Bob King

1. Rational expectations models (15 points)

The following model describes how endogenous variables (k,i,p) respond to an exogenour variable x_t which varies stochastically.

$$\begin{aligned} k_{t+1} &= i_t + (1-\delta)k_t \\ k_{t+1} &= \beta E_t p_{t+1} - p_t \\ ak_t + x_t &= i_t - \gamma p_t \end{aligned}$$

In this model, all variables are interpreted as deviations from a stationary position.

(a) (5 points) Place this model in first-order form, $AEY_{t+1} = BY_t + Cx_t$.

(b) (5 points) Explain intuitively how many (finite) eigenvalues this model should have. Derive a polynomial that restricts these eigenvalues.

(c) (5 points) Assume that $0 < \beta < 1$ and that $a - \delta > 0$. Supposing that only k_t is predetermined, indicate the conditions for a unique, stable rational expectations solution to this model. Are there the required number of stable roots?

2. Dynamic optimization (15 points)

Consider a family that maximizes a weighted sum of its members' utilities,

$$\sum_{t=0}^{T} \beta^t \theta_{it} u(c_{it}),$$

where θ_{it} is the weight attached to the momentary utility flow that family member i derives from his consumption c_{it}. In addition, assume that each household member has a momentary objective of the form

$$u(c) = \frac{1}{1-\sigma} c^{1-\sigma}$$

Suppose further that the household faces a budget constraint of the form

$$\sum_{t=0}^{T} \beta^t \sum_{i=1}^{I} c_{it} \leq \Omega$$

where Ω is the level of family wealth.

(a) (5 points) Find a general family efficiency condition for c_{it} that is directly obtained from a Lagrangian.

(b) (5 points) Explain intuitively why $\theta_{it} = \theta$ should lead to consumption that is constant across individuals and time. Show that the efficiency condition in (a) supports that finding.

(c) (5 points) Suppose that there are two individuals $i = 1, 2$ that have paths which satisfy

$$\theta_{1t} = \alpha\theta_{2t}$$

What can you say about the level of the consumption of individual 1 relative to that of individual 2 if $\alpha > 1$? What can you say about the relative growth rates of their consumptions?

3. Dynamic programming (30 points)

Suppose that there is an individual who can hold either of two risky assets. The prices of these assets are governed by

$$p_j(\varsigma_t)$$

i.e., they depend on the state of the economic system ς_t. The dividend payouts on these securities d_{jt} and the individual's income also depend on this state.

The individual has an objective of the form

$$E_t\{\sum_{t=0}^{\infty} \beta^t u(c_t)\}$$

and a budget constraint of the form

$$\sum_{j=1}^{2} [p_j(\varsigma_t) + d_j(\varsigma_t)] z_{jt} + y(\varsigma_t) \geq c_t + \sum_{j=1}^{2} p_j(\varsigma_t) z_{j,t+1}$$

where z_{jt} is the amount of asset j held from $t-1$ to t (and thus $z_{j,t+1}$ is the amount held from t to $t+1$).

(a) (8 points) Place this model in dynamic programming form by writing a Bellman equation for the individual's consumption, saving and portfolio choice problem. Please indicate the controlled and exogenous states of this problem.

(b) (8 points) Find the efficiency conditions for consumption (c_t) and portfolio holdings ($z_{j,t+1}$).

(c) (8 points) What is the marginal value of having a little bit more of z_{1t} according to the envelope theorem? What is the underlying economic reasoning behind this result?

(d) (6 points) Suppose that the individual was constrained to hold positive quantities of both assets (no short-sales were allowed). How would that alter your answers to (a)?

Sergio Rebelo

1. Short question: the neoclassical growth model (16 points) Consider the following version of the neoclassical growth model in which output is produced with labor (N_t), capital (K_t), and land (T):

$$\max U = \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

subject to,

$$Y_t = AK_t^{\alpha} N_t^{\gamma} T^{1-\alpha-\gamma},$$

$$Y_t = C_t + I_t$$

$$K_{t+1} = I_t + (1-\delta)K_t$$

a) Compute the steady state level of the stock of capital.

b) Firms in this economy make decisions about hiring labor, capital and land so as to maximize their profits, π_t:

$$\pi_t = AK_t^{\alpha} N_t^{\gamma} T_t^{1-\alpha-\gamma} - w_t N_t - R_t^K K_t - R_t^T T_t.$$

Here w_t is the real wage rate, and R_t^K and R_t^T are the rental prices of capital and land, respectively. Compute the equilibrium values of w, R^K, and R^T.

c) Suppose that there is migration of labor into this economy, so N_t goes up. What is the impact of a marginal increase in N_t on the steady-state capital stock and on w, R^K, and R^T?

2. Short question: optimal advertising for a monopolist (16 points) Consider a monopolist who faces a demand for his product given by:

$$x_t = a_t p_t^{-\gamma}.$$

Here x_t denotes the quantity sold and p_t the price chosen by the monopolist. The marginal cost of production is constant and equal to c. The monopolist can invest in advertising to increase the level of demand:

$$a_{t+1} = (I_t)^{\alpha} + (1-\delta)a_t \tag{1}$$

where I_t denotes the level of advertising investment and α and δ are parameters that take values between zero and one. The firm's objective is to maximize the present value of profits:

$$V = \sum_{t=0}^{\infty} \beta^t \left(p_t x_t - c x_t - I_t\right)$$

subject to the law of motion for a_t, (1).

a) Derive the optimal price charged by the monopolist as a function of marginal cost, c.

b) Derive the equation that characterizes the optimal behavior of investment in advertisement.

3. Long question: small open economy (28 points) Consider an economy populated by a representative agent who works one unit of time in every period. This agent seeks to maximize his or her utility defined as:

$$U = \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\sigma} - 1}{1-\sigma} dt,$$

where C_t represents the level of per capita consumption at time t.

$$\dot{a}_t = r a_t + Y - C_t$$
$$\lim_{t \to \infty} e^{-rt} a_t = 0, \tag{2}$$

$a_0 > 0$, given. Suppose that $r = \rho$.

a) Derive the first-order conditions for the planner's problem for this economy.

b) Why do we need to impose the condition (2)?

c) Compute the optimal path for consumption.

d) Compute the condition under which lifetime utility, U, is finite.

e) Suppose that at time zero, the economy learns that the level of output will increase *permanently* from Y to $Y + \phi$ from time $T > 0$ on. What is the impact of this shock on the optimal consumption path and on the economy's current account?

f) Suppose that at time zero, the economy learns that the level of output will increase *temporarily* from Y to $Y + \phi$ between time zero and time $T > 0$. What is the impact of this shock on the optimal consumption path and on the economy's current account?