King

SHORT PROBLEM

Consider the following rational expectations model

$$y_t = bE_t y_{t+1} + ax_t + c(x_t - E_{t-1}x_t)$$

This rational expectations model can be written in the standard form, $AE_tY_{t+1} = BY_t + C_0X_t + C_1E_tX_{t+1}$ if

$$Y_t = \begin{bmatrix} y_t \\ E_{t-1}x_t \end{bmatrix} \quad \text{and} \quad X_t = [x_t]$$

(a) [7 points] Write the model in this standard form

(b) [6 points] Show that this model has two roots, one that is 0 and one that is 1/b.

(c) [7 points] Using recursive forward substitution, solve the model to write y as a function of various expectations of x.

LONG PROBLEM

Consider an economy in which individuals value three commodities: leisure (l), bread (c) and cigarettes (s). Both bread and cigarettes are market goods that can be subject to sales taxes, at rates τ_c and τ_s respectively.

(a) [15 points] Suppose that the budget constraint for the individual is

$$z = l + [(1 + \tau_c)p_c]c + [(1 + \tau_s)p_s]s$$

with p_x being the market price of good x before taxation. Suppose further that the utility function takes the form

$$u(l, c, s) = \log(l) + \frac{\theta_c}{1 - \eta_c}c^{1-\eta_c} + \frac{\theta_s}{1 - \eta_s}s^{1-\eta_s}$$

where all parameters are positive. Derive the demand functions for bread and cigarettes. [Hint: it is OK to use a "lamda constant" demand function, where λ is the multiplier on the budget constraint]. Under what conditions on the θ will individuals demand more bread than cigarettes if both goods have the same price and $\eta_c = \eta_s$? What parameter restriction insures that the demand for cigarettes is less price elastic than the demand for bread? [By less price elastic, it is meant that the absolute value of the price is lower].

(b) [35 points] Now suppose that the goods are produced according to constant returns functions,

$$\begin{aligned} c &= a_c n_c \\ s &= a_s n_s \end{aligned}$$

where n_x is the amount of work allocated to production of good x. Suppose that perfect competition prevails, so that firms price at marginal cost. Finally, suppose that the government needs to raise a given amount of tax revenue (in labor units), so that its budget constraint is of the form

$$g = \tau_c p_c c + \tau_s p_s s$$

Derive implications for the optimal tax regime under the parameter restrictions developed in the last section: bread is demanded more than cigarettes at equal prices and elasticities; cigarette demand is less price elastic than bread demand.

Be as explicit as you can be about the absolute and relative levels of the taxes on the two goods.

(c) Suppose that there are external effects of cigarette consumption, so that the utility function for an individual depends on l, c, s, S, where S is the quantity of smoking by other individuals.

(c-1) [10 points] Specify the conditions for a competitive equilibrium in the presence of this consumption externality and taxation (there is no reason to use the specific parametric utility function for this purpose, you can simply write u(l,c,s,S)).

(c-2) [10 points] Describe how the presence of this externality would affect the "Ramsey problem" which you solved in part (b) and discuss the potential implications for the structure of optimal taxes which would derive from this problem.

Rebelo

Optimal oil extraction Consider the following dynamic problem. An oil producing country has Y barrels of oil reserves. Let y_t be the number of oil barrels that the country extracts and sells at time t. Since Y is the total amount of oil that can be extracted, we have:

$$\sum_{t=0}^{\infty} y_t = Y$$

The country wants to pursue an extraction policy that maximizes the present value of profits from oil extraction, given by:

$$V = \sum_{t=0}^{\infty} \beta^t p_t y_t,$$

where p_t is the time-t price of an oil barrel in terms of consumption goods. The discount factor β, is equal to $1/(1+r)$, where r is the real interest rate.

a) Suppose that p_t is constant over time, $p_t = p$. What is the optimal path for y_t?

b) Suppose that p_t grows at rate $g > r$ between time zero and time T and remains constant from time T on:

$$p_t = \begin{cases} p(1+g)^t & 0 \leq t \leq T \\ p(1+g)^T & t > T \end{cases}$$

What is the optimal path for y_t?

c) Suppose that the price of oil is constant, $p_t = p$, but there are extraction costs, given by $\frac{\phi}{2} y_t^2$. The present value of profits from oil extraction are given by:

$$V = \sum_{t=0}^{\infty} \beta^t \left(p y_t - \frac{\phi}{2} y_t^2 \right)$$

Derive the equations that characterize the optimal path for y_t. Is it optimal for y_t to increase or decrease over time?

Gali

Consider an economy with Calvo-type staggered price setting, where firms have access to a simple technology $Y_t = N_t$, where Y_t is output and N_t denotes hours of work. The representative consumer has a period utility $U(C_t, N_t) = \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}$, where C_t is a CES function of the quantities consumed of different types of goods. The discount rate varies over time according to $\rho_t = \rho + \varepsilon_t$, where ε_t is assumed to follow a white noise process. All output is consumed.

The implied equilibrium conditions take the form

$$y_t = E_t\{y_{t+1}\} - (i_t - E_t\{\pi_{t+1}\} - \rho) + \varepsilon_t \tag{1}$$

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; y_t \tag{2}$$

where y_t is (log) output, $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation, and i_t is the short-term nominal rate.

(a) (10 points) Determine the natural level of output in the above economy

(b) (10 points) Where do equilibrium conditions (1) and (2) come from? Explain *in words* but as rigorously as possible.

(c) (20 points) Suppose that the monetary authority adopts a simple Taylor rule of the form

$$i_t = \rho + \phi_\pi \; \pi_t + v_t \tag{3}$$

with $\phi_\pi > 1$ and where v_t is a white noise exogenous monetary policy shock. Determine the equilibrium path of inflation and output as a function of the two exogenous shocks.

(d) (10 points) Suppose that an econometrician attempts to estimate (3) by running an OLS regression of the nominal rate on inflation. Explain why the estimate of ϕ_π would be biased and determine the sign of that bias (and sketch how you would determine its size).