STUDY CENTER
GERZENSEE

Swiss Program for Beginning Doctoral Students in Economics 2006

Midterm Exam in Microeconomics

Saturday, July 29, 2006, 14.00h – 16.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please do **not** mention your name on top of the pages, but only your identification number. (The exams will be graded anonymously.) If you use the back side of a page, put your ID number there as well.

3. Please answer questions on the same page or the back side of the page. (The exams will be separated in order to be sent to the different professors.)

4. Please use **a pen** to guarantee that your answers can be read without problems.

5. Please **write legibly**. Your exams will be photocopied for grading.

6. Answers should be **concise and precise!** The space provided should be sufficient to answer each question.

7. No cell phones!

8. Please have your ID card ready on the desk. The TA will check your ID cards and have you sign a list.

9. Good luck!

ID-Number:_____

1. (20 points) Consider a consumer who spends his budget on three different goods in the following way:

   - At prices $p^0 = (2, 2, 4)$ she chooses consumption bundle $x^0 = (5, 19, 9)$
   - At prices $p^1 = (1, 1, 1)$ she chooses consumption bundle $x^1 = (12, 12, 12)$
   - At prices $p^2 = (1, 2, 1)$ she chooses consumption bundle $x^2 = (27, 11, 1)$

   Nothing else has changed in the three situations.

   (a) Show that these data satisfy the weak axiom of revealed preference.

(b) Show, that the revealed preferences of the consumer fail to be transitive.

2. (15 points) You represent a union where all members have the same utility function $u(x_1, x_2)$ of two aggregate goods one (housing) and two (food) given by $u(x_1, x_2) = \min\{x_1, x_2\}$. Wages are 1000 SFR/month and the prices of the aggregates one and two are one SFR per month for each unit. The management wants to move the factory to a new location where the price of the first aggregate good (housing) is four times the price at the old location while the price of the second aggregate good (food) remains the same. The union members agree that they will move if they receive a pay raise, R, that gives them the same utility at the new location as at the old.

   (a) Find the raise, R. How is this compensation called in welfare economics? After obtaining the raise, R, can the union afford the same (x_1, x_2) pair at the new location as they did at the old? Do they choose it?

(b) Now suppose that the utility function of each member is $u(x_1, x_2) = \sqrt{x_1 \cdot x_2}$. Answer the same question as above. After it receives the raise you have computed can the union afford the same (x_1, x_2) pair at the new location now? If not, why not.

3. (25 points) Consider the following utility function

$$u(w) = a + bw + cw^2$$

where $w \in IR$ is the wealth of the decision maker.

(a) Which restrictions, if any, must be imposed on the parameters a, b and c for this function to be a von Neumann-Morgenstern utility function of a risk averse decision maker?

(b) Over which domain is this vNM utility function well defined?

(c) Show that with this utility function the expected utility from a distribution is determined by the mean and variance of the distribution and, in fact, by these moments alone. Furthermore, show that his utility is increasing in the mean and decreasing in the variance of the distribution.

(d) Suppose that a decision maker with this utility function and an initial wealth w has to decide which amount $s \leq w$ to invest into a risky asset. The risky asset has an expected return $r > 1$ and a variance $v > 0$ per Dollar of investment. Assuming an interior solution compute the optimal investment s^* as a function of w and show that $s^*(w)$ is decreasing.

Problem 1 (40 points)
Consider a two-good exchange economy. Good 1 is taken as a numéraire. The price of good 2 is denoted p. There are two consumers. Their incomes are denoted I_1, I_2

1. Compute the demand function of consumer 1, whose utility function is: $u_1(x_{11}, x_{12}) = x_{11}.x_{12}$

2. Compute the demand function of consumer 2, whose utility function is: $u_2(x_{21}, x_{22}) = x_{21} + \ln x_{22}$

3. Initial endowments are $\omega_1 = (I, 0)$ $\omega_2 = (0, 1)$ where I is a parameter. Compute the equilibrium price as a function of I.

4. Compute the equilibrium allocations as a function of I.

5. Show that the equilibrium utility of both consumers increase with I. Is this surprising?

Problem 2 (20 points)

Consider a two-good economy with one firm and two consumers, with utility functions:

$u_1(x_{11}, x_{12}) = x_{11}.x_{12}$ and $u_2(x_{21}, x_{22}) = x_{21} + \sqrt{x_{22}}$

The firm produces good 2 out of good 1, with a cost function $C(y_2) = \frac{1}{2} y_2^2$. Good one is taken as a numéraire.

1. Compute the supply and the profit functions of the firm.

2. Compute the demand functions of the two consumers. Consumer 2 owns the firm. Consumer 1 owns the initial endowment of the economy $\omega_1 = (1, 0)$.

3. Form a polynomial equation that characterizes equilibrium price and show that it has a unique positive solution.