Savings, asset scarcity, and monetary policy*

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Abstract

The goal of this paper is to analyze monetary policy in a model where both money and savings are essential and asset markets matter. I build an overlapping generations model where agents need money to trade in some transactions because of a lack of double-coincidence of wants, and they need to hold assets to provide for consumption later in life because they can only work when young. The model shows that risky assets can be valued above their unconstrained prices if interest rates are low, i.e. a premium on these assets emerges through an increase in the stochastic discount factor. To increase welfare, the monetary authority should run the Friedman rule or, if that is not feasible, the fiscal authority should issue enough bonds. If that is not feasible either, the monetary authority can increase welfare by increasing inflation in good states while decreasing it in bad states. This finding goes against the conventional wisdom that inflation should be countercyclical.

Keywords: *New monetarism, overlapping generations, bubbles, optimal stabilization*

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Preliminary and incomplete - please do not circulate

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1 Introduction

After the financial crisis of 2007-2009, the zero lower bound became a major topic in monetary economics, as researchers wanted to understand what caused the decrease in interest rates. In the new monetarist literature pioneered by Lagos and Wright (2005), several studies already showed that assets can be valued above their unconstrained value (the value the assets would take if their supply was infinitely elastic) if they provide liquidity services to at least some agents in the economy. This additional valuation is typically called the liquidity premium. Typical examples are Lagos and Rocheteau (2008) for capital and Geromichalos et al. (2007) for real assets. Therefore, it was natural to explain the occurrence of the zero lower bound by a similar mechanism, namely a liquidity premia on government bonds, as in Williamson (2012, 2016) or Altermatt (2017). These studies show that the zero lower bound occurs if the liquidity premia become so high that government bonds pay no interest rates in equilibrium, which makes them perfectly replaceable by money.

Interestingly, in most of these studies the main role of assets was neglected: That these assets are used by agents to save. In this paper, I want to bridge this gap and study the savings motive in a monetary model where saving is essential, in the sense that it is impossible to achieve the first-best if agents don't save. To save, agents can use nominal, safe assets (government bonds), real, risky assets (stocks) or fiat money. In a simple model, I analyze whether assets can also be priced above their unconstrained value if they provide no liquidity services, but are essential for savings. The answer is yes, and I dub the premium that emerges in this case as the *savings premium*. Similarly to the liquidity premium, the savings premium on nominal assets can be so high that an economy ends up at the zero lower bound. A savings premium on risky assets can also be interpreted as a bubble. Interesting questions that this model is able to answer in regards to bubbles are under what circumstances bubbles emerge, whether the zero lower bound always leads to bubbles on risky assets, and whether bubbles are more or less likely with high inflation rates.

I see this paper as a first step towards making savings decisions and savings motives more relevant in monetary economics. To understand the savings premium, I study monetary policy in a model where the savings premium is the only novel mechanism compared to standard models. As a next step, the savings premium and the liquidity premium should be integrated into a single model. As this paper shows that many effects related to liquidity premia can also be generated through a savings premium, it is to be expected that in a model where both effects are present, the risk of ending up in a bubble or at the zero lower bound becomes even more severe.

To make savings essential, I put the Lagos and Wright (2005) (LW) framework into a simple overlapping-generations (OLG) model based on Wallace (1980). This approach is interesting because savings motives are easy to incorporate in OLG-models, and the LW structure allows to make money essential even if it is not used for savings. There have been a few other papers that combined the OLG structure with the money search environment of LW, such as Zhu (2008), Jacquet and Tan (2011), Waller (2009), or Hiraguchi (2017). However, the exact details of which agents (young or old, buyers or sellers) enter which markets differ in each of these papers, and their research questions have also been different. In this model, buyers live for two periods, but they can only work in the first period, while sellers are infinitely-lived. To provide for consumption in the second period, buyers have to save by holding some assets. There are three assets in the model, namely fiat money, nominal government bonds, and risky assets (equity). I first study economies with fiat money and either only government bonds or only risky assets available to understand some basic concepts, before analyzing an economy with all three assets present. Fiat money is essential for intra-period trade, but it is typically dominated in terms of the rate of return by the other assets, so it is not used for savings in equilibrium, except at the zero lower bound. Since buyers are risk-averse, they prefer holding government bonds over risky assets to save for consumption in the second period for a given rate of return. However, if the interest rate on bonds is not fully compensating for inflation and discounting (i.e. if there is a shortage of bonds), the risky assets start being used for savings as well. The higher the savings premium on bonds, the more risky assets are demanded by buyers. At some point, the demand for risky assets exceeds the supply at the unconstrained price, meaning that the price has to increase above the unconstrained level to clear the market. Thus, risky assets also pay a savings premia, which can be considered a bubble. That such savings premia, or bubbles, can exist in OLG frameworks is well known and was shown amongst others by Tirole (1985).

The model exhibits four types of equilibria, depending on parameter values. Some of them can be considered *bubble equilibria*, *zero lower bound equilibria*, or both. By issuing enough government bonds, the fiscal authority can prevent both a zero lower bound and bubbles. If for some reason the fiscal authority is not willing to issue enough bonds, the monetary authority can use an optimal stabilization policy in which it applies different inflation rates after shocks on the equity market. However, this policy is procyclical, i.e. inflation should be set higher after positive shocks on the stock market and lower after negative shocks.

This paper is closely related to the work by Caballero et al. (2017) and especially Caballero and Farhi (2017). In their paper, Caballero and Farhi show that a shortage of safe assets can lead to a situation they label the safety trap, which has similar but even more severe effects than a standard liquidity trap. Such a safety trap is deflationary and leads to sharp decreases in output in their model. To get these results, Caballero and Farhi add nominal rigidities, two types of agents and financial frictions to a perpetual youth OLG model. Compared to that, my approach is much simpler. As there are no nominal rigidities in my paper, I cannot study output reactions. However, the simple approach allows me to study the effects on prices of assets and bonds from either changes in the supply of these assets or changes in monetary policy.

Other papers studying the effects of shortages of safe assets are Caballero et al. (2008), Caballero and Krishnamurthy (2009), Bernanke et al. (2011), Barro and Mollerus (2014), and He et al. (2015). The macroeconomic effects caused by a shortage of safe assets found in these studies mostly corresponds to the results from Caballero and Farhi (2017).

The paper is also closely related to the literature on liquidity traps, e.g. Krugman et al. (1998), Eggertsson and Woodford (2003, 2004), Eggertsson and Krugman (2012), Williamson (2012, 2016), Rocheteau et al. (2016), Guerrieri and Lorenzoni (2017), Cochrane (2017), or Altermatt (2017). While some of these papers (e.g. Williamson (2012, 2016)) focus on the liquidity services provided by bonds, others like Eggertsson and Krugman (2012) explain the liquidity trap by financial frictions like tightened borrowing constraints. My model shows that you can get stuck at the zero lower bound with neither of those, but because certain assets are essential for savings and thus agents are willing to pay a premium for these assets.

The rest of the paper is organised as follows. In section 2, the basic environment is explained. Section 3 presents the economy with only government bonds and fiat money, while section 4 presents an economy with only risky assets and fiat money. In section 5, an economy with all three assets is analyzed, and section 6 concludes.

2 The basic environment

Time is discrete and continues forever. Each period is divided into two subperiods, called CM and DM. At the beginning of a period, the CM takes place, and after the CM closes, the DM opens and remains open until the period ends. In any period t, there is a measure N_t of buyers born. Buyers live for two periods. There is also a measure N_t infinitely lived sellers alive in period t. The population growth rate is defined as $\frac{N_t}{N_{t-1}} = n$. Buyers are able to produce a special good x when they are young in the CM. Sellers can produce a special good q in the DM that gives utility to buyers. In the DM, buyers and sellers are matched bilaterally, and buyers can make take it or leave it offers. In the CM, there exists a centralized market for the general goods produced by young buyers, and sellers as well as old buyers get utility from consuming them. Neither general goods nor special goods can be stored by agents. The preferences of buyers are given by:

$$\mathbb{E}_t\Big\{-h_t^y + u(q_t^y) + \beta[U(x_{t+1}^o) + u(q_{t+1}^o)]\Big\} \tag{1}$$

1 states that buyers get disutility h from producing in the CM, get utility $u(q)$ from consuming in the DM and $U(x)$ from consuming in the CM when they are old, and that they discount the second period of their life by a factor $\beta \in (0,1)$. Consumption that occurs in the first period of their life (when they are young) is indicated by a superscript y, while consumption that occurs in the second period of their life (when they are old) is indicated by a superscript o. Assumptions on the utility functions are that $U(0) = u(0) = 0$, $U'(0) = u'(0) = \infty$, $u'(q) > 0$, $u''(q) < 0$, $U'(x) > 0$, and $U''(q) < 0$.

The preferences of the sellers are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (x_t^s - h_t^s - c(q_t)) \tag{2}$$

2 states that sellers discount future periods by a factor $\beta \in (0,1)$, get linear utility from consuming x in the CM, a linear disutility h from producing in the CM, and disutility $c(q)$ from producing in the DM, with $c(0) = 0$, $c'(0) = 0$, $c'(q) > 0$, $c''(q) > 0$, and $c(\bar{q}) = u(\bar{q})$ for some $\bar{q} > 0$. Furthermore, I define q^* as $u'(q^*) = c'(q^*)$ and x^* as $U'(x^*) = 1$, i.e. as the socially efficient quantities. Variables of the sellers are indicated by a superscript s.

I assume that sellers cannot commit to any future payments. This prevents them from issuing assets to insure buyers against future consumption risk. As buyers are risk-averse towards CM consumption while sellers are risk-neutral, there would be room for such contracts in general.

The monetary authority issues fiat money M_t, which it can costlessly produce. Actions of the monetary authority always take place at the beginning of the period. The amount of general goods one unit of fiat money can buy in the CM of period t is denoted by ϕ_t, the inflation rate is defined as $\phi_t/\phi_{t+1} - 1 = \pi_{t+1}$, and the growth rate of fiat money from period $t-1$ to t is $\frac{M_t}{M_{t-1}} = \gamma_t$. Newly-printed money is distributed lump-sum to young buyers. The real value of these transfers is denoted as Δ_t. Agents' money holdings will be denoted by m_t, and agents' real money balances are denoted by $z_t = \phi_t m_t$.

The fiscal authority has to finance some spending g_t in each period, and can do so by levying lump-sum taxes T_t or issuing one-period bonds. If the government issues bonds, they are sold for the market-clearing price $\rho_{b,t}$ and redeemed for one unit of fiat money in the next period. This gives rise to the following government budget constraint:

$$\phi_t \rho_{b,t} B_t + \left(\frac{1}{n} + 2\right) N_t T_t = \phi_t B_{t-1} + g_t \tag{3}$$

It is assumed that the government exogenously decides whether to finance its expenditures by debt or taxes. Specifically, I will assume in some sections that the supply of bonds is either zero or strictly positive in all periods. I define the net real lump-sum tax to agents as $\tau_t = T_t - \Delta_t$.

In stationary equilibria, only the real supply of bonds $\phi_t B_t$ is constant. As ϕ_t is determined endogenously and depends on the money supply, it is often convenient to use the bonds-to-money ratio $\mathcal{B}_t = \frac{B_t}{M_t}$ instead of the nominal bond supply.

In section 5, I assume that there is also an endowment of risky assets with aggregate value A_t available in the economy. For simplicity, I will assume that the sellers are endowed with the risky assets at the beginning of the CM of each period. These risky assets are perfectly divisible. In the following period, the assets pay a high return κ^H with probability χ and a low return κ^L with probability $1 - \chi$. The return is an aggregate shock, i.e. when a return is realized in a given period, it is the same for all the assets in the economy. Thus, there is also no private information about the return of an asset. After the realization of the shock, the assets pay out the real return and cease to exist, i.e. they are replaced by a new set of assets that is independent of the old one in each period.

These assets should be interpreted as something like the aggregate stock market, i.e. A_t represents the unconstrained value of all outstanding equities in period t. Then, $\kappa_{t+1} A_t$ is the unconstrained value of the aggregate stock market in the next period, and an investor that held a balanced portfolio of the market realized the return κ_{t+1}[1].

In the next section, I will analyze an economy where only bonds are available for agents to save. After that, I will look at an economy where both bonds and risky assets are present.

[1] Alternatively, the asset can also be interpreted as potatoes: If there are A_0 potatoes available in the initial period, planting all of them will yield $\kappa_t A_0$ potatoes in period 1. These potatoes can then either be eaten or invested (i.e. put into the ground) again. As I will show later, agents will never prefer to reduce the stock of assets (i.e. to eat the potatoes).

3 The economy with bonds

In this section, I will assume that the fiscal authority finances some share of its expenditure according to equation 3 by issuing bonds, and the bonds are sold for the price that clears the market. Furthermore, I assume that there are no risky assets available in this economy, i.e. $A_t = 0\ \forall t$, so the only available assets are bonds and fiat money.

3.1 The buyer's problem

A buyer has to decide how much goods he wants to produce in the first CM, in order to acquire fiat money and bonds for consumption in the later parts of his life. A buyer's value function when he is young $W^{y,b}$ is:

$$W^{y,b} = \max_{h_t^y, m_t^y, b_t^y, d_t^y} -h_t^y + u(q(\phi_{t+1} d_t^y)) + \beta W^{o,b}(b_t^y, m_t^y)$$
$$s.t. \quad h_t^y - \tau_t = \phi_t (m_t^y + d_t^y + \rho_{b,t} b_t^y)$$

Here, h_t denotes the goods produced in the CM, d_t^y denotes the money holdings of a young buyer he plans to take to the DM, m_t^y denotes his money holdings used to save for the next period, and b_t^y denotes bond holdings of a young buyer. Note that the DM money and the savings money is the same object, but using two different variables for them makes it simpler to write down the problem. The function $q(\phi_{t+1} m_t^y)$ depends on the terms of trade in the DM and will be made explicit later. Finally, $W^{o,b}(b_t)$ denotes the value function of an old buyer with bond holdings b_t, which is simply:

$$W^{o,b}(b_t^y) = \max_{x_{t+1}^o} U(x_{t+1}^o)$$
$$s.t. \quad \phi_{t+1}(m_t^y + b_t^y) = x_{t+1}^o$$

Here, x_{t+1}^o denotes the CM consumption of an old buyer. The old buyer just sells all his assets in the CM and consumes the rewards.

By substituting in the constraints and the value function of the old buyer, we can get the lifetime value function of a buyer:

$$W^b = \max_{d_t^y, m_t^y, x_{t+1}^o} -\left(\phi_t d_t^y + \phi_t (1 - \rho_{b,t}) m_t^y + \frac{\phi_t}{\phi_{t+1}} \rho_{b,t} x_{t+1}^o\right) + u(q(\phi_t m_t^y)) + \beta U(x_{t+1}^o) \qquad (4)$$

3.1.1 The DM problem

To solve for the first-order conditions of the buyer's problem, we need to know the terms of trade in the DM. As stated above, it is assumed that buyers can make take it or leave it offers to sellers. Their offer has to satisfy the sellers' participation constraint. This gives rise to the following maximization problem:

$$
\begin{aligned}
&\max_{q_t,d_t} \quad u(q_t) - \beta\phi_{t+1}d_t \\
&s.t. \quad -c(q) + \beta\phi_{t+1}d_t \geq 0
\end{aligned}
$$

The sellers' participation constraint will always be binding, so the solution to this problem is:

$$
q = c^{-1}(\beta\phi_{t+1}d_t) \tag{5}
$$

3.1.2 Solution to the buyer's problem

Now we can substitute equation 5 into the buyer's lifetime value function given by 4 and solve the maximization problem to get the following first-order conditions:

$$
\begin{aligned}
d_t^y &: 1 = \beta\frac{\phi_{t+1}}{\phi_t}\frac{u' \circ c^{-1}(\beta\phi_{t+1}d_t^y)}{c' \circ c^{-1}(\beta\phi_{t+1}d_t^y)} \\
m_t^y &: 1 \geq \rho_{b,t} \\
x_{t+1}^o &: 1 = \frac{\beta}{\rho_{b,t}}\frac{\phi_{t+1}}{\phi_t}U'(x_{t+1}^o)
\end{aligned}
$$

The first-order condition for DM money is the standard result from Lagos and Wright (2005) and captures the trade-off between consumption and the inflation tax. The first-order condition for money as a means of saving says that agents only use it if the bond price equals 1. The first-order condition on consumption when old yields a pricing formula for bonds, which says that the bond price depends on inflation, discounting, and a stochastic discount factor, which is 1 if the economy is able to reach the first-best, i.e. in an unconstrained case.

3.2 Bond market clearing

For the bond market to clear, the price of bonds has to adjust such that agents are willing to hold all bonds. The demand for bonds b_t^y by buyers is given by:

$$b_t^y = \frac{x_{t+1}^o}{\phi_{t+1}} - m_t^y$$

This can be seen directly in the constraint to the value function of old buyers. Since only young buyers demand bonds, total demand for bonds by buyers is given by $N_t b_t^y$.

Sellers will only hold bonds if there is no cost to hold them. This means that sellers only hold bonds if $\rho_{b,t} \leq \frac{\beta}{1+\pi_{t+1}}$. However, if $\rho_{b,t} < \frac{\beta}{1+\pi_{t+1}}$, sellers want to hold an infinite amount of bonds. Since the supply of bonds is finite, the price of bonds will be driven up until $\rho_{b,t} = \frac{\beta}{1+\pi_{t+1}}$, which if we interpret $1/\beta$ as the real interest rate, can be called the Fisher equation (Fisher, 1930). This creates a lower bound on the price of bonds. Whenever the bonds are priced above this lower bound, they exhibit a savings premium. The amount of bonds held by an individual seller is denoted as b_t^s, so the total demand for bonds by sellers is $N_t b_t^s$. Now we can add up the total demand for bonds to determine the market clearing condition:

$$x_{t+1}^o - \phi_{t+1} m_t^y + \phi_{t+1} b_t^s = \frac{\phi_{t+1} B_t}{N_t} \tag{6}$$

$$\text{with } b_t^s = 0 \quad \text{if } \rho_{b,t} > \frac{\beta}{1+\pi_{t+1}}, \text{ and } b_t^s = \left(\frac{B_t}{N_t} + m_t^y - \frac{x_{t+1}^o}{\phi_{t+1}}\right) \text{ otherwise}$$

Equation 6 shows that an increase in the supply of bonds B_t has to be offset by an increase in x_{t+1}^o if $\rho_{b,t} > \frac{\beta}{1+\pi_{t+1}}$. From the solution to the buyer's problem, we know that x_{t+1}^o is decreasing in the price of bonds. Thus, an increase in the supply of bonds will result in a decrease in the price of bonds. This mechanism is at work until the supply of bonds is high enough for the bond price to fall to $\rho_{b,t} = \frac{\beta}{1+\pi_{t+1}}$. From that point onwards, a further increase in the supply of bonds will just lead to an increase in b_t^s. I therefore label this lower bound for bond prices as the unconstrained bond price, and it is defined by $\underline{\rho}_b \equiv \frac{\beta}{1+\pi_{t+1}}$. The corresponding minimal amount of bonds required to reach $\underline{\rho}_b$ is denoted as $\mathcal{B}^*$.

While an increase in the supply of bonds leads to a decrease in the price of bonds if that price is not yet at the lower bound, a decrease in bond supply leads to an increase in the price of bonds. However, the price of bonds is also bounded above, namely by $\rho_{b,t} = 1$. This is because at that price, holding bonds and fiat money is equally costly. Since fiat money is equally good as a means of savings as bonds, given the price of the two assets is the same, agents are never willing to pay a higher price for bonds than this. Instead, they would start using fiat money to save if the supply of bonds is not high enough for the bond market to clear at a price $\rho_{b,t} = 1$. I will denote the maximal amount of bonds that leads to a bond price of $\rho_{b,t} = 1$ as $\underline{\mathcal{B}}$.

Note that at the Friedman rule ($1 + \pi_{t+1} = \beta$), the upper and lower bound of the bond price collapse into one, leaving only $\rho_{b,t} = 1$ as a possibility.

3.3 Money market clearing

Next, we can state the market clearing condition for fiat money:

$$\phi_t M_t = N_t z^y \tag{7}$$

Here, $z^y = \phi_t(m_t^y + d_t^y)$, so the right hand side denotes the total real demand for fiat money, given by the real balances of young buyers. Sellers acquire no money in the CM. This has to equal the supply of fiat money.

3.3.1 Steady-state inflation

In a steady state, the rate of return of money is constant over time and equals:

$$\frac{1}{1+\pi_{t+1}} = \frac{\phi_{t+1}}{\phi_t} = \frac{\frac{N_{t+1}z_t^y}{M_{t+1}}}{\frac{N_t z_t^y}{M_t}} = \frac{n}{\gamma} \tag{8}$$

3.4 Steady-state equilibrium

Since the buyer's first order conditions are in terms of both period t and period $t+1$ variables, we have to adjust those that occur in terms of period $t+1$ to get the equations relevant for equilibrium in any given period. Additionally, we can also plug in the steady-state value for the rate of return of money. Thus, the relevant conditions are:

$$1 = \beta\frac{n}{\gamma}\frac{u' \circ c^{-1}(\beta\phi_{t+1}d_t^y)}{c' \circ c^{-1}(\beta\phi_{t+1}d_t^y)} \tag{9}$$

$$1 \geq \rho_{b,t} \tag{10}$$

$$1 = \frac{\beta}{\rho_{b,t-1}}\frac{n}{\gamma}U'(x_t^o) \tag{11}$$

Now we can define an equilibrium in this economy:

Definition 1. *An equilibrium is a sequence of prices $\rho_{b,t}$, and quantities $\phi_t m_t^y, \phi_t d_t^y, x_t^o$, and b_t^s, that simultaneously solve the equations 9 and 11, as well as the inequalities 10 and 6 and the corresponding complementary slackness conditions $\forall t$.*

Since the left-hand side of the bond-market clearing condition (equation 6) consists only of real variables, its right-hand side has to be constant over time in a steady state. This implies:

$$\frac{B_{t+1}}{B_t} = \frac{N_{t+1}}{N_t}\frac{\phi_{t+1}}{\phi_{t+2}} = n \cdot \frac{\gamma}{n} = \gamma$$

This shows that the growth rate of bonds has to equal the growth rate of fiat money for a steady-state to exist, which corresponds to a constant bonds-to-money ratio $\mathcal{B}_t = \mathcal{B}\ \forall t$.

As it can be seen easily in equation 9, the real balances chosen by young buyers only depend on the inflation rate prevalent in the economy. Old buyers CM consumption however does also depend on the price of bonds and thus on the bond market clearing. If the bond supply in this economy is plentiful, the demand for bonds cannot be absorbed by buyers only, and thus the bond price will be set according to the Fisher equation, which allows for first-best consumption in the CM according to equation 11. If bonds get scarce and their price increases above the Fisher equation,the CM consumption of old buyers decrease.

As just shown, the model with only bonds and fiat money is relatively simple and straightforward. However, the concepts developed about bond market clearing still hold in the full model. In the next section, I want to analyse an economy with only risky assets and fiat money, to again show some basic concepts. After that, I will put everything together.

4 The economy with risky assets

In this section, I will assume that the fiscal authority finances all its expenditures by raising taxes and does not issue any bonds. However, there is some endowment $A_t > 0$ of risky assets held by sellers each period. Thus, buyers can only use risky assets or fiat money to save in this version of the model. Before analyzing the buyers' problem, I want to analyze the nature of the risky assets a little further.

In the absence of any savings premia, risk-neutral agents would price these assets according to the following equation. Let this be called the unconstrained price of risky assets:

$$\rho_{a,t} = \beta \left(\chi \kappa^H + (1 - \chi) \kappa^L \right) \tag{12}$$

For the remainder of this paper, I will assume that $1 = \beta(\chi\kappa^H + (1 - \chi)\kappa^L)$. This normalizes the unconstrained price of the risky asset to 1, so whenever a price $\rho_{a,t} > 1$ is observed, the asset is traded above its unconstrained value and exhibits a savings premium. Note that this also implies $\kappa^H \geq \frac{1}{\beta}$ and $\kappa^L \leq \frac{1}{\beta}$.

At the unconstrained price $\rho_{a,t} = 1$, a seller is indifferent about holding the asset or selling it. Thus whenever the assets are priced at their unconstrained value, sellers are absorbing any risky assets that are not demanded by buyers. At $\rho_{a_t} > 1$, sellers strictly prefer to sell the assets, thus to observe such prices, all risky assets must be held by buyers.

4.1 The buyers' problem

The buyers' problem here is pretty similar to the one analyzed in the previous section, except that we now have to also take the riskiness of the assets into account. A young buyer solves the problem:

$$W^{y,a} = \max_{h_t^y, d_t^y, m_t^y, a_t^y} -h_t^y + u(q(\phi_{t+1} d_t^y)) + \beta \left[\chi W^{o,a}(\kappa^H a_t^y, m_t^y) + (1-\chi) W^{o,a}(\kappa^L a_t^y, m_t^y) \right]$$

$$s.t. \quad h_t^y - \tau_t = \phi_t(m_t^y + d_t^y) + \rho_{a,t} a_t^y$$

For old buyers, the problem is simply:

$$W^{o,a}(\kappa_{t+1} a_t^y, m_t^y) = \max_{x_{t+1}^o} U(x_{t+1}^o)$$

$$s.t. \quad x_{t+1}^o = \kappa_{t+1} a_t^y + \phi_{t+1} m_t^y$$

Now we can combine the two value functions into a lifetime problem by substituting out the assets[2] in the two budget constraints:

$$W^a = \max_{\substack{h_t^y, d_t^y, \\ x_{t+1}^H, x_{t+1}^L, m_t^y \geq 0}} - h_t^y + u(q(\phi_{t+1} d_t^y)) + \beta \left[\chi U(x_{t+1}^H) + (1-\chi) U(x_{t+1}^L) \right]$$

$$s.t. \quad h_t^y - \tau_t = \phi_t(m_t^y + d_t^y) + \frac{\rho_{a,t}}{\kappa^H}(x_{t+1}^H - \phi_{t+1} m_{t+1}) \qquad (\lambda^H)$$

$$s.t. \quad h_t^y - \tau_t = \phi_t(m_t^y + d_t^y) + \frac{\rho_{a,t}}{\kappa^L}(x_{t+1}^L - \phi_{t+1} m_{t+1}) \qquad (\lambda^L)$$

The buyer's maximization problem gives rise to the following first-order conditions:

[2]The problem what to substitute to combine the value functions is not trivial, as depending on the rates of return, buyers might choose to only use one asset to save. As will be shown later, buyers are willing to use a positive amount of risky assets for their savings as long as the expected return of risky assets is higher than the return of fiat money. For any inflation rate above the Friedman rule, this condition is fulfilled at the unconstrained asset price. This analysis thus does not hold at the Friedman rule, but at any other feasible rate of inflation.

$$h_t: \quad 1 = \lambda^H + \lambda^L$$

$$d_t^y: \quad \phi_t(\lambda^H + \lambda^L) = \beta \phi_{t+1} \frac{u' \circ c^{-1}(\beta \phi_{t+1} d_t^y)}{c' \circ c^{-1}(\beta \phi_{t+1} d_t^y)}$$

$$m_t^y: \quad \frac{\phi_t}{\phi_{t+1}}(\lambda^H + \lambda^L) \geq \frac{\rho_{a,t}}{\kappa^H}\lambda^H + \frac{\rho_{a,t}}{\kappa^L}\lambda^L$$

$$x_{t+1}^H: \quad \lambda^H \frac{\rho_{a,t}}{\kappa^H} = \beta \chi U'(x_{t+1}^H)$$

$$x_{t+1}^L: \quad \lambda^L \frac{\rho_{a,t}}{\kappa^L} = \beta (1-\chi) U'(x_{t+1}^L)$$

I made use of the solution to the DM problem as derived in section 3.1.1, since that problem is not affected by the type of assets present in the economy. By replacing the Lagrange multipliers, one can get the following five equilibrium conditions:

$$1 = \beta \frac{\phi_{t+1}}{\phi_t} \frac{u' \circ c^{-1}(\beta \phi_{t+1} d_t^y)}{c' \circ c^{-1}(\beta \phi_{t+1} d_t^y)} \tag{13}$$

$$\frac{1}{\beta}\frac{\phi_t}{\phi_{t+1}} \geq \chi U'(x_{t+1}^H) + (1-\chi) U'(x_{t+1}^L) \tag{14}$$

$$\frac{\rho_{a,t}}{\beta} = \kappa^H \chi U'(x_{t+1}^H) + \kappa^L (1-\chi) U'(x_{t+1}^L) \tag{15}$$

$$\phi_{t+1} m_t^y = \frac{\kappa^L x_{t+1}^H - \kappa^H x_{t+1}^L}{\kappa^L - \kappa^H} \tag{16}$$

As in the previous section, the choice of DM money holdings is independent of all other endogenous variables and determined entirely by equation 13. Equation 14 is related to the use of money as a means of savings. If it is slack, money holdings are zero, and thus equation 16 simplifies to $\kappa^L x_{t+1}^H = \kappa^H x_{t+1}^L$, which jointly with equation 15 determines consumption levels in $t+1$ in that case. If however equation 14 holds at equality, consumption levels in $t+1$ are jointly determined by equations 14 and 15, and then 16 only determines the amount of money holdings used for savings. Note that m_t^y is increasing in the price of the risky assets $\rho_{a,t}$. If $\rho_{a,t}$ is sufficiently small, equation 14 cannot hold at equality for nonnegative money balances. For some price $\rho_{a,t}$, equation 14 holds at equality with $m_t^y = 0$. I will label this price as $\tilde{\rho}_a$. For any $\rho_a \leq \tilde{\rho}_a$, buyers choose not to hold any money balances for savings, while they hold positive money balances for any $\rho_a > \tilde{\rho}_a$.
At $\bar{\rho}_a \equiv (1+\pi_{t+1})(\chi\kappa^H + (1-\chi)\kappa^L) = \frac{1+\pi_{t+1}}{\beta}$, expected returns of risky assets and fiat money are equal. Since buyers are risk-averse, they will strictly prefer to save with money at this price, and thus $x_{t+1}^H = x_{t+1}^L$ and $a_t^y = 0$ at $\bar{\rho}_a$.

4.2 Asset market clearing

To close the model, we need an asset market clearing condition. Demand for assets by buyers can be found by rearranging the budget constraint of old buyers and is thus given by:

$$a_t^y = \frac{x_{t+1}^H - \phi_{t+1} m_t^y}{\kappa^H} = \frac{x_{t+1}^L - \phi_{t+1} m_t^y}{\kappa^L}$$

Because I normalized the expected return of a risky asset to $\frac{1}{\beta}$ sellers are willing to hold any amount of risky assets at the price $\rho_{a,t} = 1$. At any price lower than that, sellers would demand an infinity of the assets, thus pushing up the price. Therefore, $\rho_{a,t} = 1$ is the lower bound for the asset price. At any price higher than that, sellers are not willing to hold any risky assets. Thus, market clearing for risky assets is given by:

$$\frac{A_t}{N_t} = \frac{x_{t+1}^H - \phi_{t+1} m_t^y}{\kappa^H} + a_t^s \tag{17}$$

$$\text{with } a_t^s = 0 \quad \text{if } \rho_{a,t} > 1, \text{ and } a_t^s = \frac{A_t}{N_t} - \frac{x_{t+1}^H - \phi_{t+1} m_t^y}{\kappa^H} \text{ otherwise}$$

Equation 17 shows that an increase in the supply of assets leads to either an increase in x_{t+1}^o or a decrease in money savings m_t^y as long as $\rho_{a,t} > 1$. It can be seen directly from equation 15 that an increase in CM consumption has to lead to a decrease in the asset price $\rho_{a,t}$, and we already established that m_t^y is increasing in the price of risky assets, thus a decrease in money savings also forces the asset price to go down. Therefore, an increase in the asset supply unambiguously leads to a decrease in asset prices, and vice versa. However, if the price hits the lower bound, a further increase in asset supply does not have an effect on prices, as the sellers absorb any additional risky assets at a price $\rho_{a,t} = 1$. Thus, I denote the corresponding quantity of assets that is at least required to reach $\rho_{a,t} = 1$ as A^*.

Similarly, a further decrease in asset supply has no effect on asset prices once the price of assets gets infinitely close to $\bar{\rho}_a$. For any positive asset supply, this price will not be reached, thus $\bar{\rho}_a$ corresponds to $A_t = 0$. Finally, there is the threshold $\tilde{\rho}_a$ at which buyers start using both savings instruments. I denote the corresponding quantity of assets that leads to $\tilde{\rho}_a$ as $\tilde{A}$. Note that $\tilde{\rho}_a \lesseqgtr 1$ in principle, so it is not clear whether $\tilde{\rho}_a$ is a feasible price. If $\tilde{\rho}_a < 1$, buyers are using money to save even if the asset price is unconstrained. Higher inflation and less variance $\kappa^H - \kappa^L$ both make it more likely that $\tilde{\rho}_a > 1$.

As the previous section on an economy with only bonds and fiat money, this section mainly serves the purpose of establishing some concepts about the risky assets as a means of saving and especially the market clearing and the existing thresholds on the price of risky assets. The next section presents the full model where buyers can use all three assets to save.

5 The economy with government bonds and risky assets

In this section, both government bonds and risky assets are present, as is fiat money. The optimal portfolio choice of any agent thus consists of three different assets.

5.1 The buyer's problem

With all assets present, the buyer's decision becomes more involved, but is similar to the two previous cases. The buyers' first-period problem is as follows:

$$W^{y,ab} = \max_{h_t^y, d_t^y, m_t^y, a_t^y, b_t^y} -h_t^y + u(q(\phi_{t+1} d_t^y)) + \beta \left[\chi W^{o,ab}(\kappa^H a_t^y, m_t^y, b_t^y) + (1-\chi) W^{o,ab}(\kappa^L a_t^y, m_t^y, b_t^y)\right]$$

$$s.t. \quad h_t^y - \tau_t = \phi_t(m_t^y + d_t^y + \rho_{b,t} b_t^y) + \rho_{a,t} a_t^y$$

The notation follows that of the previous two sections. The old buyer's value function is:

$$W^{o,ab}(\kappa_{t+1} a_t^y, m_t^y, b_t^y) = \max_{x_{t+1}^o} U(x_{t+1}^o)$$

$$s.t. \quad x_{t+1}^o = \kappa_{t+1} a_t^y + \phi_{t+1} m_t^y + \phi_{t+1} b_t^y$$

Again, the value functions can be combined. It is best to combine them by substituting out bonds, because since agents prefer to use bonds to save over money and risky assets, they will always hold a positive amount of bonds.

$$W^a b = \max_{\substack{h_t^y, d_t^y, x_{t+1}^H, x_{t+1}^L, \\ m_t^y \geq 0, a_t^y \geq 0}} - h_t^y + u(q(\phi_{t+1} d_t^y)) + \beta \left[\chi U(x_{t+1}^H) + (1-\chi) U(x_{t+1}^L) \right]$$

$$s.t. \quad h_t^y - \tau_t = \rho_{a,t} a_t^y + \phi_t(m_t^y + d_t^y) + \frac{\phi_t}{\phi_{t+1}} \rho_{b,t}(x_{t+1}^H - \kappa^H a_t^y - \phi_{t+1} m_t^y) \qquad (\lambda^H)$$

$$s.t. \quad h_t^y - \tau_t = \rho_{a,t} a_t^y + \phi_t(m_t^y + d_t^y) + \frac{\phi_t}{\phi_{t+1}} \rho_{b,t}(x_{t+1}^L - \kappa^L a_t^y - \phi_{t+1} m_t^y) \qquad (\lambda^L)$$

The buyer's maximization problem gives rise to the following first-order conditions:

$$h_t: \qquad 1 = \lambda^H + \lambda^L$$

$$d_t^y: \qquad \phi_t(\lambda^H + \lambda^L) = \beta\phi_{t+1}\frac{u' \circ c^{-1}(\beta\phi_{t+1}d_t^y)}{c' \circ c^{-1}(\beta\phi_{t+1}d_t^y)}$$

$$m_t^y: \qquad \phi_t(\lambda^H + \lambda^L) \geq \rho_{b,t}\phi_t(\lambda^H + \lambda^L)$$

$$a_t^y: \qquad \rho_{a,t}(\lambda^H + \lambda^L) \geq \frac{\phi_t}{\phi_{t+1}}\rho_b(\lambda^H\kappa^H + \lambda^L\kappa^L)$$

$$x_{t+1}^H: \qquad \lambda^H\frac{\phi_t}{\phi_{t+1}}\rho_{b,t} = \beta\chi U'(x_{t+1}^H)$$

$$x_{t+1}^L: \qquad \lambda^L\frac{\phi_t}{\phi_{t+1}}\rho_{b,t} = \beta(1-\chi)U'(x_{t+1}^L)$$

I made use of the solution to the DM problem as derived in section 3.1.1, since that problem is not affected by the type of assets present in the economy. By replacing the Lagrange multipliers, one can get the following five equilibrium conditions:

$$1 = \beta\frac{\phi_{t+1}}{\phi_t}\frac{u' \circ c^{-1}(\beta\phi_{t+1}d_t^y)}{c' \circ c^{-1}(\beta\phi_{t+1}d_t^y)} \tag{18}$$

$$1 \geq \rho_{b,t} \tag{19}$$

$$\frac{\phi_t}{\phi_{t+1}}\frac{\rho_{b,t}}{\beta} = \chi U'(x_{t+1}^H) + (1-\chi)U'(x_{t+1}^L) \tag{20}$$

$$\frac{\rho_{a,t}}{\beta} \geq \kappa^H\chi U'(x_{t+1}^H) + \kappa^L(1-\chi)U'(x_{t+1}^L) \tag{21}$$

$$x_{t+1}^H - x_{t+1}^L = a_t^y(\kappa^H - \kappa^L) \tag{22}$$

Equation 18 shows that the choice of money holdings used in DM meetings is independent of other decisions and depends only on the terms of trade and inflation rates. Condition 19 shows that agents only want to save with money if the price of bonds equals one. Equation 20 sets the cost of acquiring and holding bonds equal to the benefit of holding more bonds, which is more consumption in both the high and the low state. If condition 21 holds with equality, agents acquire risky assets such that the cost of acquiring them is equal to the benefit agents can draw from them. If asset prices are too high, condition 21 will not hold at equality and agents thus acquire no risky assets. Finally, equation 22 says that any difference in consumption levels in the second period is caused by asset holdings. Consequently, consumption in the low and the high state will be equal if the return of the asset is not risky or if agents do not hold any assets.

5.2 Bond market clearing

The bond market clearing works similarly to what I showed in section 3.2, but some variables change due to the presence of risky assets. Thus I will just briefly summarize the main takeaways from the analysis done earlier, updated with the new variables:

$$b_t^y = \frac{x_{t+1}^H - \kappa^H a_t^y - \phi_{t+1} m_t^y}{\phi_{t+1}} = \frac{B_t}{N_t} - b_t^s \tag{23}$$

$$\text{with } b_t^s = 0 \quad \text{if } \rho_{b,t} > \frac{\beta}{1+\pi_{t+1}}, \text{ and } b_t^s = \frac{B_t}{N_t} - \frac{x_{t+1}^H - \kappa^H a_t^y - \phi_{t+1} m_t^y}{\phi_{t+1}} \text{ otherwise}$$

Equation 23 states that all bonds have to be held by young buyers unless the bonds are priced at the Fisher equation. Further, it shows that an increase in the supply of bonds leads to a decrease in their price. From condition 19 and the analysis done in section 3.2, we know that the bond price is bounded above by one, which means that the bond price can lie in the range:

$$\beta \frac{\phi_{t+1}}{\phi_t} \leq \rho_{b,t} \leq 1$$

5.3 Asset market clearing

The asset market clearing works just like in section 4.2, just with some slight changes to the variable that determines the buyer's asset holdings. From equation 22, we know these are given by:

$$a_t^y = \frac{x_{t+1}^H - x_{t+1}^L}{\kappa^H - \kappa^L}$$

So for the asset market to clear, the following condition needs to hold:

$$\frac{A_t}{N_t} = \frac{x_{t+1}^H - x_{t+1}^L}{\kappa^H - \kappa^L} + a_t^s \tag{24}$$

$$\text{with } a_t^s = 0 \quad \text{if } \rho_{a,t} > 1, \text{ and } a_t^s = \frac{A_t}{N_t} - \frac{x_{t+1}^H - x_{t+1}^L}{\kappa^H - \kappa^L} \text{ otherwise}$$

5.4 Equilibrium

Definition 2. *An equilibrium is a sequence of prices $\rho_{b,t}, \rho_{a,t}$, and quantities $d_t^y, x_{t+1}^H, x_{t+1}^L, m_t^y, a_t^y, b_t^s$, and a_t^s, that simultaneously solve the equations 18, 20, 22, and the following inequalities with complementary slackness conditions:* $19, 21, 23$, *and* $24 \;\; \forall t$.

The equilibrium conditions give rise to a number of different regions in the parameter space for which the resulting equilibrium differs. In the following, I will characterize the different possible equilibria. For a given inflation rate, the selection of an equilibrium region depends on the supply of risky assets A_t and the bonds-to-money ratio $\mathcal{B}_t$. Figures 1 and 2 depict the equilibrium regions for high and low inflation rates, respectively.

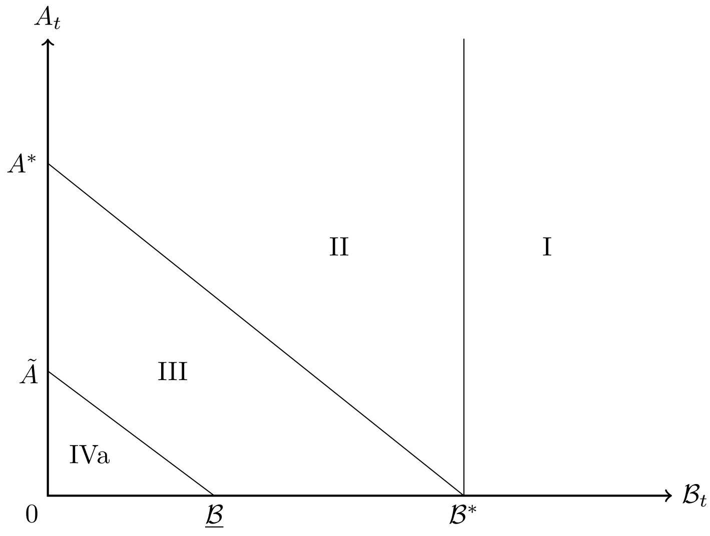

Figure 1: Equilibrium regions for $\tilde{\rho}_a > 1$.

Proposition 1. *If the supply of government bonds exceeds $\mathcal{B}^*$, buyers will only hold bonds, while sellers hold all the risky assets and the remaining government bonds.*

$\mathcal{B}^*$ is the amount of bonds that allows buyers to do all their savings through bonds, as defined in section 3. If the supply of government bonds is at least that large, the bonds will be priced at their lower bound, i.e. $\rho_{b,t} = \frac{\beta}{1+\pi_{t+1}}$. This means that a bond pays the same expected return as a risky asset, but since buyers are risk-averse, they will only hold the riskless asset, i.e. the bond. It can be shown that at this bond price, condition 21 can only hold at equality for $x^H_{t+1} = x^L_{t+1}$, which then leads to a contradiction with equation 22, thus proving the proposition. Since buyers are not buying risky assets, the price of the risky assets is 1 independent of the supply of risky assets. Because bonds perfectly compensate for discounting and inflation, buyers consume $x^H = x^L = x^*$.

I will denote the parameter region for which proposition 1 holds as region I. It is defined by $\rho_{b,t} = \underline{\rho}_b$ while $\rho_{a,t} = 1$.

Proposition 2. *If $\mathcal{B}_t < \mathcal{B}^*$, but the combined supply of $\mathcal{B}_t$ and A_t is sufficiently large, buyers will hold all the bonds and some risky assets. Sellers hold the remaining risky assets. There is a risk premium paid on risky assets compared to government bonds, but risky assets are still priced at their unconstrained value.*

Once the price of bonds is lifted above the lower bound, the expected return of bonds becomes lower than the expected return of a risky asset priced at its unconstrained value. Buyers can thus increase their utility by holding some risky assets. The amount of risky assets they are willing to hold depends on their risk aversion. Whenever $\rho_{b,t} > \underline{\rho}_b$ and $\rho_{a,t} = 1$, for condition 21 to be a

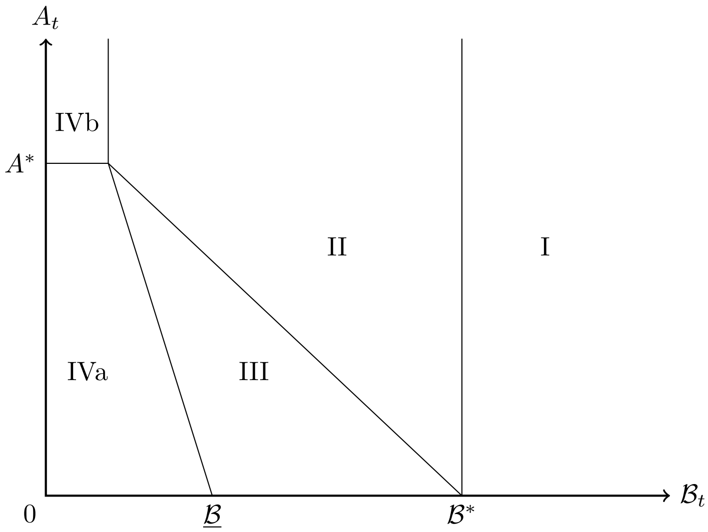

Figure 2: Equilibrium regions for $\tilde{\rho}_a \leq 1$.

strict inequality, $x^H_{t+1} > x^L_{t+1}$ is required. However, this leads to a contradiction with equation 22, showing that buyers will always hold some risky assets in this case. Note that in this region, we will typically observe $x^H > x^* > x^L$.

I will denote the parameter region for which proposition 2 holds as region II. It is defined by $1 > \rho_{b,t} > \underline{\rho}_b$ while $\rho_{a,t} = 1$. This means that there is a savings premium on bonds in this region, but none on risky assets.

Proposition 3. *If the combined supply of bonds and risky assets is scarce, the price of both assets will be above its lower bound and buyers will hold the entire supply of both assets.*

In region II, buyers are demanding some risky assets. The lower the buyer's risk aversion and the higher the price of government bonds, the more risky assets they demand. At some point, demand exceeds supply at the unconstrained price, thus lifting the price of risky assets above its unconstrained level. This discourages sellers from holding risky assets.

I will denote the parameter region for which proposition 3 holds as region III. It is defined by $1 > \rho_{b,t} > \underline{\rho}_b$ while $\rho_{a,t} > 1$, so there are risk premia on both kind of assets in this region. Because of the savings premium on risky assets, this region can be considered a *bubble equilibrium.*

Proposition 4. *If the combined supply of bonds and risky assets is severely scarce, bonds will yield the same return as fiat money and buyers will also use fiat money to save.*

Once the bond price is driven up to $\rho_{b,t} = 1$, buyers will not demand more of bonds or risky assets, but start to save with fiat money instead. Depending on risk aversion parameters and the supply of risky assets, the price of risky assets can be at its unconstrained value or above it. In an economy without risky assets, $\rho_{b,t} = 1$ if $\mathcal{B}_t \leq \underline{\mathcal{B}}$. With risky assets available, the amount of bonds required to get away from the zero lower bound gets smaller as buyers also use risky assets to save.

I will denote the parameter region for which proposition 4 holds as region IV. It is defined by $\rho_{b,t} = 1$ and can therefore be considered a *zero lower bound equilibrium*. Note that it can exist with either $\rho_{a,t} > 1$ or $\rho_{a,t} = 1$. I denote the parameter space where we simultaneously observe a *bubble* and *zero lower bound* as region IVa, while a zero lower bound without a bubble is denoted as a IVb equilibrium.

A IVb equilibrium can only exist if buyers are using money to safe even when they are satiated with risky assets. Thus, this equilibrium exists only if $\tilde{\rho}_a \leq 1$. This situation is depicted in figure 2.

5.5 Risk premium

The risk premium is typically defined as the difference in expected return between a risky and a safe asset that both deliver the same services. Thus, the difference in expected return between the risky asset and the government bond is a natural candidate for the risk premium in this model, which I therefore define as:

$$\mathcal{R}_t = \frac{1}{\beta}/\rho_{a,t} - \frac{\phi_{t+1}}{\phi_t}/\rho_{b,t}$$

Note that if both assets are priced at their unconstrained value, i.e. in region I, $\mathcal{R}_t = 0$. However, the risk premium measures the difference in expected returns that makes a single agent indifferent between the two assets. Since buyers are not willing to hold risky assets in region I, the risk premium cannot be measured in that region. In all other regions however, buyers hold a positive amount of both assets and are thus indifferent between the two on the margin. Interestingly however, $\mathcal{R}_t$ is not a constant: In region II, $\rho_{a,t} = 1$ everywhere while $1 > \rho_{b,t} > \beta\frac{\phi_{t+1}}{\phi_t}$, depending on $\mathcal{B}_t$. Thus, the risk premium can also vary according to these bounds on $\rho_{b,t}$. In region III, $\mathcal{R}_t$ can go up or down depending on whether $\rho_{a,t}$ or $\rho_{b,t}$ increase faster, but in region IV, the risk premium is decreasing again as $\rho_{b,t} = 1$ while $\rho_{a,t}$ is increasing the further the economy goes into region IV.

5.6 Comparative statics

In this subsection, I want to analyze the comparative statics of inflation, the supply of both risky assets and bonds, as well as the riskyness of the asset in all equilibrium regions.

5.6.1 Inflation

From equation 18, it is obvious that inflation reduces d_t^y and thus DM consumption of young agents. Since this is true for all regions, it is clear that inflation always has negative effects. As it turns out, it doesn't have positive effects on the CM consumption either, thus making the Friedman rule optimal.

In region I, the bond price is compensating fully for inflation, thus an increase in inflation leads to a decrease in bond prices in this region, without affecting second-period consumption or other real variables. In region II, the bond price is not compensating fully for inflation because of the scarcity of bonds. Without scarcity, the bond price would drop exactly as much as inflation increases, allowing buyers to get the same level of consumption after the change, just like in region 1. However, to get the same level of consumption as before the increase in inflation, buyers would need to hold more bonds, which puts upward pressure on the bond price. Thus, the bond price is dropping somewhat (at the previous bond price, agents would demand less bonds than before), but not compensating fully for inflation. Asset prices stay at their lower bound in this region because there is no asset scarcity. This means that an increase in inflation leads to more asset holdings of buyers (assets become relatively more attractive because the difference in returns increases) and thus also the difference $x_{t+1}^H - x_{t+1}^L$ increases. All in all, buyers are worse off.

In region III, both assets are scarce and thus all of them are held by buyers. The effect on bonds is thus similar as in region II, but since buyers cannot compensate by holding more risky assets now, the price of risky assets increases too. Thus, buyers have to work more to purchase the assets, and they get less consumption in both states because the real rate of return of bonds decreases, making them clearly worse off. In region IV, bonds are so scarce that fiat money is a perfect substitute for saving. Thus, an increase in inflation does not affect bond prices, which stay at $\rho_{b,t} = 1$, meaning that the real return of bonds decreases even more than in regions II and III. Additionally, buyers will reduce their money savings somewhat because the real return of money also decreases. In region IVb, buyers can substitute away into risky assets, thus the overall effects are similar to region II, i.e. consumption variance increases, but expected consumption stays constant. In region IVa, buyers would also like to purchase more risky assets, but because they are scarce in this region, they merely drive up asset prices, making the overall effects in this region similar to those in region III. Clearly, buyers are again made worse off by an increase in inflation in region

IV.

5.6.2 Decrease in the supply of risky assets

In regions I, II, and IVb, such a change has no real effect, as at least some risky assets are held by sellers, and the sellers are indifferent about holding them. In region III, a decrease in the supply of risky assets leads to an increase in the price of risky assets. As this makes bonds relatively more attractive, bond prices increase too. Thus, second-period consumption falls in both states because less risky assets can be held by buyers, but also the difference $x^H_{t+1} - x^L_{t+1}$ decreases as a^y_t falls.
In region IVa, the effect is similar, except that instead of increasing bond prices, money savings increase. This increase reduces the difference $x^H_{t+1} - x^L_{t+1}$. However, expected consumption also falls somewhat, as buyers are not willing to fully replace the missing assets with money savings due to the inflation tax. Thus, buyers are worse off.

5.6.3 Decrease in the supply of government bonds

As bonds are not scarce in region I, a marginal decrease in their supply has no real effect. In region II, bonds are scarce, thus a further decrease in supply leads to an increase in the bond price. This makes risky assets relatively more attractive, and since they are not scarce, buyers can easily substitute into them. Thus, their consumption variance increases, but their expected consumption does not decrease. In region III, similar mechanisms are at play as in region II, but since risky assets are also scarce, buyers cannot hold more of them and instead drive up their price. This makes purchasing the assets more expensive, and drives down CM consumption in both states as less assets are available. In region IV, a decrease in bonds has no real effect again, as bonds are already priced at their upper bound, and a further decrease in supply thus only increases money savings.

5.6.4 Riskyness of the asset

In general, one can say that the less risky the asset is, i.e. the smaller $\frac{\kappa^H}{\kappa^L}$, the better risky assets become as a substitute for bonds.
In region I, the riskiness of the asset does not matter as only risk-neutral agents (sellers) are willing to hold them. In region II, less riskiness makes the assets more attractive, thus increasing a^y_t, which lowers pressure on bond prices and thus leads to lower $\rho_{b,t}$. Agents consume more in the low state and welfare increases. In region III, the higher attractiveness of the assets increases their scarcity, thus increasing asset prices. However, the pressure on bond prices is reduced, which leads bond prices to fall. Overall, agents are better off because the variance of their consumption decreases. In

region VIb, the effects are similar to region 2, but without a change in bond prices. In region IVa, asset prices increase, but welfare still increases as buyers get less variance in their second-period consumption.

5.7 Policy

In terms of policy, it is easy for the authorities to achieve the first-best: Running the Friedman rule ensures both d^* and x^*. If this is not an option, it is still possible to achieve x^* by issuing enough bonds. However, if both of these policies are not feasible for some reason, the monetary authority can improve welfare through stabilization policy.

5.7.1 Optimal stabilization policy

For this, I will assume that the long-term, expected inflation rate is fixed, but that the monetary authority can inject a higher or lower money supply depending on the state. In fact, we will see that it is optimal to inject more money in the high state and less money in the low state, thus we can write the expected price level of next period as:

$$\mathbb{E}_t[\phi_{t+1}] = \chi\phi^L_{t+1} + (1-\chi)\phi^H_{t+1} \tag{25}$$

Thus, the value of money is low in the high state, because the monetary authority injected more money than expected, and vice versa. This is a similar policy as the one studied in Berentsen and Waller (2011).

With this additional policy tool, the equilibrium conditions change slightly:

$$1 =\beta\frac{\mathbb{E}_t[\phi_{t+1}]}{\phi_t}\frac{u'\circ c^{-1}(\beta\mathbb{E}_t[\phi_{t+1}]d^y_t)}{c'\circ c^{-1}(\beta\mathbb{E}_t[\phi_{t+1}]d^y_t)} \tag{26}$$

$$1 \geq\rho_{b,t} \tag{27}$$

$$\frac{\rho_{b,t}}{\beta} =\chi U'(x^H_{t+1})\frac{\phi^L_{t+1}}{\phi_t} + (1-\chi)U'(x^L_{t+1})\frac{\phi^H_{t+1}}{\phi_t} \tag{28}$$

$$\frac{\rho_{a,t}}{\beta} \geq\kappa^H\chi U'(x^H_{t+1}) + \kappa^L(1-\chi)U'(x^L_{t+1}) \tag{29}$$

$$\frac{\phi_t}{\phi^L_{t+1}}x^H_{t+1} - \frac{\phi_t}{\phi^H_{t+1}}x^L_{t+1} =a^y_t(\frac{\phi_t}{\phi^L_{t+1}}\kappa^H - \frac{\phi_t}{\phi^H_{t+1}}\kappa^L) \tag{30}$$

Equations 27 and 29 remain unchanged compared to the general problem, and equation 26 only changes such that now the expected inflation is relevant instead of the actual inflation, as this decision is taken before the agents know the realization of the asset return. The most significant change occurs in equation 28, as now the marginal utilities in the two states have to be discounted by the actual inflation rate. From here, it can already be seen that this policy is welfare-improving:

While before we had overconsumption in the high state and underconsumption in the low state respectively in some regions, the marginal utilities in these two states now get discounted by the actual inflation rate, which allows agents to consume more in the low and less in the high state. In fact, what happens is that through the different inflation rates in the two states, the monetary authority makes bonds (and also fiat money) an asset that is perfectly negatively correlated with the risky asset, which allows agents to reduce the overall riskyness of their portfolio.
In region II, the correct choice of ϕ_{t+1}^L and ϕ_{t+1}^H allows to achieve the first best, i.e. $x^L = x^H = x^*$. This is not feasible in regions III and IV, but the policy can reduce consumption variance without affecting expected consumption, and thus increase welfare.
These results are true as long as the central bank has access to both lump-sum taxes and transfers. If taxes are not feasible, there is an upper bound on next period's value of money, namely $\phi_{t+1} \leq \phi_t$. This means that the optimal ϕ_{t+1}^H might not be available. In this case, $x^L = x^H = x^*$ is not achievable even in region II, but setting $\phi_{t+1}^H = \phi_t$ and ϕ_{t+1}^L according to equation 25 still improves the outcome.

However, note that to achieve the optimal stabilization, the monetary authority is increasing inflation in good states while it is lowering it in bad states. This goes against the conventional wisdom that inflation should be countercyclical to prevent 'overheating' in good times or to 'kickstart' an economy that is in a slump.

6 Conclusion

This paper shows that savings premia on risky assets can occur naturally in asset markets when there is a shortage of savings instruments for agents. Savings premia on risky assets are especially likely to arise at the zero lower bound, but it is also possible for a zero lower bound to occur while risky assets are priced according to their unconstrained value. If the long-run inflation target is positive and the fiscal authority is politically limited to issue the optimal amount of government bonds, the best available policy is a varying inflation rate that is positively correlated with the stock market - i.e. if the stock market returns are above average, inflation should be high, while inflation should be low when stock market returns are low. This is because the return of government bonds is inversely correlated to the inflation rate, and thus this policy makes stocks and bonds negatively correlated, which allows agents to better balance their portfolio.

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