Short Question 1: Dynamic Programming (15 points)

Consider the following dynamic programming problem. The value function takes the form

$$V = \phi + \gamma \log k.$$

The momentary objective takes the form

$$u(c) = \log(c)$$

and there is a constraint of the form

$$c + k' = ak^{\alpha}$$

with $a > 0$ and $\alpha < 1$. Finally, the Bellman equation takes the form

$$V(k) = \max_{c,k'}\{u(c) + \beta V(k')\}$$

with β being a discount factor between 0 and 1, where the maximization is subject to the constraint.

(a) Find conditions which restrict the optimal c and k' choices.

(b) Find the optimal policy functions $c = f(k)$ and $k' = g(k)$.

(c) Find the value function (that is, determine the coefficients ϕ and γ).

Short question 2: Rational Expectations Models (15 points)

Suppose that you are given a discrete time perfect foresight model which contains an equation of the form

$$r_t = \lambda_t - \lambda_{t+1}$$

with both r and λ being endogenous variables.

(a) Give an example of an economic relationship studied in weeks 1 and 2 that fit into this framework. (There are several: do not worry about which one you provide.)

(b) Now write the above equation as $r_t = \lambda_t - E_t\lambda_{t+1}$, with E_t being the rational expectations operator. Suppose that the above equation is supplemented with an additional equation

$$r_t = \phi E_t\lambda_{t+1} + gx_t$$

where x_t is some exogenous variable which obeys a stationary stochastic difference equation. Finally, suppose that λ is not predetermined. Under what conditions on ϕ and g is there a unique stationary solution for λ_t?

(c) Suppose that $x_t = \rho x_{t-1} + e_t$ with $|\rho| < 1$. Show that the solution takes the form

$$\lambda_t = f \; x_t$$

and determine the coefficient f.

Longer question: Optimal Consumption Choice with Habits (30 minuets)

Consider an individual that has a utility function of the form

$$u(c, l, z) = \log(c_t + \kappa z_t) + \gamma l_t \tag{1}$$

where c_t is the flow of consumption, l_t is the amount of leisure, and z_t is an endogenous state variable, described further below. Suppose further that the consumption good is produced according to

$$c_t = a(\varsigma_t)(1 - l_t) \tag{2}$$

where $a(\varsigma_t) > 0$ is productivity, which depends on a Markov random variable ς_t.

(a) Supposing that $\kappa = 0$, determine the optimal level of consumption and leisure. How do these depend on work effort. Why?

(b) Suppose now that $\kappa \neq 0$ and that

$$z_{t+1} = \delta z_t + (1 - \delta) c_t \tag{3}$$

with $0 < \delta < 1$. What is a possible economic interpretation of z_t if κ is negative?

(c) Write a Bellman equation for the maximization of

$$U_t = E_t[\sum_{j=0}^{\infty} \beta^j u(c_{t+j,} l_{t+j}, z_{t+j})$$

subject to (2) and (3).

(d) What are the state variables on which date t optimal policies will depend? Why?

(e) Attaching a multiplier θ_t to (2) and λ_t to (3), display efficiency conditions for c, l, and z.

(f) What are the envelope theorem results for this problem?