The Risk-Taking Channel of Liquidity Regulations and Monetary Policy

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Abstract

We study the implications of liquidity regulations and monetary policy on deposit-making and risk-taking. Banks give risky loans by creating deposits that firms use to pay suppliers. Firms and banks can take more or less risk. In equilibrium, higher liquidity requirements always lower risk at the cost of lower investment. Nevertheless, a positive liquidity requirement is always optimal. Monetary conditions affect the optimal size of liquidity requirements, and the optimal size is countercyclical. It is only optimal to impose a 100% liquidity requirement when the nominal interest rate is sufficiently low.

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1 Introduction

Typically, financial crises build up from risk-taking in the financial sector and culminate in liquidity crises. Lax monetary policy is one of the factors contributing to the buildup of financial risk. When rates are low, banks lend more, thus increasing their liquidity risk, and they lend to riskier borrowers.[1] Following the recent crisis, the Basel Committee on Banking Supervision was quick to recognize that some banks mismanaged their liquidity risk, and it adopted new regulations with two objectives in mind. The first objective is to promote short-term resilience of a bank's liquidity risk profile by ensuring that a bank always has enough liquid assets to sustain operations during a stress scenario lasting one month. In Basel III, this objective is achieved with the liquidity coverage ratio. The second objective is to promote resilience over a longer time horizon by incentivizing banks to fund their activities with more stable sources of funding. In Basel III, this second objective is achieved with the net stable funding ratio.

While these liquidity regulations make sure there is sufficient liquidity in crisis periods and may reduce risk-taking incentives, they also limit the creation of liquidity by banks. Much like maturity transformation, liquidity creation (or transformation) is a fundamental aspect of banking. Generations of students have been taught that banks fund illiquid assets using liquid, and even money-like, liabilities and that it is socially optimal that they do so although it may induce financial fragility (e.g., Diamond and Rajan, 2001).

Hence, at the heart of liquidity requirements is the issue of limiting money creation by banks. Banks that cannot create money because of 100% liquidity requirements, e.g., narrow banks, would always be resilient to any liquidity stress scenario. This resilience brings us to a fundamental question in monetary economics and banking: Is an economy that allows banks to freely create (inside) money, such as tradable deposits, more or less stable, more or less productive, and more or less efficient than the same economy constrained by higher liquidity requirements? The famous Chicago plan called for 100% reserve requirements and narrow banking at a time when the Great Depression gave ammunition to those arguing

[1] There is growing empirical evidence that banks' loan portfolios tend to be riskier when interest rates are low. For example, see Jimenez, et. al (2014) or Dell'Ariccia, et. al. (2017).

for limiting the creation of deposits. Under this plan, banks would hold $1 in reserve for each $1 of deposits on their balance sheet. The Great Recession revived the academic debate (see for instance Chari and Phelan, 2014 or Cochrane, 2014), and policy makers imposed new liquidity regulations on banks, although timid compared with 100% reserves.[2] However, by imposing stricter liquidity requirements, regulations may well limit the modus operandi of banks, increase the costs of funds, and in the end penalize the real economy. This paper studies the tension between liquidity creation and risk-taking of the banking sector under different monetary policy conditions. We analyze the effects of imposing liquidity requirements – including reserve requirements on deposits – and we solve for the optimal level of liquidity requirements as a function of monetary policy.

The theoretical and empirical literature is large. Below, we review only its most recent developments, but first, let us mention two recurrent themes. A system relying on the free creation of deposits is arguably more efficient because banks have more flexibility to respond to loan demand (e.g., Williamson, 1999). However, this system is inherently unstable because it may, for instance, allow multiple equilibria, which opens the door to exotic dynamics, cycles, and crashes (e.g., Sanches, 2015). One puzzling aspect of the literature is that risk is missing from the analysis: banks and their borrowers do not engage in risk-taking activities. Rather, as in the seminal paper by Diamond and Dybvig (1981), it is the source of funding that is fragile. Here, instead, we analyze the effect of the risk-taking decision of borrowers on financial stability.

More precisely, we introduce moral hazard in an otherwise standard monetary model with banks. A bank's reserves are remunerated, but maybe at a rate lower than the prevailing market rate. We say that monetary conditions are tighter when the spread between the interest rate paid on reserves and market rates is larger. Risk is related to the probability of success of the investment. Borrowers can choose this rate of success but the higher the rate, the more costly it becomes for them. Still, it is optimal that borrowers take no risk at all. However, moral hazard and limited liability implies that borrowers will take risk in

[2]In countries with less developed financial markets, reserve requirement is an even more frequently used monetary policy instrument (see for instance Chang et al, 2018 on reserve requirement and stabilization policy in China).

equilibrium. As is standard, the more indebted they are, the more risk they take. When borrowers face a low loan rate, maybe due to lax monetary policy, they will tend to borrow more, increasing their indebtedness, thus taking more risks. In this context, we study if and how liquidity requirements can help achieve the (constrained) optimal level of debt and risk-taking.

We show that liquidity requirements combined with tighter monetary policy exploit a trade-off between risk and investment. When a bank fails, it can have no resources to pay its creditors (e.g. depositors, interbank lenders). So Requiring banks to hold liquid assets, e.g. central bank reserves, ensures that inside money (deposits, loans, and interbank loans) pays something in case the bank fails. Tighter monetary policy or more stringent liquidity requirements, thus make holding liquid assets more costly, and banks pass on this cost to firms by charging higher loan rates. As a consequence, firms take smaller loans, thus reducing their leverage. As a result, they make safer investments. Similarly, we show that when banks' funding cost is low, firms take on higher leverage and make riskier investments. In this sense, optimal monetary policy trades off risk and investment. So the Friedman rule and a zero-liquidity requirement is not necessarily optimal, as it would induce too high leverage and too much risk-taking. In spite of being the safest system, fully backed deposits may not be optimal as it can reduce leverage and investment too much.

Our paper sheds light on the effects of the current policy trend to increase liquidity requirements for banks. Bech and Keister (2012) have shown the effects of such an increase on the functioning of the interbank market. We look at the macroeconomic effects of liquidity requirements. While we do not model the bank run scenario justifying the Basel III Liquidity Coverage Ratio, our results support the view that liquidity requirements will make the overall financial system safer. The different components of the liquidity requirement however cannot be set independently, and most importantly they cannot be set independently of prevailing monetary conditions. High liquidity requirements can be optimal when interest rates are low. However, they should be set lower when interest rates are higher: liquidity requirements force banks to hold low (real) yield assets, which are costly when policy rates are high. In that case, the requirements become costly, and banks would reduce their loans

too much. Finally, we find conditions for which liquidity requirements and leverage ratio requirements can achieve the same allocation.

The rest of the paper is organized as follows. We present the model in Section 2 and we derive the equilibrium in Section 3. Section 4 contains several extensions, such as the effect of bail-out policies and deposit insurance, and the consequence of capital requirements. We place our results in the recent literature in Section 5. The last section summarizes the findings and concludes.

2 The Model

The model is a version of Rocheteau, Wright, and Zhang (2016). Time $t = 1, 2, ...$ is discrete and continues forever. There are two goods, a capital good, which fully depreciates at the end of each period, and a perishable consumption good. There are three types of risk-neutral agents, each of measure one: short-lived firms and bankers, and long-lived suppliers. Each period is divided into three subperiods. In the first subperiod, there are two loan markets. One loan market is where firms can borrow from bankers, and the other is for interbank loans. In the second subperiod, there is a market for capital, and finally a market for the consumption good in the third subperiod.

Firms are born at the beginning of each period with no resources and perish after consumption in the third subperiod. A fraction $\lambda \in (0, 1]$ of firms is productive. When they invest k units of capital in the second subperiod, their technology returns $F(k)$ units of consumption in the third subperiod with probability q and nothing otherwise. The output realization is idiosyncratic across firms. $F(k)$ is a neoclassical production function homogeneous of degree $\sigma < 1$. Each productive firm chooses $q \in [0, 1]$ by bearing a cost $q^2 F(k)/2$. We think of q as the quality of the project but also as inversely related to the firm's default risk.

Suppliers produce capital using a technology that transforms hours worked one-for-one into capital in the second subperiod.[3] As firms are short-lived and cannot commit towards suppli-

[3]Conveniently, this act implies that we can also interpret capital as labor.

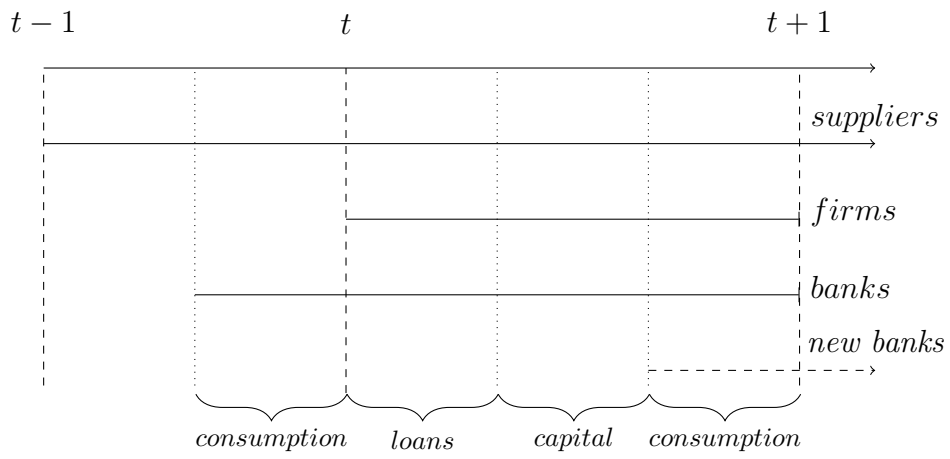


Figure 1: Births and deaths.

ers, they cannot obtain credit directly from them. Hence, firms need a means to buy capital from suppliers. There are two possible means of payments, money and deposits (banknotes), that firms can obtain from bankers. Deposits are similar to bearer notes: a bank issuing a deposit promises 1 unit of money in the consumption market. Deposits are subject to reserve requirements: making deposits requires banks to set money aside. We let r be the interest rate banks earn on required and excess reserves.

Bankers are born at the beginning of the consumption market and live through one period. When born, young bankers can transform hours worked one-for-one into consumption that they can sell for money.[4] This is one way banks can satisfy their reserve requirements. Alternatively they can borrow reserves in the interbank market. Old bankers cannot work, but they are committed to repay their debt if they can. Finally, banks can only lend to one firm and so they cannot diversify their risk.[5] Figure 1 shows the life span of each type of agents in the economy.

Preferences of suppliers and bankers are represented by the utility function $U(c,h) = c - h$

[4]Young bankers can raise equity by producing and selling the numeraire for money. In Section 4.4, we show how young bankers can raise equity by selling shares of the bank instead of producing.

[5]It is not necessary that banks are short lived. It would suffice to assume that they can have difficulty raising equity, so that default is a possible event. In our model banks would diversify if they could. However the following model with only aggregate, non diversifiable risk is isomorphic to ours. Suppose there is a continuum of aggregate states, $s \sim U[0,1]$. The bank has to choose from a continuum of projects of type q. With k investment, a risky project of type q yields $F(k)$ in all states $s \leq q$, and fails otherwise. But the cost of choosing projects of quality q is $q^2k/2$. In this model, the equilibrium q coincides with ours and banks will fail.

where $c \geq 0$ is consumption and $h \geq 0$ is hours worked. Firms' preferences for consumption are $u(c) = c$. Banks and suppliers discount the future at a rate $\beta \in (0, 1)$.

Finally, we denote the stock of money at time t as M_t. A central bank controls the stock of money, which evolves according to $M_{t+1} = (1 + \pi)M_t$. The price of money in terms of consumption is v_t. In stationary equilibrium $v_{t+1}M_{t+1} = v_t M_t$, so $v_t = (1 + \pi)v_{t+1}$. The nominal rate of interest is $i = (1 + \pi)/\beta - 1$. This is the rate on a fictitious nominal bond that cannot be used as a means of payment. The central bank also remunerates reserves at rate $r \geq 0$.[6] We assume that the rate on the illiquid nominal bond is strictly greater than the interest rate on reserves, $i > r$. So the cost of holding reserves is the spread $i - r > 0$.

Timing and markets The timing is as follows. In the first subperiod of each period, the loan markets between bankers and firms open. Firms are randomly matched pairwise with bankers. We assume all bankers are matched with a firm. If the firm is productive, it bargains with the matched banker over the terms of the loan in a way we describe below. Concurrently, all bankers have access to a competitive interbank market. Bankers with too little reserves can borrow from banks with too much reserves in the interbank market.

In the second subperiod, productive firms who managed to obtain a bank loan use it to purchase capital from suppliers in the capital market. Then, they invest capital and choose the quality of their project.

In the third subperiod, successful firms repay their bank loans using their output.[7] The bank redeems its deposits using reserves and some of the output from the firm. At this stage, suppliers and successful bankers may hold reserves or money but have little use for it. They can sell them to a young banker seeking to build equity in the form of reserves for the upcoming loan market.[8]

Figure 2 shows the circulation of deposits, capital, and consumption in all three markets.

[6] The case where $r < 0$ is available from the authors.

[7] While it would be natural (and feasible) to assume that firms sell their output for deposits/cash and then repay their loans, it is equivalent and simpler to assume that firms repay their (now old) banker by transferring some of their output. See Rocheteau, Wright, and Zhang (2017) for details.

[8] Stephen Williamson refers to this form of equity as "sweat equity."

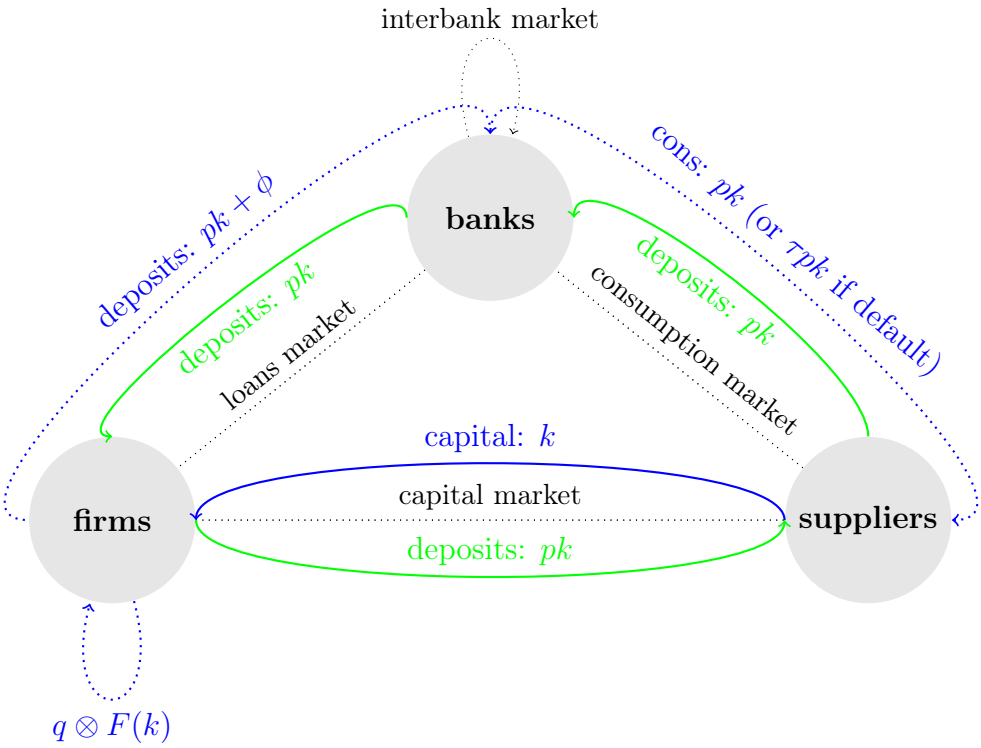


Figure 2: deposits and consumption circulation

The dotted blue lines show where there could be a default. We now describe each market in more detail and in chronological order.

2.1 Markets for loans and interbank loans

Bankers and firms are randomly matched. Since a measure $1 - \lambda$ of firms is not productive, a measure $1 - \lambda$ of bankers will lend reserves on the interbank market. In Appendix I we show banks want to economize on their reserve holdings whenever $i - r > 0$, so that their liquidity requirement (if any) will bind. Intuitively, when this spread is higher, the liquidity offered by the interbank loan is more valuable. In that case, and a measure λ of banks will borrow on in the interbank market. A banker grants a loan using deposits. In the market for capital, let p denote the amount of deposits needed to purchase 1 unit of capital. So p is

the deposit price of capital,[9] and firms need pk units of deposits to buy k units of capital. The banker charges a net fee, ϕ, for this loan. We assume that the quality of the project is contractible.[10] Hence, a bank loan is a list (pk, ϕ, q). To simplify notation, we write a bank loan as a list (k, ϕ, q).

Reserve/liquidity requirements Banks may face reserve requirements. Formally, a young banker who issued deposits pk has to set aside enough reserves (in the form of money) to be able to pay at least $\tilde{\tau}pk$ when old to its depositors, where $\tilde{\tau} \in [0, 1]$ is a policy variable.

Banks can change their current holding of reserves m in the interbank market. This market is organized as a Walrasian market, and i_m is the market clearing rate. Banks can increase their reserve holdings by borrowing b on the interbank market, or they can lend an amount ℓ of reserves if they have excess reserves. We assume that any interbank borrowing b is subject to a liquidity requirement $\tilde{\tau}_m b$. Since reserves are the only liquid asset in our model, banks satisfy this liquidity requirement by holding reserves. Therefore, a borrowing bank is always able to pay at least this amount to its interbank creditors. Reserve and liquidity requirements differ from capital requirements insofar as the reserves are not invested and a bankrupt bank can use these reserves to (partially) pay holders of deposits or interbank lenders. In an extension, we analyze how capital requirements affect the results.

Banks who extend a contract (k, ϕ, q) must hold enough reserves R to satisfy the constraint (in real terms)

$$\tilde{\tau}pk + \tilde{\tau}_m b \leq (1 + r)R, \tag{1}$$

where $\tilde{\tau}, \tilde{\tau}_m \in [0, 1]$.[11] We specify the reserve requirement taking into account the interest rate paid on reserves $(1 + r)R$. One could make the case that the reserve constraint should not account for the interest rate on reserves, i.e., $\tilde{\tau}pk + \tilde{\tau}_m b \leq R$. However, in this case, banks with a binding reserve requirement may still be able to pay their liabilities in case the

[9] $p > 1$ is well possible because we do not assume deposit insurance (however, see an extension) and banks may default on their deposits.

[10] We worked out a version of this model where the quality of the project is not contractible. The results, available from the authors, are qualitatively identical.

[11] A requirement higher than 100% would not make the economy safer than a requirement of just 100%; but it would impose additional costs on banks.

firm defaults whenever r is large enough. This outcome complicates the analysis without adding substantive value, and so we opted for (1). Since $i > r$ it is costly to hold reserves and the reserve constraint (1) binds. Additionally, for simplicity we use the normalization $\tau \equiv \tilde{\tau}/(1+r)$ and $\tau_m \equiv \tilde{\tau}_m/(1+r)$. Since $R = m + b$ (where m is the amount of reserves held by banks), we end up with $\tau pk + \tau_m b = m + b$ so that the interbank borrowing is

$$b = \frac{\tau pk - m}{1 - \tau_m}. \tag{2}$$

Later, we will verify that $b > 0$ in equilibrium.

Banks' participation constraint We can now define the bank's participation constraint given contract (k, ϕ, q), as

$$\begin{aligned} q \quad & [pk + \phi + (1+r)(\tau pk + \tau_m b) - pk - (1+i_m)b] \\ & \geq Q(1+i_m)m + (1-Q)(1+r)\tau_m m. \end{aligned} \tag{3}$$

The left hand side of (3) is the bank's expected payoff from lending to the firm. If the firm succeeds, the bank gets paid the principal pk and ϕ, as well as the interest rate on its reserves. From this amount, the bank redeems its deposits pk and pays its interbank loans, if any. If the firm fails, the bank (partially) pays its liabilities and retains nothing. The right hand side of (3) is the bank's outside option when holding m: it is the expected payoff from lending these reserves on the interbank market. When (1) binds, banks expect their interbank loans to fail with probability $1-Q$, in which case they get $(1+r)\tau_m$ per unit lent. Then, using the expression for b in (2) we can simplify the participation constraint as

$$\begin{aligned} & q\left[\phi + (1+r)\left(\tau pk + \tau_m \frac{\tau pk - m}{1-\tau_m}\right) - (1+i_m)\frac{\tau pk - m}{1-\tau_m}\right] \\ & \geq Q(1+i_m)m + (1-Q)(1+r)\tau_m m \end{aligned} \tag{4}$$

We should stress that the banker fails when its borrowing firm fails *and* she does not hold enough reserves to pay the par-value of its deposits. In this case, the holders of deposits are

paid pro rata from τpk, while interbank creditors are paid pro rata from $\tau_m b$. Therefore, deposits and interbank loans can be risky if the banker does not hold enough reserves.

Loan contract The choice of q is contractible by the banker (but not by suppliers) so that there is no moral hazard between the firm and the banker in this version of our model. In addition, we assume the banker has no bargaining power, so a loan contract is a tuple (k, ϕ, q) that maximizes the firm's payoff,

$$\max_{k,\phi,q} q[F(k) - \phi - pk] - \frac{1}{2}q^2 F(k),$$

subject to the bank's participation constraint, (4).

Introducing moral hazard between the firm and the bank would reinforce our result but does not qualitatively change our results. Our assumption that loan contracts give the entire surplus to firms implies that the moral hazard problem originating from lending is minimized. Any other surplus sharing rule would make the moral hazard problem even more acute and would reinforce our result. Once loan contracts are agreed upon, deposits are granted, and required reserves are set aside, and the market for capital opens.

2.2 Capital market

The demand for capital is given by the bank loan contract. To determine the supply of capital, we turn to the problem of suppliers in the capital market. Obviously, suppliers are aware of the moral hazard problem, and they expect each firm (and their bank) to fail with probability $1 - Q$. In addition, when the bank fails, they expect to get the reserves held by the bank. The capital market being Walrasian, suppliers are able to perfectly diversify the risk by selling capital to every productive firm.[12] Hence, the problem of a supplier in the capital market is

$$\max_{k \geq 0} -k + (1 - Q)(1 + r)\tau pk + Qpk, \tag{5}$$

[12]The ability to diversify plays no role in this model where suppliers are risk neutral.

where k is the capital sold for deposits at (real) price p. Also as reserves are costly to hold, we already integrated the result that suppliers do not hold any reserves across periods. The first order condition gives

$$p(Q;\tau,r) = \frac{1}{Q + (1-Q)(1+r)\tau}. \tag{6}$$

Clearly, deposits carry a risk premium unless the banks hold 100% reserves, that is $\tilde{\tau} = \tau(1+r) = 1$.[13]

2.3 Consumption market (CM)

In the last subperiod, successful firms settle their debt towards their bank with their production and their banks redeem their deposits with goods but also reserves. They consume whatever is left. Suppliers consume their real net worth. It is standard to show that when $i > r \geq 0$, suppliers choose to hold no real balances, as their reserves are not remunerated and they have no liquidity needs. Young bankers will choose real balances m to maximize their lifetime net worth, given they obtain no surplus when they are lending to a firm:

$$\max_{m} -(1+\pi)m + \beta\left[Q(1+i_m)m + (1-Q)(1+r)\tau_m m\right].$$

The reader should notice the existence of a hold-up problem when there is a reserve requirement and $i > r$: While bankers incur the cost of bringing real balances in the loan market, they do not obtain any surplus from it. Therefore, if the interbank market were absent, there would be no equilibrium with lending because banks would never incur the cost of acquiring real balances, knowing firms would grab the surplus real balances generate. However, the interbank market gives bankers a viable (outside) option. The firm has to guarantee the bank at least this level of expected payoff, and it is sufficient for an equilibrium with lending to exist. Nevertheless, since the lifetime net worth is linear in m, in an equilibrium with

[13] If we allowed for $\tilde{\tau} > 1$, we would need to require $p(Q;\tau,r) \geq 1$ as banks would always be able to make depositors whole. This requirement complicates the analysis without adding much value.

lending, a bank must be indifferent as to the amount of real balances it brings so that

$$1 + i_m = \frac{1 + i - (1 - Q)(1 + r)\tau_m}{Q}. \tag{7}$$

where $\frac{1+\pi}{\beta} = 1 + i$. Notice that $1 + i_m \geq 1 + i$ because $(1 + r)\tau_m \leq 1$.

2.4 Moral hazard

There are two sources of moral hazard in our model. The first one is related to the terms of trade when a firm trades its deposits for capital. The real value p assigned to the deposit depends on the average default probability of a bank rather than the default probability of that specific firm's bank. In other words, suppliers form their own beliefs about a bank's default. If the firm were to choose a higher quality q, its increased probability of success implies that the bank deposit it holds is less likely to default. If suppliers recognized that fact, the price p of capital when purchased with that particular deposit would decrease. If it did, it is straightforward to show that the efficient levels of effort and investment are achieved in equilibrium and there is no need for reserve requirements. However, a firm instead takes p as given when choosing its effort level q. As a result, the equilibrium quality level will be inefficiently low, as will the levels of investment, output and welfare.

The second source of moral hazard is related to the pricing of interbank loans. Banks that are not matched with productive firms will choose to lend out their reserves to banks that are making loans and facing a reserve requirements. These interbank loans are risky because a bank will not repay its interbank loan if the bank's firm is unsuccessful. In equilibrium, the interest rate on these interbank loans adjusts to compensate the lender for this risk. However, the interest rate a borrowing bank pays depends on the average probability of default and not the bank's individual probability of default. In other words, a bank that contracts with its firm to exert a higher level of effort q will be more likely to repay its interbank loan, but it will not be rewarded with a lower interest rate. As with the first externality, this one will tend to lead firms to choose lower levels of effort.

2.5 Equilibrium

We can now define a symmetric stationary equilibrium.

Definition. Given the policy variables i, r, τ, and τ_m, a symmetric stationary equilibrium is a list consisting of loan contracts (k, ϕ, q), project quality Q, prices p, i_m, choice of real balances m, such that: given the policy variables and prices p and i_m, the contract (k, ϕ, q) solves the bargaining problem, m is given by market clearing, p is given by (6), i^m is given by (7), the market for balances clears $m = M$, and aggregate quality is consistent with individual choices $Q = q$.

Since we consider a symmetric equilibrium, the interbank market clearing condition is

$$\underbrace{(1-\lambda)m}_{\substack{\text{reserves from} \\ \text{non-lending banks}}} = \lambda \underbrace{b}_{\substack{\text{reserve deficit} \\ \text{of a lending bank}}},$$

so that

$$m = \frac{\lambda \tau p k}{1-(1-\lambda)\tau_m}, \text{ and } b = \frac{(1-\lambda)\tau p k}{1-(1-\lambda)\tau_m}. \tag{8}$$

3 Equilibrium characterization

In this section, we characterize the equilibrium when reserves are costly $i > r$. To do this, we first solve the bargaining problem determining the equilibrium loan contract. Since the participation constraint of the bank binds, we can eliminate ϕ from the firm's problem to obtain the following problem

$$\begin{aligned}\max_{k,q} q[F(k) - pk + (1+r)\left(\tau pk + \tau_m \frac{\tau pk - m}{1-\tau_m}\right) - (1+i_m)\frac{\tau pk - m}{1-\tau_m}] \\ -Q(1+i_m)m - (1-Q)(1+r)\tau_m m - \frac{1}{2}q^2 F(k),\end{aligned}$$

and the first order conditions for k and q are, respectively,

$$k: \quad \left(1-\frac{1}{2}q\right)F'(k) = p+(i_m-r)\frac{\tau p}{1-\tau_m}, \tag{9}$$

$$q: \quad (1-q)\,F(k) = pk+(i_m-r)\frac{\tau pk-m}{1-\tau_m}-(1+r)m. \tag{10}$$

On the right hand side of (9) is the bank's funding cost when it lends enough deposits for the firm to purchase one unit of capital. This funding cost consists of the cost of redeeming the additional deposits p, as well as the marginal opportunity cost of the required reserves τ: they earn r from lending to firms but could instead earn i_m from the interbank market. The left hand side of (10) is the net marginal benefit of increasing quality. The right hand side is the marginal cost of higher quality, which consists of a more frequent payment to depositors, and to other interbank lenders, net of the interest rate paid on borrowed and own reserves. (10) clearly shows that q is decreasing with the bank/firm's level of indebtedness as measured by $pk+(1+i_m)(\tau pk-m)/(1-\tau_m)$ but increasing with the interest rate on reserves.

Replacing the equilibrium value for the interbank rate, the price of deposits, as well as m, and arranging we obtain how the (optimal) contract responds to changes in policy variables, i and τ but also in the market perception of risk Q.

$$\left(1-\frac{1}{2}q\right)F'(k) = \frac{1}{Q}\left\{1+\frac{(i-r)}{[Q+(1-Q)(1+r)\tau]}\frac{\tau}{(1-\tau_m)}\right\}, \tag{11}$$

$$(1-q)\,\frac{F(k)}{k} = \frac{1}{Q}\left\{1+\frac{(i-r)-\lambda(1+i)}{[Q+(1-Q)(1+r)\tau]}\frac{\tau}{[1-(1-\lambda)\tau_m]}\right\}. \tag{12}$$

Equations (11) and (12) say that investment will increase with market perception Q, but will decrease with inflation and reserve requirements τ and τ_m, while the firm's quality choice q is decreasing with Q, and decreasing with inflation (recall that $\tau(1+r)\leq 1$). Thus, we have the following result:

Lemma 1. *The firm's investment level k is increasing and the firm's choice of quality q is always decreasing with market perception Q.*

The intuition is simple: an improvement in the market perception of quality reduces the risk premium on deposits and interbank loans. This reduced risk premium makes investing

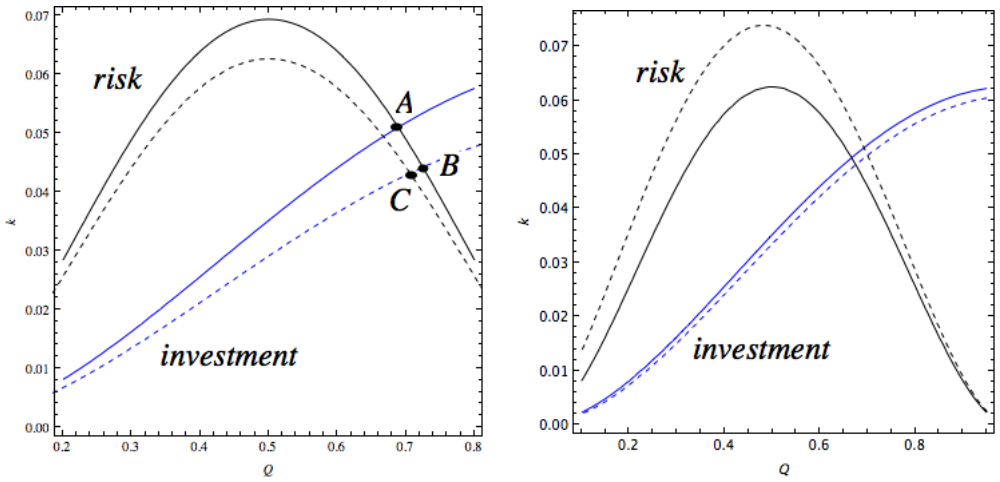
cheaper, and investment increases. As a result, the firm becomes more indebted to the bank. Therefore, the firm takes more risk. This outcome is a key mechanism of our paper: by reducing the risk premium, the market perception makes funding cheaper, which induces firms to invest more and to take more risk. Of course in equilibrium $q = Q$.

Proposition 1. *If $i > r$, there exists a unique symmetric stationary equilibrium where the reserve requirement always binds and the interbank market is active. The equilibrium quality Q and investment k solve equations (11) and (12) with $q = Q$.*

Figure (3a) shows both curves and how they shift following a rise in inflation. The equilibrium with little inflation is at point A. As the spread $i - r$ increases, the marginal cost of holding reserves increases, everything else constant. Hence, given Q, the investment-curve shifts down and investment drops. If only investment were to drop, this would induce a move down on the risk-curve from A to B: leverage would decline, quality increase and risk drop. However, the increased cost of holding reserves induces the bank to charge a higher interest rate to the firm and as a result, the risk-curve shifts down. Because its debt increases, the firm chooses a slightly lower quality. The market perception of risk adjusts and the equilibrium moves from B to C. In the new equilibrium, investment is lower but quality is still higher than in the original equilibrium A. The reduction in investment is sufficient to undo the increase in the funding cost, so that leverage still declines. Our result implies that the investment effect due to the change in inflation is always stronger than the direct effect on quality; as a result, Q always increases with inflation. Intuitively, this is due to the presence of a positive feedback loop: Because quality increases, deposits are now safer, so the risk premium in p declines. This decreased risk contributes to a further reduction in leverage and higher average quality of active firms.

In addition, there are several remarks worth making on Proposition 1.

- When $i > r \geq 0$ and $\tau > 0$, reserve requirements and inflation are substitutes in affecting risk. However, it must be that $\tau > 0$ for inflation to impact risk. Indeed, the bank only lends deposits to the firm and keeps just enough reserves to satisfy its requirements. Setting $\tau = 0$, it is obvious that Q is independent of inflation or the interest



(a) Equilibrium Q, k (increase in i) (b) Equilibrium Q, k (increase in τ)

rate on reserves. Additionally, in this model, banks engage in interbank activities only to satisfy their reserve requirements. Hence, interbank liquidity requirements impact risk-taking insofar as $\tau > 0$.

- When $\tau > 0$, the comparative statics of Q with respect to its arguments are

$$\frac{\partial Q}{\partial i} \geq 0, \ \frac{\partial Q}{\partial \tau} \geq 0, \ \frac{\partial Q}{\partial \tau_m} \geq 0 \text{ and } \frac{\partial Q}{\partial r} \leq 0.$$

  So, a higher cost of holding reserves, and higher reserve requirement or a higher liquidity requirement all reduce risk-taking, but a higher interest rate on reserves increases risk-taking. The intuition for the last result is simple: When the interest rate on reserves is higher, the bank has more to lose by lending to the firm (e.g., if the firm fails, the bank loses the interest on the reserves it holds) and so requires a higher payment. This reduces the firm's incentives to exert an effort and Q drops.

- From (12), we can write $q = 1 - (\text{effective rate} \times k)/F(k)$,[14] Therefore, the sensitivity of quality with respect to investment is related to σ since

$$\frac{\partial q}{\partial k} = -\frac{(\text{effective rate})}{F(k)}(1 - \sigma).$$

  Hence, as σ increases towards 1, the choice of quality becomes less (negatively) sensitive

[14] Where $effective\ rate = \frac{1}{Q} + [(i - r) - \lambda(1 + i)] \frac{\tau p(Q)}{Q(1-(1-\lambda)\tau_m)}$.

to investment. In the limit, q is totally insensitive to the investment level. As is expected, the higher the effective rate, the more sensitive is q to investment. Since the effective rate is a function of aggregate quality, the general equilibrium effect affects the sensitivity of quality to investment: Increasing investment lowers project quality. This in turn increases the effective lending rate, which induces a higher sensitivity of quality to investment. It is worth noting here that there is no equilibrium if $\sigma = 1$.[15]

- As Figure 3a shows, the equilibrium level of investment always drops with the cost of holding reserves. For example, take a rise in the rate of inflation. This increases the rate i which induces a large drop in investment. When this drop is large enough, debt declines and firms choose higher quality levels. This increase in quality induces cheaper funding conditions for firms, which may in turn invest a little more, but not enough to induce investment to raise above its initial level. In general, when the cost of holding reserves is small, we can show that investment is decreasing in i when $\tau = \tau_m = 0$. However, as Figure 3b shows, investment can be increasing in liquidity requirements τ, or τ_m starting from $\tau = \tau_m = 0$, if the cost of reserves is small, but it will be decreasing if τ becomes too large. This is intuitive: keeping the cost of reserves small, the cost of increasing liquidity requirement is tiny. The price of deposits however can drop by an order of magnitude. Therefore, the cost of funds decline, and firms are willing to invest more. This willingness, however, does not induce more risk taking when the cost of funding drops more than the increase in investment. The effects of an increase in liquidity requirements τ_m are qualitatively similar to the ones shown in Figure 3b for an increase in τ.

3.1 Welfare

In this section, we study the welfare consequences of the risk-investment trade-off. As all agents are risk neutral, welfare is given by aggregate output net of the cost of producing the

[15] More precisely, when $\sigma = 1$, there is no real solution to (11) and (12) where $q \in [0, 1]$.

investment good and the firm's cost of effort,

$$\mathcal{W} = \lambda \left[Q \left(1 - \frac{1}{2} Q \right) F(k) - k \right].$$

A planner seeking to maximize welfare will choose investment k^* and quality Q^* to maximize $\mathcal{W}$. The first order conditions are

$$\begin{aligned} Q^* \left(1 - \frac{1}{2} Q^* \right) F'(k^*) &= 1, \\ \text{and} \qquad\qquad Q^* &= 1, \end{aligned} \tag{13}$$

so that $F'(k^*) = 2$.

We now determine an expression for welfare in equilibrium. First notice that there is room for policy actions.

Corollary 1. *In equilibrium, there is too much risk-taking $Q < Q^*$, and there is too little investment $k < k^*$.*

We want to know how welfare moves with $\tilde{\tau}$, i.e., we want to compute $\partial \mathcal{W} / \partial \tilde{\tau}$. This is

$$\frac{\partial \mathcal{W}}{\partial \tilde{\tau}} \quad \propto \quad (1 - Q) \, F(k) Q'(\tilde{\tau}) + \left[Q \left(1 - \frac{1}{2} Q \right) F'(k) - 1 \right] k'(\tilde{\tau}),$$

and using the investment curve, we obtain

$$\frac{\partial \mathcal{W}}{\partial \tilde{\tau}} \propto (1 - Q) \, F(k) Q'(\tilde{\tau}) + \frac{(i - r)}{[Q + (1 - Q)(1 + r)\tau]} \frac{\tau}{(1 - \tau_m)} k'(\tilde{\tau}).$$

As $Q'(\tilde{\tau}) > 0$ for all $\tilde{\tau} \in [0, 1]$, we obtain that $\frac{\partial \mathcal{W}}{\partial \tilde{\tau}} \mid_{\tilde{\tau}=1} > 0$ whenever $i \approx r$, so that in this case $\tilde{\tau} = 1$ is optimal. However, if the spread $i - r$ is large enough, then the (negative) second term in the welfare derivative dominates and $\tilde{\tau} < 1$ is optimal. We summarize this discussion in the following proposition.

Proposition 2. *Suppose $i \simeq r$ then $\tilde{\tau} = 1$ is optimal. Suppose $i - r > 0$ and large, then $\tilde{\tau} < 1$ is optimal.*

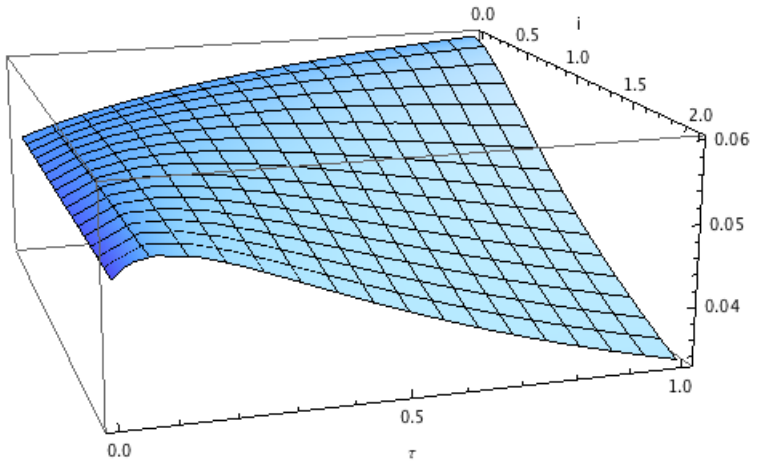


Figure 3: Welfare, $\sigma = 0.5$.

Proposition 2 highlights the trade-off between quality and output: When the cost of holding reserves is high, investment is already low and increasing reserve requirements, while improving quality would also make investment more costly. Thus, it is optimal to reduce reserve requirements. Alternatively, when holding reserves is not so costly increasing reserve requirement is optimal as the increase in quality dominates the possible decline in investment. Figure 3 illustrates the optimal level of reserve requirement $\tilde{\tau}$ as a function of the spread $i - r$. For small spread levels, the optimal reserve requirements is 100% or close to 100%. However, as the spread increases the optimal level of reserve requirements falls, but always remains positive, even for spreads of 200%. In words, if money is relatively cheap for banks to hold ($i \approx r$), it is optimal to raise the level of reserve requirements. This result is intuitive: With low inflation rate, the cost to raise reserve requirements is small for banks. However, higher reserves imply that deposits are safer. Hence, the lower risk premium on deposits dampens the increase in the firms' funding cost due to higher reserve requirements. The effect on investment of higher reserve requirements then is small (if negative) or positive, and for welfare, the higher quality dominates the possibly lower investment.

Now, suppose $\tilde{\tau} > 0$ so that banks may want to borrow on the interbank market. We study the welfare implications of increasing $\tilde{\tau}_m$. Again,

$$\frac{\partial \mathcal{W}}{\partial \tilde{\tau}_m} \propto (1-Q)\, F(k) Q'(\tilde{\tau}_m) + \frac{(i-r)}{[Q+(1-Q)(1+r)\tau]} \frac{\tau}{(1-\tau_m)} k'(\tilde{\tau}_m),$$

so by the same argument as above, we obtain the following result,

Proposition 3. *If* $(i-r)\tau \leq \bar{c}$ *then* $\tau_m > 0$ *is optimal.*

If the cost of the reserve requirement $(i-r)\tau$ is too low but cannot be changed, it is optimal to require liquidity requirement for interbank market borrowing ($\tau_m > 0$) to increase the cost of banks' lending. The level $\bar{c}$ is given by the optimal choice for i or τ when $\tau_m = 0$.

4 Extensions

In these extensions, we assume there is no interbank liquidity requirement and set $\tilde{\tau}_m = 0$ for simplicity.

4.1 Operating banks α

Most, if not all, regulators will agree that it is socially costly for any bank to fail. One such cost is the disruption of the payment system, and some even argue that there is also a loss of expertise. In this section, we get to the idea that bank failure is costly by assuming it takes time to unwind a failing bank. As a result, a bank that fails is replaced by a new bank, but with a one-period lag. Bankers being short-lived will not fully internalize the cost of their default. Of course, our qualitative results that reserve requirements can be welfare improving do not depend on this assumption, although it may affect the quantitative predictions of the model.

We now compute the number of operating banks in period t. Since banks that financed a failing firm lose their license for one period, the number of operating banks in period t is α_t:

$$\alpha_t = Q \underbrace{\lambda\alpha_{t-1}}_{\text{\#b with a match}} + \underbrace{(1-\lambda)\alpha_{t-1}}_{\text{\#b with no match}} + \underbrace{(1-\alpha_{t-1})}_{\text{\#b not operating last period}}, \text{ or}$$

$$1-\alpha_t = (1-Q_{t-1})\lambda\alpha_{t-1}.$$

Therefore, in steady state $\alpha \equiv \alpha_t = \alpha_{t-1}$,

$$\alpha = \frac{1}{1+\lambda(1-Q)}. \tag{14}$$

Naturally, if $Q = 1$, all banks are operating, while less banks are operating otherwise. Then, all of our equilibrium analysis goes through. Welfare is given by the aggregate output net of the cost of producing the investment good and the firm's cost of effort,

$$\mathcal{W} = \alpha\lambda \left[Q(1 - \frac{1}{2}Q)F(k) - k \right],$$

where α is now given by (14). Therefore, optimal investment k^* and quality Q^* solve (13) and $Q^* = 1$.[16] The unit upper bound on quality binds because the planner would require a higher quality since she internalizes the cost of bank failure. If anything, the cost of failure would reinforce the need for reserve requirement.

4.2 (Anticipated) bail-outs

In this section, we consider the case where the government decides to bail-out failing banks. This bailout means that the government will pay off all of the failing bank's liabilities by taxing suppliers in a lump-sum way. When banks are bailed out, all of their liabilities are always repaid so none of their liabilities carries a risk premium. Hence, the market does not perceive or price any risk. More precisely, the real price of deposits becomes $p = 1$ while the interbank market rate is

$$1 + i_m = \frac{1+\pi}{\beta} = (1+i).$$

[16] The first order condition is

$$\frac{\lambda}{[1+\lambda(1-Q)]^2} \left[Q(1 - \frac{1}{2}Q)F(k) - k \right] + \frac{1}{1+\lambda(1-Q)}(1-Q)F(k) = 0,$$

which simplifies in the expression above.

The rest of the model is as before. In particular, and in addition to using (8), the first order conditions (9) and (10) become

$$k: \quad F'(k)\left(1-\frac{1}{2}q\right) = 1+(i-r)\tau, \tag{15}$$

$$q: \quad (1-q)\,F(k) = k+(i-r)\tau k-(1+i)\lambda\tau k. \tag{16}$$

An equilibrium with bail-out is a list (p, i_m, k, q, Q, m) such that given the bail-out policy, and policies i, τ, prices p and i_m, and aggregate risk $1-Q$, banks optimally choose m^b, firms choose k and q to maximize their surplus, $p=1$, i_m clears the interbank market and $q=Q$.

Combining (15) and (16), we obtain

$$q = 1-\frac{\sigma\left[1+(i-r)\tau-(1+i)\lambda\tau\right]}{(2-\sigma)\left(1+(i-r)\tau\right)+\sigma(1+i)\lambda\tau}, \tag{17}$$

$$F'(k) = (2-\sigma)\left(1+(i-r)\tau\right)+\sigma(1+i)\lambda\tau. \tag{18}$$

It should be clear that increasing τ and/or i will always decrease k and increase q.

Proposition 4. *In an equilibrium with bail-out, the investment level is k^b given by (18) and the quality of projects is $q=Q^b$ given by (17). k^b always declines with inflation or reserve requirements. Q^b increases with i and τ. For any $i \geq r$ and $\tau > 0$, an anticipated bail-out policy increases risk-taking and investment.*

To complete the proof of Proposition 4, we now compare welfare under bail-out and no bail-out. First we concentrate on the quality choice as given in (17). Inspecting (29) (the equilibrium quality without bailout derived from the investment and risk curves) notice that increasing p has the same effect as increasing τ: it increases Q. But comparing (17) is the same expression as (29) except for setting p to its minimum level $p=1$. Therefore, Q^b given by (17) will be lower than Q given by (29). We conclude that bail-out increases risk-taking.

By comparing the levels of investment as given by (30) (the investment level without insurance) and (18) with $\tilde{\tau}=\tau$, we can show that $k^b > k$. An (anticipated) bail-out policy increases investment. Whether the bailout policy is welfare improving depends on the level of initial reserve requirement: If it is was too high relative to the optimum (but less than

one), then the bailout can be welfare improving. Of course, this result hinges on the fact that a bail-out does not involve any distortion (such as the use of distortionary taxes).

4.3 Deposit insurance

In this section, we analyze whether a well designed insurance scheme for deposits can do better than reserve requirements, or the bail-out policies we analyzed in the previous section. We now assume that banks have to work in the CM when they are born and pledge resources to the deposit insurance fund, as a fraction δ of their deposits pk. In this sense, the insurance scheme is similar to a lending tax. However, it is more than a tax, as the deposit insurance company would then tap into the funds to guarantee deposits (but not the interbank market loans). Sustainability of the deposit insurance mechanism requires that it has enough resources to cover the shortfall, i.e., $\delta pk = (1-Q)pk(1-\tau(1+r))$ – where we still assume that all banks will hold τpk in reserves. Since in an equilibrium with deposit insurance deposits are as safe as money, $p = 1$ and we obtain $\delta = (1-Q)\left(1-\tau(1+r)\right)$. Notice that banks always lose their contributions to the deposit insurance fund.

With such an insurance scheme in place, the bank's incentive constraint becomes

$$-\delta k + q\left[\phi + \tau(1+r)k - (1+i_m)(\tau k - m)\right] \geq Q(1+i_m)m,$$

so that

$$\phi = \frac{\delta k + Q(1+i_m)m}{q} - (1+i_m)m + (i_m - r)\tau k.$$

Then, the bargaining problem becomes

$$\max_{k,q} qF(k) - \delta k - Q(1+i_m)m + q(1+i_m)m - q(i_m - r)\tau k - qk - \frac{1}{2}q^2F(k),$$

with first order conditions

$$q\left(1 - \frac{1}{2}q\right)F'(k) = \delta + q\left(i_m - r\right)\tau + q, \tag{19}$$

$$(1-q)F(k) = \left(i_m - r\right)\tau k + k - (1+i_m)m. \tag{20}$$

An equilibrium with deposit insurance is a list (p, i_m, k, q, Q, m) such that given the deposit insurance policy δ, and policies π, τ, prices p and i_m, and aggregate risk $1-Q$, banks optimally choose m, the contract k and q maximize the bank/firm's surplus, $p=1$, i_m clears the interbank market and $q=Q$. Market clearing condition still requires $\lambda\tau k = m$ and from the banks' demand for money,

$$1+i_m = \frac{1+\pi}{Q\beta} = \frac{1+i}{Q}.$$

To solve for the equilibrium quality choice Q, we use $q=Q$, $\delta = (1-Q)\left(1-\tau(1+r)\right)$ and the expressions for m and i_m in the first order conditions to obtain

$$\left(1-\frac{1}{2}Q\right)F'(k) = \frac{(1-Q)}{Q}\left[1-\tau(1+r)\right] + \left[\frac{1+i}{Q} - (1+r)\right]\tau + 1,$$
$$(1-Q)F(k) = \left[\frac{1+i}{Q} - (1+r)\right]\tau k + k - \frac{1+i}{Q}\lambda\tau k.$$

The fact that interbank market exposures are not covered by the deposit insurance scheme is reflected by the risk premium $\frac{1+i}{Q}$, while the contribution to the deposit insurance – increasing in the aggregate risk – is captured by the term $1/Q$. We can now find k and Q from re-arranging the first order conditions,

$$Q\left(1-\frac{1}{2}Q\right)F'(k) = 1+\tau(i-r), \tag{21}$$

where $Q = Q^d$,

$$Q^d = \frac{2\left\{(1-\sigma)\left[1+(i-r)\tau\right] + \sigma(1-Q^d)\left(1-(1+r)\tau\right) + \sigma\left(1+i\right)\lambda\tau\right\}}{(2-\sigma)\left[1+(i-r)\tau\right] + \sigma(1-Q^d)\left(1-(1+r)\tau\right) + \sigma\left(1+i\right)\lambda\tau}. \tag{22}$$

There are two solutions to (22). We can show that the unique equilibrium is one with the negative root, as the other root is always greater than one.

We can now compare the project's quality without insurance (29) with the one with insurance (22). We want to check conditions for which $Q^d > Q$. However, replacing the expression for

$p(Q)$ we obtain $Q^d > Q$ whenever

$$(1-Q)\sigma(1-\tilde{\tau})\left[(1+i)(1-\lambda)\tau + Q(1-\tilde{\tau})\right] > 0,$$

which is always satisfied. Finally, comparing the equilibrium level of investment without deposit insurance (11) we find that k is higher with deposit insurance.[17] We summarize this discussion in the following proposition.

Proposition 5. *Suppose $\tau(1+r) < 1$. There exists a unique equilibrium with deposit insurance, in which the investment level is k given by (21) and the quality of projects is Q given by (22). The average quality as well as the level of investment, and welfare are higher with deposit insurance than without deposit insurance.*

The deposit insurance scheme we impose achieves a better allocation because it is a direct function of the bank's loan size. As such, the bank and the firm internalize the cost of making larger loans. To see this, assume that each bank has to make a fixed contribution Δ to the deposit insurance scheme, independent of their current loans. Then, the first order conditions from the bargaining problem give (9) and (10) with $p = 1$, which we can simplify as

$$\tilde{Q}^d = 1 - \frac{\sigma\left[1 + (i-r)\tau - (1+i)\lambda\tau\right]}{(2-\sigma)\left[1+(i-r)\tau\right] + \sigma(1+i)\lambda\tau}. \tag{23}$$

Since Q in (23) is increasing in p, we obtain $\tilde{Q}^d < Q$. Turning to investment, $\tilde{k}^d$ solves

$$\tilde{Q}^d\left(1 - \frac{1}{2}\tilde{Q}^d\right)F'(\tilde{k}^d) = \tilde{Q}^d\left(1-(1+r)\tau\right) + (1+i)\,\tau, \tag{24}$$

while without deposit insurance,

$$Q\left(1-\frac{1}{2}Q\right)F'(k) = \frac{Q\left(1-(1+r)\tau\right) + (1+i)\tau}{Q + (1-Q)(1+r)\tau}.$$

[17] As $Q^d > Q$, we have $Q^d(1-Q^d/2) > Q(1-Q/2)$ is also higher. Since $Q + (1-Q)\tilde{\tau} < 1$, we obtain

$$F'(k^d) = \frac{1+(i-r)\tau}{Q^d\left(1-\frac{1}{2}Q^d\right)} < \frac{1+\frac{(i-r)\tau}{Q+(1-Q)(1+r)\tau}}{Q\left(1-\frac{1}{2}Q\right)} = F'(k)$$

As $\tilde{Q}^d < Q$, we have $Q^d(1 - Q^d/2) < Q(1 - Q/2)$, so in general it is difficult to say if $k > \tilde{k}^d$. However, when $(1 + r)\tau \to 1$, $F'(k) \to \frac{(1+i)\tau}{Q\left(1-\frac{1}{2}Q\right)}$ while $F'(\tilde{k}^d) \to \frac{(1+i)\tau}{\tilde{Q}^d\left(1-\frac{1}{2}\tilde{Q}^d\right)}$ so that $\tilde{k}^d < k$ whenever $(1 + r)\tau$ is large enough. In those cases, a fixed deposit insurance makes the economy worse, as it decreases quality as well as investment.

4.4 Capital requirements

In this section, we show conditions for which capital requirements are equivalent to liquidity requirements. As will become clear, capital and liquidity requirements are equivalent whenever, as in this model, raising equity is as "easy" as getting an interbank loan.

To show the equivalence result, we replace the liquidity requirement constraint by an equity constraint. To be precise, we assume that for any loan size k, the bank has to have at least a fraction $\varepsilon \in [0, 1]$ of its investment in own equity. Therefore, the bank can finance a fraction $(1-\varepsilon)$ of its loan with deposits and the remaining fraction with equity. We relax the assumption that banks can produce sweat equity in the centralized market, and we instead assume that young banks raise equity by selling shares to suppliers. A share is a claim to the bank's future profit. Also, banks can sell additional shares to other banks once they meet a productive firm. This second equity market replaces the interbank market in the case with reserve requirements.

When there is inflation, it is costly to hold (unused) capital and so the equity requirement will bind. Then, when a bank investing k raises ek equity from suppliers and Ek equity to other banks, it must be that $e + E = \varepsilon$. The bank's participation constraint given contract (k, ϕ), is

$$q\left[p(1-\varepsilon)k + \varepsilon k + \phi - (1+\varrho)Ek - p(1-\varepsilon)k\right] \;\geq\; Q(1+\rho)ek. \tag{25}$$

The left hand side shows the expected profit of the bank that accrues to shareholders. If the firm fails, the bank's equity is wiped out. If the firm succeeds, it pays the principal $p(1-\varepsilon)k+\varepsilon k$ back to the bank plus ϕ, and the bank redeems deposits with a cost $-p(1-\varepsilon)k$. The bank also pays ϱ to other banks holding its equity. The right-hand side is the outside

option of the bank: It gets an expected return $Q(1+\rho)$ on its (existing) equity by buying other productive banks equity. Then, we can simplify the participation constraint as

$$\phi + \varepsilon k \geq \left[\frac{Q(1+\rho)e}{q\varepsilon} + (1+\varrho)\frac{E}{\varepsilon}\right]\varepsilon k \tag{26}$$

The left-hand side is the bank's resources when the firm succeeds, and the right-hand side is the return paid to equity holders. In a symmetric equilibrium, $\varrho = \rho$ and $q = Q$.

We now consider the choice of contract (k, ϕ, q). Again, it maximizes the firm's payoff,

$$\max_{k,\phi,q} q[F(k) - \phi - p(1-\varepsilon)k - \varepsilon k] - \frac{1}{2}q^2 F(k),$$

subject to the bank's participation constraint (26). Using the expression for ϕ, this problem becomes

$$\max_{k,q} qF(k) - \frac{1}{2}q^2 F(k) - q[(1+\varrho)Ek + p(1-\varepsilon)k] - Q(1+\rho)ek.$$

In the capital market, suppliers no longer expect banks to hold reserves, so that the price of deposits fully reflects the risk of bank's failure,

$$p = \frac{1}{Q}. \tag{27}$$

This is one difference with reserves requirement: where liquidity requirement helps reduce the risk premium of deposits, capital requirement does not. Then, the first order conditions of the firm's problem are

$$\begin{aligned} q\left(1 - \frac{1}{2}q\right)F'(k) &= q(1+\varrho)E + Q(1+\rho)e + \frac{q}{Q}(1-\varepsilon), \\ (1-q)\,F(k) &= (1+\varrho)Ek + \frac{1}{Q}(1-\varepsilon)k. \end{aligned}$$

Therefore, we can already conclude that full equity requirement $\varepsilon = 1$ achieves the first best allocation whenever $\rho = 0$ and $E = 0$. The reason is that cash is cash: suppliers do not require a risk premium when they are paid with cash and there is no distortion of the allocation coming from this margin. Also, when $\rho = 0$, it is costless to build equity.

Therefore, there is no distortion out of this margin either. Finally, when $E = 0$, the firm itself is not being held-up by the bank having to raise new equity, and it chooses the first best quality level.

We now solve for ρ. Since ρ is the real return on equity, we normalize the price of a bank share in the centralized market to one. When they consider how many shares s of a generic bank to purchase, suppliers solve the following problem in the centralized market,

$$\max_s -(1+\pi)s + \beta Q \left[\frac{p(1-\varepsilon)k + \varepsilon k + \phi - (1+\varrho)Ek - p(1-\varepsilon)k}{ek}\right] s,$$

where Q is the supplier's belief about investment risk, and the term in square bracket is the real return on each share sold to suppliers, given the bank raises ek of equity from suppliers. In equilibrium $q = Q$, and (25) implies

$$1 + i = Q(1+\rho).$$

In case $i > \rho$, the holdup problem implies that there is no equilibrium, as the bank has no incentive to build equity ahead of its lending activity.[18] In addition, the higher i the more expensive it is to raise equity (the higher the return on equity has to be, so the lower investment is. Finally, whenever $i > 0$ banks have to raise equity from other banks, as otherwise there would not be any equilibrium (because of the hold up problem). Hence, it must be that $e < \varepsilon$ (unless $i = 0$.) The equilibrium condition for the (inter)bank market for equity gives $(1-\lambda)e = \lambda E$. Together with $e + E = \varepsilon$, we obtain $e = \lambda\varepsilon$. Therefore, in equilibrium

$$\begin{aligned} Q\left(1 - \frac{1}{2}Q\right)F'(k) &= 1 + i\varepsilon, \\ Q\left(1-Q\right)\frac{F(k)}{k} &= 1 + i\varepsilon - (1+i)\lambda\varepsilon. \end{aligned}$$

Comparing these two equations with (11) and (12), it is straightforward to see that setting $\varepsilon = \frac{\tau}{Q^\tau + (1-Q^\tau)(1+r)\tau}$ gives us the same equilibrium condition as in the case with reserve re-

[18] This outcome would change if banks had some bargaining power.

quirements, where Q^τ is the equilibrium quality choice with reserve requirement τ. Therefore, in this model, liquidity requirements are equivalent to capital requirement.

5 Literature review

The idea that interest rate policy affects risk-taking by intermediaries also referred to as the risk-taking channel of monetary policy, a term coined by Borio and Zhu (2012) prompted a recent empirical literature. One main finding of this literature is a negative relationship between the level of interest rates and bank risk-taking. In light of this observation, it has been argued that central banks could have prevented the buildup of risk in the run-up to the recent financial crisis and the ensuing negative consequences for the macroeconomy by raising interest rates. Dell'Ariccia, Laeven, and Marquez (2014) and de Nicolò, Dell'Ariccia, Laeven, and Valencia (2010) document a negative relationship between the *real* fed funds rate and the riskiness of U.S. banks assets. Others use *nominal* interest rate data to establish a negative relationship to bank risk-taking in different countries.[19]

On the theoretical side, Williamson (1999) argues that the creation of tradable deposit allows productive intermediation and is thus desirable. Using a similar angle of attack, Chari and Phelan (2016) argue that the creation of deposits has the (private) benefits of insuring against liquidity shocks, while at the same time imposing a pecuniary externality by raising the price level. This outcome implies that the social benefits of deposit creation can even be negative. As a result, 100% reserve requirement can be desirable. Our mechanism also plays through a pecuniary externality, but while Chari and Phelan study the effect of consumption loans, we study the effect of corporate credit lines on the production process. Then, we can show that deposits possibly increase leverage beyond its optimal level and increasing risk (in addition to the price level). Monnet and Sanches (2015) also show that 100% reserve requirements may be undesirable because bankers cannot commit to repay deposits. Instead, our results are driven by limited liability. Still, with limited commitment but with moral hazard for banks

[19]For example, Gambacorta (2009), Ioannidou, Ongena, and Peydró (2015), Jiménez, Ongena, Peydró, and Saurina (2014), Delis and Kouretas (2011) and Altunbas, Gambacorta, and Marques-Ibanes (2014) and Cociuba, Shukayev and Ueberfeldt (2016).

decision, Hu and Li (2017) analyze the effect of capital regulation. Instead, we concentrate on the effects of monetary policy on banks' balance sheet risk. Sanches (2015) argues that a purely private monetary regime is inconsistent with macroeconomic stability. The result hinges on endogenously determined limits on private money creation and the presence of self-fulfilling equilibrium characterized by monetary collapse.

Jakab and Kumhof (2015) remark that, with a few exceptions, the academic literature has focused on a debatable model of banks, namely, the "intermediation of loanable funds" model. In this model, banks intermediate funds from savers to borrowers. A prime example of such a model is Diamond and Dybvig (1983), or Berentsen, Camera, and Waller (2007). Calomiris, Heider, and Hoerova (2015) is also using the "intermediation" model but is more related to our question, as they analyze the need for liquidity requirements for banks. In their model, liquidity requirements act as collateral a disciplining device for bankers who otherwise would engage in moral hazard. Instead, we investigate another channel: liquidity requirement reduces leverage and thus risk taking, by increasing the cost of firm's funding. Also we provide a general equilibrium model where banks can create deposits that circulate as the means of payments, and we can analyze the effect of monetary policy on bank risk-taking. Jakab and Kumhof argue that banks' main activity is to finance firms through the creation of money (or deposits). Among many other results, they show that the "financing" model of banking explains why leverage is pro-cyclical. Our model belongs to the financing view of banking and we concentrate on risk-taking, the optimal reserve requirement policy and its interaction with monetary policy when banks issue tradable deposits.[20] Our paper is also related to Williamson (2016) that features the moral hazard problem of creating low quality collateral when the interest rate is low.

6 Conclusion

We presented a model to study the implications of deposit-making on risk-taking. We believe our arguably simple model captures several important features of bank lending activities:

[20] We refer the reader to Bigio and Weill (2016) for a recent theory of banks balance sheet and why banks are useful in providing liquid assets.

Firms need funding and they obtain it from banks. Banks finance firms by creating deposits. Deposits are used as a means of payments. Deposits carry a risk premium as long as they are not insured or only partially backed by liquid assets. We find that borrower's quality is an increasing function of inflation: The increased cost of liquid asset induces banks to charge higher rates to borrowers. As a consequence, they borrow less, their debt level falls, and they take less risk.

The model is simple, and we chose to abstract from many relevant aspects. Let us mention the four most obvious: First, banks have no bargaining power. This may seem unrealistic, but this assumption implies that the firms' incentives are most aligned with the one of a planner. If banks had some bargaining power, it would only deter firms from choosing higher quality as banks would capture some of the surplus. Second, banks do not take deposits from depositors and they finance their liquidity requirements using only (sweat) equity. Hence, we cannot study issues such as bank runs in the current version of the model. Still let us stress that the risk premium on deposits is getting to the idea of a run on banks: If the risk premium increases to infinity, firms cannot trade deposits. We think it would be interesting to extend the model in this direction. The third aspect that is missing from the model is the cost of raising equity. Here, we modeled equity as an effort level that banks have to exert in order to get started. Finally, analyzing growth should yield interesting insights in particular regarding the debate on growth versus stability of the financial system. Overall, we expect the mechanism we highlighted to be robust to these four and other extensions.

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A Proof of Lemma 1

Proof. Replace (11) in (12). Making use of $F'(k) = \sigma F(k)/k$ and arranging, we obtain an expression for k as a function of Q,

$$F'(k) = \frac{2}{Q}\left[1 + (i-r)\frac{\tau p(Q)}{1-\tau_m}\right] - \frac{\sigma}{Q}\left[1 + (i-r-(1+i)\lambda)\frac{\tau p(Q)}{1-(1-\lambda)\tau_m}\right]$$

where we use $p(Q)$ to ease notation. Since $p'(Q) \leq 0$ and $2/(1-\tau_m) > \sigma/(1-(1-\lambda)\tau_m)$, the right-hand side is decreasing with Q. Hence, k is always increasing with Q. Using this expression back in (12), we find

$$(1-q) = \frac{\sigma\left\{1 + [(i-r) - \lambda(1+i)]\frac{\tau p(Q)}{1-(1-\lambda)\tau_m}\right\}}{(2-\sigma) + (i-r)\tau p(Q)\left[\frac{2}{(1-\tau_m)} - \frac{\sigma}{1-(1-\lambda)\tau_m}\right] + \sigma(1+i)\lambda\frac{\tau p(Q)}{1-(1-\lambda)\tau_m}} \tag{28}$$

Simplifying, we find that the derivative of the right-hand side with respect to Q is

$$-p'(Q)\frac{\sigma\left[(i-r)\left[2\frac{(1-(1-\lambda)\tau_m)}{(1-\tau_m)} - (1-\lambda)\sigma\right] + 2(1+r)\lambda\right]\tau(1-(1-\lambda)\tau_m)}{(\cdot)^2}$$

which is always positive since $2\left(1-(1-\lambda)\tau_m\right) > \sigma(1-\tau_m)$ and $p'(Q) \leq 0$. Hence, (28) defines $q(Q)$ with $q'(Q) < 0$.

□

B Proof of Proposition 1:

Proof. To show Proposition 1, it is convenient to rewrite (11) and (12) as

$$Q = 1 - \frac{\sigma\left\{1 + [(i-r) - \lambda(1+i)]\frac{\tau p(Q)}{1-(1-\lambda)\tau_m}\right\}}{(2-\sigma) + (i-r)\tau p(Q)\left[\frac{2}{(1-\tau_m)} - \frac{\sigma}{1-(1-\lambda)\tau_m}\right] + \sigma(1+i)\lambda\frac{\tau p(Q)}{1-(1-\lambda)\tau_m}}, \tag{29}$$

and

$$F'(k) = \frac{2}{Q}\left[1 + (i-r)\frac{\tau p(Q)}{(1-\tau_m)}\right] - \frac{\sigma}{Q}\left[1 + (i-r-(1+i)\lambda)\frac{\tau p(Q)}{[1-(1-\lambda)\tau_m]}\right]. \tag{30}$$

Existence requires that the RHS of (29) is between $[0,1]$, that is,

$$0 \leq \frac{\sigma\left\{1-(1-\lambda)\tau_m + [(i-r) - \lambda(1+i)]\tau p(Q)\right\}}{(2-\sigma)\left(1-(1-\lambda)\tau_m\right) + (i-r)\tau p(Q)\left[2\frac{1-(1-\lambda)\tau_m}{(1-\tau_m)} - \sigma\right] + \sigma(1+i)\lambda\tau p(Q)} \leq 1.$$

The constraint for which this expression is less than unity is obviously satisfied since $2-\sigma \geq 1 \geq \sigma$.

To guarantee that this value is positive, we need the nominator to be positive, which is easily checked by using the expression for $p(Q)$ and the fact that $\tilde{\tau} = (1+r)\tau \in [0,1]$ and $\tau_m \in [0,1]$.

□

C Proof of Corollary 1

Proof. Comparing (13) and (11), it would be optimal to set $i = \tau = 0$ if this was not affecting risk-taking. However, there is too much risk-taking when $\tau = 0$ since then

$$Q = 1 - \frac{\sigma}{2-\sigma} < 1.$$

In addition, (11) implies that the market equilibrium is characterized by underinvestment. Indeed, (11) shows that $F'(k) \geq \left[\left(1 - \frac{1}{2}Q\right)Q\right]^{-1}$. Since the RHS is decreasing in Q, it reaches a minimum when $Q = 1$ with value 2.

□

D Proof of $Q'(i) \geq 0$

The derivative of Q gives

$$Q'(i) = \frac{\sigma}{B^2} 2\lambda\tau(1-\tau_m)\left[1-(1-\lambda)\tau_m\right]\left[p(Q) - (1+r)\tau p(Q)^2 + (1+i-(1+r)\tau_m)p'(Q)Q'(i)\right],$$

where

$$B = (1-\tau_m)(1-(1-\lambda)\tau_m)(1+r)\left\{(2-\sigma) + \frac{(i-r)\tilde{\tau}}{1+r}p(Q)\left[\frac{2}{(1-\tau_m)} - \frac{\sigma}{1-(1-\lambda)\tau_m}\right]\right.$$
$$\left. +\sigma\frac{(1+i)}{1+r}\lambda\frac{\tilde{\tau}p(Q)}{1-(1-\lambda)\tau_m}\right\}.$$

Hence,

$$Q'(i) = \frac{2\lambda\tau(1-\tau_m)(1-(1-\lambda)\tau_m)\left(1-(1+r)\tau p(Q)\right)p(Q)}{\frac{B^2}{\sigma} - 2\lambda\tau(1-\tau_m)(1-(1-\lambda)\tau_m)(1+i-(1+r)\tau_m)p'(Q)}.$$

Since $p'(Q) < 0$ the denominator is positive. The numerator is also positive whenever

$$\begin{aligned} 1 &\geq (1+r)\tau p(Q), \\ 1 &\geq \frac{(1+r)\tau}{Q + (1-Q)(1+r)\tau}, \end{aligned}$$

which is always the case (as the denominator is an average of $(1+r)\tau$ and $1 \geq (1+r)\tau$. Hence, $Q'(i) > 0$.

E Proof of $Q'(\tilde{\tau}) \geq 0$

Q can be written alternatively as

$$Q = 1 - \frac{\sigma\left\{1 + \left[\frac{(i-r)}{1+r} - \lambda\frac{(1+i)}{1+r}\right]\frac{\tilde{\tau}p(Q)}{1-(1-\lambda)\tau_m}\right\}}{(2-\sigma) + \frac{(i-r)}{1+r}\tilde{\tau}p(Q)\left[\frac{2}{(1-\tau_m)} - \frac{\sigma}{1-(1-\lambda)\tau_m}\right] + \sigma\frac{(1+i)}{1+r}\lambda\frac{\tilde{\tau}p(Q)}{1-(1-\lambda)\tau_m}}. \quad (31)$$

Therefore, it is easy, although cumbersome, to check that

$$Q'(\tilde{\tau}) = \frac{\sigma}{B^2}2(1+r)\lambda(1-\tau_m)\left[1+i-(1+r)\tau_m\right]\left[1-(1-\lambda)\tau_m\right]\left[p(Q) + \tilde{\tau}p'(Q)Q'(\tilde{\tau})\right].$$

Hence,

$$Q'(\tilde{\tau}) = \frac{2(1+r)\lambda(1-\tau_m)(1+i-(1+r)\tau_m)(1-(1-\lambda)\tau_m)p(Q)}{\frac{B^2}{\sigma} - 2(1+r)\lambda(1-\tau_m)(1+i-(1+r)\tau_m)(1-(1-\lambda)\tau_m)\tilde{\tau}p'(Q)}.$$

The denominator is positive since $p'(Q) < 0$, and the nominator is positive as $(1+r)\tau_m \leq 1$. Hence, $Q'(\tilde{\tau}) \geq 0$ for all $\tilde{\tau}$.

F Proof of $Q'(\tilde{\tau}_m) \geq 0$

It is a little less cumbersome to check the sign of $Q'(\tau_m)$. We have

$$Q = 1 - \frac{\sigma\left\{1-(1-\lambda)\tau_m + \left[(i-r) - \lambda(1+i)\right]\tau p(Q)\right\}}{(2-\sigma)\left(1-(1-\lambda)\tau_m\right) + (i-r)\tau p(Q)\left[2\frac{1-(1-\lambda)\tau_m}{(1-\tau_m)} - \sigma\right] + \sigma(1+i)\lambda\tau p(Q)}. \quad (32)$$

Therefore, it is easy although cumbersome to check that,

$$\begin{aligned} Q'(\tau_m) &= \frac{\sigma}{C^2}2\lambda\tau\left\{\left[(1+i)(1-\lambda)(1-2\tau_m) + i - r + (1+r)(1-\lambda)\tau_m^2\right]p(Q)\right. \\ &\quad \left. -(i-r)(r - i(1-\lambda) + \lambda)\tau p(Q)^2 + (1-\tau_m)(1+i-(1+r)\tau_m)(1-(1-\lambda)\tau_m)p'(Q)Q'(\tau_m)\right\} \\ &= 2\lambda\tau\sigma\frac{\left[(1+i)(1-\lambda)(1-\tau_m)^2 + (i-r)\left(1-(1-\lambda)\tau_m^2\right)\right]p(Q) - (i-r)\left[r - i(1-\lambda) + \lambda\right]\tau p(Q)^2}{C^2 - 2\sigma\lambda\tau(1-\tau_m)\left[1+i-(1+r)\tau_m\right]\left[1-(1-\lambda)\tau_m\right]p'(Q)} \end{aligned}$$

where

$$C \equiv (1-\tau_m)\left((2-\sigma)(1-(1-\lambda)\tau_m) + (i-r)\tau p(Q)\left[2\frac{1-(1-\lambda)\tau_m}{(1-\tau_m)} - \sigma\right] + \sigma(1+i)\lambda\tau p(Q)\right).$$

Notice that

$$\left[(1+i)\lambda - (i-r)\right]\tau p(Q) \leq \left[(1+i)\lambda - (i-r)\lambda\right]\tau p(Q) = \lambda(1+r)\tau p(Q) \leq \lambda.$$

Hence,

$$\begin{aligned} Q'(\tau_m) &\geq 2\lambda\tau\sigma\frac{\left((1+i)(1-\lambda)(1-\tau_m)^2+(i-r)\left[1-(1-\lambda)\tau_m^2\right]\right)p(Q)-(i-r)\lambda p(Q)}{C^2-2\sigma\lambda\tau(1-\tau_m)(1+i-(1+r)\tau_m)(1-(1-\lambda)\tau_m)p'(Q)} \\ &\geq 2\lambda\tau\sigma(1-\lambda)p(Q)\frac{(1+i)(1-\tau_m)^2+(i-r)(1-\tau_m^2)}{C^2-2\sigma\lambda\tau(1-\tau_m)(1+i-(1+r)\tau_m)(1-(1-\lambda)\tau_m)p'(Q)}, \end{aligned}$$

which is always positive.

G Comparative statics $k'(i)<0$ whenever $i\approx r$ and $\sigma<2/3$.

We showed that $Q'(i)>0$. Then, from

$$Q\left(1-\frac{1}{2}Q\right)F'(k)=1+(i-r)\frac{\tau}{(1-\tau_m)}p(Q), \tag{33}$$

we have

$$(1-Q)\,F'(k)Q'(i)+Q\left(1-\frac{1}{2}Q\right)F''(k)k'(i)=(i-r)\frac{\tau}{(1-\tau_m)}p'(Q)Q'(i)+\frac{\tau}{(1-\tau_m)}p(Q).$$

Therefore,

$$Q\left(1-\frac{1}{2}Q\right)F''(k)k'(i)=\underbrace{\frac{\tau}{(1-\tau_m)}p(Q)}_{\text{direct effect}}+\underbrace{(i-r)\frac{\tau}{(1-\tau_m)}p'(Q)Q'(i)}_{\text{effect on deposits value}}-\underbrace{(1-Q)\,F'(k)Q'(i)}_{\text{effect on effort}}.$$

There are three effects that determine how capital moves with i. The first is the direct effect of the rise in i which makes banks' loans more expensive and so contributes to reducing k. The second effect is the indirect effect of i on deposit prices: Since average quality increases, the deposits are worth more, so $p'(Q)$ drops. This outcome contributes to making banks loans cheaper and to increase investment. The third effect is the direct effect of i on the firm's chance to succeed. Since $Q'(i)>0$, this third effect also contributes to increase investment. Overall, $k'(i)<0$ whenever

$$\frac{\tau}{(1-\tau_m)}p(Q)+(i-r)\frac{\tau}{(1-\tau_m)}p'(Q)Q'(i)-(1-Q)\,F'(k)Q'(i)>0,$$

where $p'(Q)=-(1-(1+r)\tau)p(Q)^2$ and $F'(k)$ is given by (33). Hence, $k'(i)<0$ iff

$$\frac{\tau}{(1-\tau_m)}p(Q)-(i-r)\frac{\tau}{(1-\tau_m)}(1-(1+r)\tau)p(Q)^2Q'(i)-\frac{(1-Q)}{Q\left(1-\frac{1}{2}Q\right)}\left[1+(i-r)\frac{\tau}{(1-\tau_m)}p(Q)\right]Q'(i)>0.$$

With $i\approx r$, this condition becomes

$$\frac{\tau}{(1-\tau_m)}p(Q)-\frac{(1-Q)}{Q\left(1-\frac{1}{2}Q\right)}Q'(i)>0.$$

Turning to $Q'(i)$, when $i \approx r$, we have

$$\begin{aligned} Q'(i) &= \frac{2\lambda\sigma\tau\left[1-(1-\lambda)\tau_m\right]\left\{p(Q)\left[1-(1+r)\tau p(Q)\right]+(1+r)(1-\tau_m)p'(Q)Q'(i)\right\}}{(1-\tau_m)\left[(2-\sigma)(1-(1-\lambda)\tau_m)+(1+r)\lambda\sigma\tau p(Q)\right]^2} \\ &= \frac{2\lambda\sigma\tau\left[1-(1-\lambda)\tau_m\right]p(Q)\left[1-(1+r)\tau p(Q)\right]}{(1-\tau_m)\left\{\left[(2-\sigma)(1-(1-\lambda)\tau_m)+(1+r)\lambda\sigma\tau p(Q)\right]^2-2\lambda\sigma\tau(1+r)\left[1-(1-\lambda)\tau_m\right]p'(Q)\right\}} \\ &= \frac{2\lambda\sigma\tau(1-(1-\lambda)\tau_m)(1-(1+r)\tau p(Q))p(Q)}{(1-\tau_m)\left\{\begin{array}{c}\left[(2-\sigma)\left[1-(1-\lambda)\tau_m\right]+(1+r)\lambda\sigma\tau p(Q)\right]^2 \\ +2\lambda\sigma\tau(1+r)\left[1-(1-\lambda)\tau_m\right]\left[1-(1+r)\tau\right]p(Q)^2\end{array}\right\}}, \end{aligned}$$

where we used $p'(Q) = -\left[1-(1+r)\tau\right]p(Q)^2$. Hence, checking $\frac{\tau}{(1-\tau_m)}p(Q) - \frac{(1-Q)}{Q\left(1-\frac{1}{2}Q\right)}Q'(i) > 0$ amounts to verifying that

$$\begin{aligned} &\left\{\begin{array}{c}\left[(2-\sigma)(1-(1-\lambda)\tau_m)+(1+r)\lambda\sigma\tau p(Q)\right]^2 \\ +2\lambda\sigma\tau(1+r)(1-(1-\lambda)\tau_m)(1-(1+r)\tau)p(Q)^2\end{array}\right\} \\ > \;& \frac{(1-Q)}{Q\left(1-\frac{1}{2}Q\right)}\left\{2\lambda\sigma(1-(1-\lambda)\tau_m)(1-(1+r)\tau p(Q))\right\}. \end{aligned}$$

If $\tau = \tau_m = 0$, then we need to verify

$$(2-\sigma)^2 \quad > \quad \frac{(1-Q)}{Q\left(1-\frac{1}{2}Q\right)}2\lambda\sigma.$$

Since $i \approx r$ and $\tau = \tau_m = 0$, we have

$$Q = \frac{2(1-\sigma)}{(2-\sigma)}.$$

Hence, we need to verify

$$\begin{aligned} (2-\sigma)^2 \quad &> \quad \frac{\left(\frac{\sigma}{2-\sigma}\right)}{(1-\sigma)}(2-\sigma)^2\lambda\sigma, \\ \text{or, } 2-3\sigma+(1-\lambda)\sigma^2 \quad &> \quad 0. \end{aligned}$$

This condition will be satisfied whenever $\sigma < 2/3$. We conclude that $k'(i) < 0$ for when $i \approx r$, $\tau_m = 0$ and $\tau = 0$ if $\sigma < 2/3$. However, as the proof shows, this outcome is a sufficient condition and not at all necessary.

H Comparative statics $k'(\tilde{\tau}) \geq 0$ whenever $i \approx r$.

The derivative $k'(\tilde{\tau})$ has the same sign as $k'(\tau)$. We showed that $Q'(\tau) > 0$. Then, from

$$Q\left(1-\frac{1}{2}Q\right)F'(k) = 1 + (i-r)\frac{\tau}{(1-\tau_m)}p(Q),$$

we have

$$(1-Q)\,F'(k)Q'(\tau)+Q\left(1-\frac{1}{2}Q\right)F''(k)k'(\tau)=(i-r)\frac{\tau}{(1-\tau_m)}p'(Q)Q'(\tau)+\frac{(i-r)}{(1-\tau_m)}p(Q).$$

So

$$Q\left(1-\frac{1}{2}Q\right)F''(k)k'(\tau)=\underbrace{\frac{(i-r)}{(1-\tau_m)}p(Q)}_{\text{direct effect}}+\underbrace{(i-r)\frac{\tau}{(1-\tau_m)}p'(Q)Q'(\tau)}_{\text{effect on deposits value}}-\underbrace{(1-Q)\,F'(k)Q'(\tau)}_{\text{effect on effort}}.$$

With $\tau_m=0$ we have

$$Q=1-\frac{\sigma\left\{1+\left[(i-r)-\lambda(1+i)\right]\tau p(Q)\right\}}{(2-\sigma)+(i-r)\tau p(Q)\,(2-\sigma)+\sigma(1+i)\lambda\tau p(Q)},$$

and

$$Q'(\tau) \;=\; \sigma\frac{2(1+i)\lambda\left(p(Q)+\tau p'(Q)Q'(\tau)\right)}{\left[\left[1+(i-r)\tau p(Q)\right](2-\sigma)+\sigma(1+i)\lambda\tau p(Q)\right]^2}.$$

So

$$Q'(\tau)=\frac{2\sigma(1+i)\lambda p(Q)}{A^2-2\sigma(1+i)\lambda\tau p'(Q)},$$

where

$$A=\left[1+(i-r)\tau p(Q)\right](2-\sigma)+\sigma(1+i)\lambda\tau p(Q)$$

The denominator is positive since $p'(Q)<0$. Additionally, the nominator is positive and most importantly bounded away from zero, even when $i\approx r$. Hence, when $i\approx r$, only the effect on effort is important to determine $k'(\tau)$. Since $Q'(\tau)\geq 0$ and $F''(k)<0$, we obtain $k'(\tau)\geq 0$.

I Financing firms with deposits

In this appendix, we show that banks only finance firms with deposits whenever $i>r$ and that they only demand central bank money in order to satisfy their reserve requirement. To do so, we set $\tau_m=0$. We need to consider four cases: (1) banks do not borrow on the interbank market. Second banks borrow on the interbank market, and (2) banks do not default on any of their liabilities, (3) banks do not default on their deposits but partially default on their junior interbank liabilities, and (4) banks default on their deposits (and so on their interbank liabilities). We let k^o and k^n be the levels of the firm's investment financed with central bank money (k^o) and deposits (k^n). Additionally, we let p^o (respectively p^n) be the amount of central bank money (respectively deposits) that suppliers require to produce one unit of capital.

I.1 Banks do not borrow on the interbank market

Then, the outside option of banks is to earn interest on reserves:

$$\mathcal{O}(m^b)=(1+r)m^b.$$

As the firm takes all, the bank expects to receive a payoff equal to $(1+r)m^b$. Therefore, since $i > r$, the bank will not bring any cash to the banking market. With positive reserve requirements, the only equilibrium is autarky.

I.2 Banks borrow on the interbank market

Since the interbank market is active, the outside option of banks is to lend on the interbank market. Then, we have to consider three cases.

I.2.1 No default

The no-default condition is

$$(1+r)\left(m^b + b - k^o\right) - (1+i_m)\, b \quad \geq \quad p^n k^n. \tag{34}$$

In this case, banks do not default on their deposits so that $p^n = p^0 = 1$. Therefore, banks' outside is to lend risk free on the interbank market, with a return

$$\mathcal{O}(m^b) = (1+i_m)\, m^b.$$

Thus, it follows that the bank's problem with respect to m^b is

$$\max_{m^b} -(1+\pi)m^b + \beta\,(1+i_m)\, m^b,$$

where $1 + i_m = \frac{1+\pi}{\beta} = 1 + i > 1 + r$. Using $p^n = 1$, the bargaining problem is

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^o,b,\phi} \left\{ q\left[\left(1 - \frac{1}{2}q\right) F(k^n + k^o) - \phi - k^n - k^o\right]\right\},$$

subject to

$$\tilde{\tau} k^n \leq (1+r)\left(m^b + b - k^o\right),$$

$$q\left[k^n + k^o + \phi\right] + (1+r)\left(m^b + b - k^o\right) - (1+i_m)\, b - k^n = (1+i_m)m^b,$$

and $k^o, k^n \geq 0$.

Given (34) and $p^n = 1$, the reserve constraint cannot bind as $\tilde{\tau} \leq 1$. However, then the FOC with respect to b gives $1 + r = 1 + i_m$, which contradicts $i > r$. Therefore there cannot be an equilibrium where banks do not default on their interbank loans and $i > r$.

I.2.2 No default on deposits, but default on interbank loans

Banks do not default on their deposits whenever

$$p^n k^n \leq (1+r)(m^b + b - k^o).$$

However, they default on their interbank loans whenever

$$(1+r)(m^b+b-k^o) < p^n k^n + (1+i_m)\, b.$$

Since banks do not default on their deposits, $p^n = 1$. Therefore banks' outside option is

$$\mathcal{O}(m^b) = Q\,(1+i_m)\,m^b + (1-Q)\left[(1+r)\left(\tilde{m}^b + \tilde{b} - \tilde{k}^o\right) - \tilde{k}^n\right]\frac{m^b}{\tilde{b}}.$$

Plugging this expression back in the problem of the young banker in the centralized market, we obtain $i_m > i$ (which only holds with equality if banks do not default on their interbank loans). Using $p^n = 1$, the bargaining problem is given by

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^0,b^1,\phi} \left\{ q\left[\left(1-\frac{1}{2}q\right)F(k^n+k^o) - \phi - k^n - k^o\right]\right\},$$

subject to

$$\tilde{\tau}k^n \leq (1+r)\left(m^b + b - k^o\right),$$

$$q\left[k^n + k^0 + \phi + (1+r)\left(m^b+b-k^o\right) - (1+i_m)\,b - k^n\right] = \mathcal{O}(m^b),$$

and $k^0, k^n \geq 0$. Again, as the bank does not default on deposits, the reserve requirement constraint is not binding. Using the IC of the bank to replace for ϕ in the objective function, it becomes

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^0,b^1,\phi} q\left(1-\frac{1}{2}q\right)F(k^n+k^o) - \mathcal{O}(m^b) + q\left[(1+r)\left(m^b+b-k^o\right) - (1+i_m)\,b - k^n\right].$$

and the FOC with respect to b gives $r = i_m$. However, this value cannot be an equilibrium because $i_m > i > r$.

I.2.3 Default on deposits and interbank loans

Banks default on their deposits whenever

$$p^n k^n \quad \geq \quad (1+r)\left(m^b + b^1 - k^0\right).$$

The outside option of banks is

$$\mathcal{O}(m^b) = Q\,(1+i_m)\,m^b.$$

Therefore, the banks problem yields $1 + i_m = (1+i)/Q$. The bargaining problem is given by

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^o,b,\phi} \left\{ q\left[\left(1-\frac{1}{2}q\right)F(k^n+k^o) - \phi - p^n k^n - k^o\right]\right\}$$

subject to

$$\tilde{\tau}p^n k^n \leq (1+r)\left(m^b + b - k^0\right),$$

$$q\left[p^n k^n + k^o + \phi + (1+r)\left(m^b+b-k^o\right) - (1+i_m)\,b - p^n k^n\right] = (1+i)m^b,$$

and $k^o, k^n \geq 0$. Isolating ϕ in the last constraint and replacing in the objective function yields

$$\mathcal{P}(m^b) \equiv \max_{q,k^n,k^o,b,\phi} q\left(1-\frac{1}{2}q\right)F(k^n+k^o)-(1+i)m^b+q\left[(1+r)\left(m^b+b-k^o\right)-(1+i_m)\,b-p^nk^n\right],$$

subject to

$$\tilde{\tau}p^nk^n \leq (1+r)\left(m^b+b-k^0\right).$$

Since $i > r$, it must be that the reserve requirement constraint is binding. Otherwise, the first order condition with respect to b would give $i = r$ (see above). Therefore, we can use the constraint to replace b in the objective function. Then, it becomes,

$$\begin{aligned}\mathcal{P}(m^b) \equiv \max_{q,k^n,k^o,\phi} & \; q\left(1-\frac{1}{2}q\right)F(k^n+k^o)-(1+i)m^b \\ & + q\left[\tilde{\tau}p^nk^n-(1+i_m)\,\frac{\tilde{\tau}p^nk^n-(1+r)(m^b-k^0)}{(1+r)}-p^nk^n\right].\end{aligned}$$

The first order conditions are

$$\begin{aligned}\boldsymbol{q}:\;(1-q)\,F(k^n+k^o)+\tilde{\tau}p^nk^n-(1+i_m)\,\frac{\tilde{\tau}p^nk^n-(1+r)(m^b-k^0)}{(1+r)}-p^nk^n=0,\\ \boldsymbol{k^n}:\;q\left(1-\frac{1}{2}q\right)F'(k^n+k^o)-q\left[\frac{(1+i_m)}{(1+r)}\tilde{\tau}p^n+(1-\tilde{\tau})p^n\right]+\lambda_{k^n}=0,\\ \boldsymbol{k^0}:\;q\left(1-\frac{1}{2}q\right)F'(k^n+k^o)-q\,(1+i_m)+\lambda_{k^o}=0.\end{aligned}$$

Since the reserve requirement binds, suppliers charge a risk premium, and

$$p^n \;=\; \frac{1}{Q}-\frac{1-Q}{Q}\,(1+r)\,\frac{m^b+b^1-k^0}{k^n}=\frac{1}{Q}-\frac{1-Q}{Q}\tilde{\tau}p^n,$$

so that

$$p^n=\frac{1}{Q+(1-Q)\tilde{\tau}},$$

and the FOC with respect to k^n becomes

$$\boldsymbol{k^n}:\qquad q\left(1-\frac{1}{2}q\right)F'(k^n+k^o)-\frac{q}{Q+(1-Q)\tilde{\tau}}\left[\frac{(1+i)}{Q(1+r)}\tilde{\tau}+1-\tilde{\tau}\right]+\lambda_{k^n} \;=\; 0,$$

while the FOC with respect to k^0 becomes

$$\boldsymbol{k^0}:\qquad q\left(1-\frac{1}{2}q\right)F'(k^n+k^o)-\frac{q}{Q}(1+i)+\lambda_{k^o} \;=0.$$

Therefore, $\lambda_{k^0} > \lambda_{k^n} = 0$ whenever

$$\begin{aligned} \frac{q}{Q}(1+i) &> \frac{q}{Q+(1-Q)\tilde{\tau}}\left[\frac{(1+i)}{Q(1+r)}\tilde{\tau}+1-\tilde{\tau}\right], \\ \text{or, } (1+i) &> \frac{1}{Q+(1-Q)\tilde{\tau}}\left[\frac{(1+i)}{(1+r)}\tilde{\tau}+Q(1-\tilde{\tau})\right]. \end{aligned}$$

This inequality is always true. Indeed, consider the upper bound of the RHS by setting $r = 0$. Then, rewrite the RHS upper bound as $\frac{1}{Q+(1-Q)\tilde{\tau}}\left[i\tilde{\tau}+Q+(1-Q)\tilde{\tau}\right]$. Since $\tilde{\tau} \leq 1$ we obtain that $\tilde{\tau} < Q+(1-Q)\tilde{\tau}$. This shows that the upper bound of the RHS, and so the RHS itself is less than $1+i$. Therefore, we conclude that whenever $i > r \geq 0$ then $k^0 = 0$ and $k^n > 0$. $k'(\tau) \geq 0$ whenever $i \approx r$.