Final Exam - Micro I

Parts (a) and (b) of this problem can be solved independently of each other.

(a) Suppose that a firm can use two different plants with different production functions. Keeping input prices fixed, these production functions give rise to cost functions $c_1(y_1) = y_1^2$ and $c_2(y_2) = 3y_2$. Let $y = y_1 + y_2$ and suppose that the firm is free how to allocate production between the two plants (i.e., the two production functions). How does the cost function $c(y)$ of the firm look like?

(b) Let $c(w_1, w_2, y)$ be the cost function of a firm, where w_1 and w_2 are the prices of input factors 1 and 2, and y is the quantity to be produced. Let $z_1(w_1, w_2, y)$ denote the conditional factor demand function of this firm for input factor 1. Show that

$$\frac{\partial c(w_1, w_2, y)}{\partial w_1} = z_1(w_1, w_2, y) .$$

Rochet

Consider a production economy with 2 consumption goods $(\ell = 1, 2)$ and 2 factors (capital K and labor L). These goods are produced by competitive firms, with Leontief production functions:

$$f_1(K_1, L_1) = Min(K_1, L_1),$$
$$f_2(K_2, L_2) = Min(K_2, L_2 / 4).$$

1. Characterize the optimal production plans, given that total endowment is 22 units of labor and K units of capital (K is taken as a parameter). Consider first the case where there is excess of labor and then the case where there is excess of capital.
2. Represent the aggregate production set of this economy in the (C_1, C_2) plane.
3. There is one consumer with a utility function: $u(C_1, C_2) = C_1 C_2$. Show that there is a unique competitive equilibrium and compute the corresponding allocation when there is excess of labor. Show that this happens when K<44/5.
4. For which values of K is there an excess of capital?

Dewatripont

Question 1: Infinitely repeated Cournot Oligopoly with n firms

Assume inverse demand in the stage game is given by $p(Q) = a - Q$, where $Q = q_1 + q_2 + ... + q_n$. Assume firms have zero costs. Assume the discount factor for each firm is $\delta < 1$.

- What is the lowest value of δ such that the firms can use trigger strategies (return to the static Cournot equilibrium forever) to sustain the monopoly output level in a subgame-perfect equilibrium?
- How does the answer vary with n, and why?
- If δ is too small for the firms to use trigger strategies to sustain the monopoly output, what is the most-profitable symmetric subgame-perfect equilibrium that can be sustained using trigger strategies?

Question 2: Bertrand Competition with Asymmetric Information.

Consider a Bertrand duopoly with differentiated complementary products, where demand for firm i is $q_i(p_i , p_j) = a - p_i - b_i p_j$. Costs are zero for both firms. The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L, where $b_H > b_L > 0$. For each firm, $b_i = b_H$ with probability θ and $b_i = b_L$ with probability $1 - \theta$, independent of the realization of b_j. Each firm knows its own b_i but not its competitor's. All of this is common knowledge.

- What conditions define a symmetric pure-strategy Bayesian equilibrium in this game (assume an interior solution exists)?
- Solve for such an equilibrium.
- What happens when θ increases and why? Relate your answer to the property of strategic substitutability/complementarity.

Moore (Solve 2 out of 5 problems only!)

1. Consider an Akerlof model in which workers' individual productivities θ are uniformly distributed on [1, 6]. Each worker privately knows her own θ. A worker with productivity θ has an opportunity cost $r(\theta) = (5\theta/4) - 1$. Firms are risk neutral and have additive technologies.

   a) What would be a first-best allocation of labour?
   b) In equilibrium, what is the wage and allocation of labour?
   c) The government levies a tax t on every worker who does not work. The tax revenue is used to finance a subsidy s, paid to every worker who works. Bearing in mind that the government must break even, find the values s^* and t^* that implement first best.
   d) If the choice of s and t – respecting the government's budget constraint – were put to a vote of the entire workforce (working or not), speculate on whether the outcome of the vote would be higher or lower values than s^* and t^* respectively.

2. Consider the following Spence signalling model where workers are of two types, $\theta_L = 3$ and $\theta_H = 9$. A worker privately knows his own type. There is a fraction λ of type θ_H. A worker of type θ has cost e^2/θ of acquiring a level of university education $e \geq 0$. Competitive firms earn θ from a worker of type θ (i.e. education is intrinsically useless). The workers vote over the proposition to shut down universities in order to stop signalling. Voting is a one-voter-one-vote, with a majority rule. What is the outcome of the vote, as a function of λ?

3. Consider a competitive screening model with many firms and workers. There are equal numbers of two types of worker, respectively with productivities $\theta = 1$ and $\theta = H > 1$. And there are two possible tasks, $t = 0$ and $t = T$, where $0 < T < H(H - 1)$ (that is, T is not so high that it could never have a screening role). A worker with productivity θ has a cost of t/θ of undertaking task t, and the return from such a worker to a firm is θ (the task is intrinsically useless). The timing of the model is as follows. First, each worker privately learns her own type θ. Then, the firms offer contracts comprising a wage $w(t)$ for undertaking task t. Finally, each worker chooses which contract to accept. For which combinations of H and T (if any) is it the case that, whilst no pure-strategy separating equilibrium exists, nevertheless a pure-strategy pooling equilibrium *does* exist.

4. A monopolist, with 600 units of an indivisible good available, serves three types of customer, $i = 1, 2$ and 3, and there are 100 customers of each type. The monopolist cannot observe a customer's type. A type i customer derives monetary benefit of v_i per unit purchased, up to a capacity of 2 units, where $v_1 > v_2 > v_3 = 1$. Find the monopolist's revenue maximizing selling policy, as a function of v_1 and v_2, if she is unrestricted – i.e. she offers a menu of contracts, $\{(m_i, x_i)\}$, where a customer of type i pays a total amount m_i for x_i units of the good, and $x_i = 0, 1$ *or* 2.

5. A risk-neutral Principal contracts to hire an Agent on a project. The project either succeeds and yields $\pi > 0$ dollars revenue, or fails and yields nothing. The Agent is risk-neutral with respect to non-negative income, but has no money of his own with which to pay anything out (i.e. negative wages aren't feasible). The Agent has a reservation utility (an outside opportunity) equivalent to v dollars. If the Agent accepts the Principal's contract, he then chooses an effort level which determines the probability f that the project succeeds. When the Agent exerts 1 dollar's worth of effort (a privately-observed non-cash expenditure, netted from his wage to calculate his utility), $f = 2/3$; whereas when the Agent exerts no effort, $f = 1/3$. (Only these two effort levels are feasible.) Find the optimal contract – that is, the wage w that the Principal pays the Agent if the project succeeds – as a function of π and v.