2008 Midterm Exam Questions: Mark W. Watson
(Points/Minutes are given in Parentheses).

(22) 1. Suppose X has probability density function (pdf) $f(x) = 1/x^2$ for $x \geq 1$, and $f(x) = 0$ elsewhere.

(6) (a) Compute $P(X \geq 5)$.

(4) (b) Derive the CDF of X.

(4) (c) Show that the mean of X does not exist.

(5) (d) Let $Y = 1/X$. Derive the probability density function of Y.

(3) (e) Compute E(Y).

(31) 2. Suppose $X_i \sim iid$ U[0,θ], for i =1, …, n. (That is, X_i is *iid* and uniformly distributed between 0 and θ). Let $\hat{\theta} = 2\,\overline{X}$, where $\overline{X}$ is the sample mean of the X_i's.

(5) (a) Show that $\hat{\theta}$ is a consistent estimator of θ.

(8) (b) Show that $\sqrt{n}(\hat{\theta} - \theta) \xrightarrow{d} N(0,V)$, and derive an expression for V.

(10) (c) Suppose that $n = 100$, $\overline{X} = 0.45$, and $n^{-1}\sum_{i=1}^{n} X_i^2 = 0.3$. Use the approximation in (b) to test H_o: $\theta = 1$ versus H_a: $\theta < 1$.

(8) (d) Derive the maximum likelihood estimator of θ.

(7) 3. $Y_i \sim iid$ N(0, σ^2). Show that $S = \sum_{i=1}^{n} Y_i^2$ is a sufficient statistic for σ^2.

Bo Honoré

Problem 4 (25 points)

The tables on the last page show output from running the same regression,

$$\ln(y) = \beta_0 + \beta_1 x_1 + \beta_2 \ln(x_2) + \beta_3 x_3 + \beta_4 x_4 + \varepsilon,$$

on two different independent data sets representing two regions of the same country. In answering the following assume that the asymptotic properties derived in Chapter 2 of Hayashi provide good approximations.

1. (5 points) Find the t–statistic for β_0 and the standard error for β_4 for Region 1.

2. (5 points) Consider an individual in Region 1 with $y = 25$ and $x_1 = 1$. What is the estimated effect on y of changing x_1 to 1.02?

3. (5 points) Construct a 95% confidence interval for the difference in β_2 across the regions.

4. (10 points) Perform a joint test on the hypothesis that β_3 is equal to 0 in both of the regions. (Test at a 5% level of significance).

Problem 5 (15 points)

Suppose that you have n independent and identically distributed observations of the vector (y_i, x_{1i}', x_{2i}'). Suppose that you run two regressions. First you regress y_i on x_{1i}, then you regress y_i on x_{2i}. Note that you do not include constants in either of the regressions. Let $\widehat{\beta}_1$ and $\widehat{\beta}_2$ denote the two OLS estimators. Explain how you would find the joint asymptotic distribution of $\left(\widehat{\beta}_1, \widehat{\beta}_2\right)$ (under the assumption that all relevant moments exist).

Problem 6 (20 points)

Suppose that

$$y_i = z_i\delta + \varepsilon_i,$$
$$E\left[\varepsilon_i \middle| x_{1i}, x_{2i}\right] = 0$$

and

$$E\left[\varepsilon_i^2 \middle| x_{1i}, x_{2i}\right] = \sigma^2,$$

where y_i, z_i, x_{1i} and x_{2i} are observed one–dimensional variables.

Let $\widehat{\delta}_1$ be the Two Stage Least Squares estimator that uses x_{1i} as instrument, and let $\widehat{\delta}_2$ the Two Stage Least Squares estimator that uses both x_{1i} and x_{2i}.

1. (10 points) Assume that $E[z_i x_{2i}] = 0$ and $E[x_{1i} x_{2i}] = 0$. How do the asymptotic variances of $\widehat{\delta}_1$ and $\widehat{\delta}_2$ compare?

2. (10 points) Assume that $E[z_i x_{2i}] = 0$ and $E[x_{1i} x_{2i}] \neq 0$. How do the asymptotic variances of $\widehat{\delta}_1$ and $\widehat{\delta}_2$ compare?

Regression Results for Problem 4.

REGION 1.

Included observations: 100
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.
C	1.060612	0.204637		0.0000
X1	1.361730	0.233552	5.830513	0.0000
LN_X2	0.675347	0.135795	4.973274	0.0000
X3	0.707917	0.618196	1.145133	0.2550
X4	1.456821		2.010054	0.0473

R-squared	0.527476	Mean dependent var	2.533448
Adjusted R-squared	0.507580	S.D. dependent var	1.031477
S.E. of regression	0.723814	Akaike info criterion	2.240143
Sum squared resid	49.77119	Schwarz criterion	2.370402

REGION 2.

Included observations: 100
White Heteroskedasticity-Consistent Standard Errors & Covariance

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.803798	0.221998	3.620748	0.0005
X1	1.581766	0.253286	6.244990	0.0000
LN_X2	0.011249	0.135684	0.082904	0.9341
X3	1.089469	0.615191	1.770944	0.0798
X4	-1.022954	0.672734	-1.520591	0.1317

R-squared	0.325427	Mean dependent var	1.846091
Adjusted R-squared	0.297024	S.D. dependent var	0.860210
S.E. of regression	0.721232	Akaike info criterion	2.232995
Sum squared resid	49.41667	Schwarz criterion	2.363253