys Seasoned Baa Corporate Bond Yield (Percent)	lv
138	BAA10YM	Moodys Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity (Percent)	lv
139	TB6M3Mx	6-Month Treasury Bill Minus 3-Month Treasury Bill, secondary market (Percent)	lv
140	GS1TB3Mx	1-Year Treasury Constant Maturity Minus 3-Month Treasury Bill, secondary market (Percent)	lv
141	GS10TB3Mx	10-Year Treasury Constant Maturity Minus 3-Month Treasury Bill, secondary market (Percent)	lv
142	CPF3MTB3Mx	3-Month Commercial Paper Minus 3-Month Treasury Bill, secondary market (Percent)	lv
143	AMBSLREALx	St. Louis Adjusted Monetary Base (Billions of 1982-84 Dollars), deflated by CPI	fl
144	M1REALx	Real Ml Money Stock (Billions of 1982-84 Dollars), deflated by CPI	fl
145	M2REALx	Real M2 Money Stock (Billions of 1982-84 Dollars), deflated by CPI	fl
146	MZMREALx	Real MZM Money Stock (Billions of 1982-84 Dollars), deflated by CPI	fl
147	BUSLOANSx	Real Commercial and Industrial Loans, All Commercial Banks (Billions of 2009 U.S. Dollars), deflated by Core PCE	fl
148	CONSUMERx	Real Consumer Loans at All Commercial Banks (Billions of 2009 U.S. Dollars), deflated by Core PCE	fl
149	NONREVSLx	Total Real Nonrevolving Credit Owned and Securitized, Outstanding (Billions of Dollars), deflated by Core PCE	fl
150	REALLNx	Real Real Estate Loans, All Commercial Banks (Billions of 2009 U.S. Dollars), deflated by Core PCE	fl
151	TOTALSLx	Total Consumer Credit Outstanding, deflated by Core PCE	fl
152	TABSHNOx	Real Total Assets of Households and Nonprolit Organizations (Billions of 2009 Dollars), deflated by Core PCE	fl
153	TLBSHNOx	Real Total Liabilities of Households and Nonprolit Organizations (Billions of 2009 Dollars), deflated by Core PCE	fl
154	LIABPIx	Liabilities of Households and Nonprolit Organizations Relative to Personal Disposable Income (Percent)	fl
155	TNWBSHNOx	Real Net Worth of Households and Nonprolit Organizations (Billions of 2009 Dollars), deflated by Core PCE	fl
156	NWPIx	Net Worth of Households and Nonprolit Organizations Relative to Disposable Personal Income (Percent)	lv
157	TARESAx	Real Assets of Households and Nonprolit Organizations excluding Real Estate Assets (Billions of 2009 Dollars), deflated by Core PCE	fl
158	HNOREMQ027Sx	Real Real Estate Assets of Households and Nonprolit Organizations (Billions of 2009 Dollars), deflated by Core PCE	fl
159	TFAABSHNOx	Real Total Financial Assets of Households and Nonprolit Organizations (Billions of 2009 Dollars), deflated by Core PCE	fl
160	VXOCLSX	CB OE S&P 100 Volatility Index: VXO	lv
161	EXSZUSx	Switzerland / U.S. Foreign Exchange Rate	lv
162	EXJPUSx	Japan /U.S. Foreign Exchange Rate	lv
163	EXUSUKx	U.S. / U.K. Foreign Exchange Rate	lv

Table F.1: Time series, continued. Transformations: level (lv), first difference (fd), first log difference (fl).

ID	MNEMONIC	Description	TCode
164	EXCAUSx	Canada / U.S. Foreign Exchange Rate	lv
165	UMCSENTx	University of Michigan: Consumer Sentiment (Index Ist Quarter 1966=100)	lv
166	B020RE1Q156NBEA	Shares of gross domestic product: Exports of goods and Services (Percent)	fd
167	B021RE1Q156NBEA	Shares of gross domestic product: Imports of goods and Services (Percent)	fd
168	IPMANSICS	Industrial Production: Manufacturing (SIC) (Index 2012=100)	fl
169	IPB51222S	Industrial Production: Residential Utilities (Index 2012=100)	fl
170	IPFUELS	Industrial Production: Fuels (Index 2012=100)	fl
171	NAPMPI	ISM Manufacturing: Production Index	lv
172	UEMPMEAN	Average (Mean) Duration of Unemployment (Weeks)	fd
173	CES0600000007	Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing	fd
174	NAPMEI	ISM Manufacturing: Employment Index	lv
175	NAPM	ISM Manufacturing: PMI Composite Index	lv
176	NAPMNOI	ISM Manufacturing: New Orders Index	lv
177	NAPMII	ISM Manufacturing: Inventories Index	lv
178	TOTRESNS	Total Reserves of Depository Institutions (Billions of Dollars)	fl
179	GS5	5-Year Treasury Constant Maturity Rate	lv
180	TB3SMFFM	3-Month Treasury Constant Maturity Minus Federal Funds Rate	lv
181	T5YFFM	5-Year Treasury Constant Maturity Minus Federal Funds Rate	lv
182	AAAFFM	Moodys Seasoned Aaa Corporate Bond Minus Federal Funds Rate	lv
183	PPICRM	Producer Price Index: Crude Materials for Further Processing (Index 1982=100)	fl
184	PPICMM	Producer Price Index: Commodities: Metals and metal products: Primary nonferrous metals (Index 1982=100)	fl
185	CPIAPPSL	Consumer Price Index for All Urban Consumers: Apparel (Index 1982-84=100)	fl
186	CPITRNSL	Consumer Price Index for All Urban Consumers: Transportation (Index 1982-84=100)	fl
187	CPIMEDSL	Consumer Price Index for All Urban Consumers: Medical Care (Index	fl
188	CUSR0000SAC	Consumer Price Index for All Urban Consumers: Commodities (Index 1982-84=100)	fl
189	CUUR0000SAD	Consumer Price Index for All Urban Consumers: Durables (Index 1982-84=100)	fl
190	CUSR0000SAS	Consumer Price Index for All Urban Consumers: Services (Index 1982-84=100)	fl
191	CPIULFSL	Consumer Price Index for All Urban Consumers: All Items Less Food (Index 1982-84=100)	fl
192	CUUR0000SA0L2	Consumer Price Index for All Urban Consumers: All items less shelter (Index 1982-84=100)	fl
193	CUSR0000SA0L5	Consumer Price Index for All Urban Consumers: All items less medical care (Index 1982-84=100)	fl
194	CES0600000008	Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing (Dollars per Hour)	fl
195	DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding Owned by Finance Companies (Millions of Dollars)	fl
196	DTCTHFNM	Total Consumer Loans and Leases Outstanding Owned and Securitized by Finance Companies (Millions of Dollars)	fl
197	INVEST	Securities in Bank Credit at All Commercial Banks (Billions of Dollars)	fl
198	CLAIMSx	Initial Claims	fl
199	BUSINVx	Total Business Inventories (Millions of Dollars)	fl
200	ISRATIOx	Total Business: Inventories to Sales Ratio	fd
201	CONSPI	Nonrevolving consumer credit to Personal Income	fd
202	CP3M	3-Month AA Financial Commercial Paper Rate	fd
203	COMPAPFF	3-Month Commercial Paper Minus Federal Funds Rate	lv
204	PERMITNE	New Private Housing Units Authorized by Building Permits in the Northeast Census Region (Thousands, SAAR)	fl
205	PERMITMW	New Private Housing Units Authorized by Building Permits in the Midwest Census Region (Thousands, SAAR)	fl
206	PERMITS	New Private Housing Units Authorized by Building Permits in the South Census Region (Thousands, SAAR)	fl
207	PERMITW	New Private Housing Units Authorized by Building Permits in the West Census Region (Thousands, SAAR)	fl

Table F.1: Time series, continued. Transformations: level (lv), first difference (fd), first log difference (fl).

ID	MNEMONIC	Description	TCode
208	NIKKEI225	Nikkei Stock Average	fl
209	TLBSNNCBx	Real Nonfinancial Corporate Business Sector Liabilities (Billions of 2009 Dollars), Deflated by Implicit Price Deflator for Business Sector IPDBS	fl
210	TLBSNNCBBDIx	Nonfinancial Corporate Business Sector Liabilities to Disposable Business Income (Percent)	lv
211	TTAABSNNCBx	Real Nonfinancial Corporate Business Sector Assets (Billions of 2009 Dollars), Deflated by Implicit Price Deflator for Business Sector IPDBS	fl
212	TNWMVBSNNCBx	Real Nonfinancial Corporate Business Sector Net Worth (Billions of 2009 Dollars), Deflated by Implicit Price Deflator for Business Sector IPDBS	fl
213	TNWMVBSNNCBBDIx	Nonfinancial Corporate Business Sector Net Worth to Disposable Business Income (Percent)	fd
214	NNBTILQ027Sx	Real Nonfinancial Noncorporate Business Sector Liabilities (Billions of 2009 Dollars), Deflated by Implicit Price Deflator for Business Sector IPDBS	fl
215	NNBTILQ027SBDIx	Nonfinancial Noncorporate Business Sector Liabilities to Disposable Business Income (Percent)	lv
216	NNBTASQ027Sx	Real Nonfinancial Noncorporate Business Sector Assets (Billions of 2009 Dollars), Deflated by Implicit Price Deflator for Business Sector IPDBS	fl
217	TNWBSNNBx	Real Nonfinancial Noncorporate Business Sector Net Worth (Billions of 2009 Dollars), Deflated by Implicit Price Deflator for Business Sector IPDBS	fl
218	TNWBSNNBBDIx	Nonfinancial Noncorporate Business Sector Net Worth to Disposable Business Income (Percent)	fd
219	CNCFx	Real Disposable Business Income, Billions of 2009 Dollars (Corporate cash flow with IVA minus taxes on corporate income, deflated by Implicit Price Deflator for Business Sector IPDBS)	fl
220	SP500	S&P Common Stock Price Index: Composite	fl
221	SPIndust	S&P Common Stock Price Index: Industrials	fl
222	SPDivYield	S&P Composite Common Stock: Dividend Yield	fd
223	SPPERatio	S&P Composite Common Stock: Price-Earnings Ratio	fl
224	TFP	Total Factor Productivity	fl