Midterm Exam - Micro I

1. (20 points) Show that if the demand function $x(p,w)$ is generated by a rational preference relation, then it must satisfy the weak axiom.

2. (20 points) Let $v(p,x)$ be the indirect utility function of a consumer.

   (a) Show that the Marshallian demand is invariant to arbitrary monotonic transformations of $v(p,x)$.

   (b) Suppose that the consumer consumes two goods and that his Hicksian demand functions are given by:

$$h_1(p,u) = \left(\frac{\alpha}{1-\alpha}\frac{p_2}{p_1}\right)^{1-\alpha} \cdot u$$

$$h_2(p,u) = \left(\frac{1-\alpha}{\alpha}\frac{p_1}{p_2}\right)^{\alpha} \cdot u$$

   Derive the Walrasian demand function for good 1.

   (c) Show that the Walrasian demand function for good 1 does not violate homogeneity of degree 0 in (p,w) if $\alpha = \alpha(\frac{p_1}{p_2})$, but that it does violate homogeneity of degree 0 in p if $\alpha = \alpha(p_1)$.

3. (20 points) Consider a decision maker whose preferences can be represented by the following von Neumann Morgenstern utility function: $u(x) = x^{1-\rho}$, with $0 < \rho < 1$.

   (a) What can you say about the degrees of absolute and relative risk aversion of this decision maker?

   (b) Let W denote the level of wealth of the agent. The agent has to decide which fraction, a, of W to invest in a risky asset that yields a random return $\tilde{\pi}$ per unit of investment. The rest of his wealth has to be held in cash yielding no interest. Show that the optimal level of a is independent of W.

Swiss Doctoral Program
Midterm Exam 2007
Microeconomics 2

Problem 1 (30 marks)

Consider an economy with 2 consumers (i =1, 2), 2 firms (j =1, 2) and 3 goods (l =1, 2, 3). Firm 1 transforms good 3 into good 1 with a constant marginal cost 2. Firm 2 transforms good 3 into good 2 with a cost function $C(q_2) = \frac{1}{2}q_2^2$ (q_2 denotes the quantity of good 2 produced by firm 2). Both consumers have the same utility function: $u_i = x_{1,i}^2.x_{2,i}$, i =1, 2. Notice that good 3 is not consumed. The initial endowment of the economy consists of 10 units of good 3: it is owned by consumer 1, who also owns firm 1. Consumer 2 owns firm 2. Good 3 is taken as the numéraire.

1. Without computation, determine the competitive equilibrium price of good 1.
2. Compute the supply and profit functions of firm 2.
3. Compute the demand functions of the two consumers
4. Compute the prices and allocations at the competitive equilibrium.
5. Characterize the Pareto Optima of this economy.
6. Explain why they are all associated with the same productions of good 1 and 2.

Problem 2 (30 marks)

Consider an exchange economy with uncertainty. There is only one consumption good but 2 dates (t =0, 1) and two possible states of the world (s =1, 2) at date 2. There are 2 consumers with identical utility functions: $u_i = x_{oi} + \sqrt{x_{1i}} + \sqrt{x_{2i}}$, and initial endowments: $\omega_1 = (1,2,1)$ and $\omega_2 = (1,1,1)$.

1. By equalizing marginal rates of substitution across consumers for all couples of states of the world, find a simple characterization of the Pareto Optima of this economy.

There is only one financial asset (a riskless bond) giving one unit of money at date 2 (independently of the state of the world s). Its price (in terms of consumption at date 0) is denoted p

2. Compute the demand function of consumer 2 for the bond.
3. Find a relation between p and the demand of consumer 1 for the bond.
4. Show that the competitive equilibrium price satisfies the equation:

$$2 = \frac{1}{\sqrt{3p^2 - 1}} + \frac{1}{\sqrt{2p^2 - 1}}$$

5. Show that this equation has a unique solution (do not compute it)
6. Is the competitive equilibrium allocation Pareto optimal?
7. Explain this result.