Midterm Exam - Micro I

1. (20 points) Suppose $L = 3$, and consider the demand function $x(p, w)$ defined by

$$
\begin{aligned}
x_1(p,w) &= \frac{p_1^{\alpha} w}{p_1^{\delta} + p_2^{\delta} + p_3^{\delta}} \\
x_2(p,w) &= \frac{p_2^{\alpha} w}{p_1^{\delta} + p_2^{\delta} + p_3^{\delta}} + \beta \frac{p_1}{p_3} \\
x_3(p,w) &= \frac{\gamma p_3^{\alpha} w}{p_1^{\delta} + p_2^{\delta} + p_3^{\delta}}
\end{aligned}
$$

   For what values of α, β and γ does this demand function satisfy

   (a) homogeneity of degree zero?

   (b) Walras law?

2. (20 points) A consumer has expenditure function

$$
e(p_1, p_2, u) = \frac{u p_1 p_2}{p_1 + p_2} .
$$

   (a) What is indirect utility function?

   (b) Derive the Walrasian demand functions.

   (c) Compute the elasticity of substitution between good 1 and good 2.

   (d) Find a direct utility function $u(x_1, x_2)$ that rationalizes the person's demand behavior.

3. (20 points) Consider a price change from the initial price vector p^0 to a new price vector $p^1 \leq p^0$ in which only the price of good l changes. Show that $EV \leq CV$ if good l is inferior.

Jean-Charles Rochet

Problem 1 (40 marks)

Consider an exchange economy with 2 consumers (ι =1; 2),and 2 goods (λ =1; 2). Consumers have the following utility functions:

$u_1 = x_{1,1}.x_{2,1}$, $u_2 = \sqrt{x_{1,2}} + x_{2,2}$,

Initial endowments are (1,0) for consumer 1 and (0,1) for consumer 2.

1. Characterize the interior Pareto Optima of this economy.
2. Show that there are other Pareto Optima on the boundary of the Edgeworth box.
3. Compute the demand functions of the two consumers.
4. Show that there is a unique competitive equilibrium.
5. Compute the associated price and allocations.

Problem 2 (20 marks)

Consider an exchange economy with uncertainty. There is only one consumption good but 2 dates (τ =0; 1) and three possible states of the world (σ =1; 2,3) at date 1. There are 3 financial assets:

- ☐ A riskless bond (of price T at date 0) that delivers one unit of good at date 1, irrespective of the state of the world.
- ☐ A stock (of price S at date 0) that delivers 2 units of the good in state 1, and 0 in states 2 and 3
- ☐ A risky bond (of price B at date 0) that delivers 1 unit of the good in states 1 and 2 and zero in state 3.

1. Show that there is a unique vector (p_1, p_2, p_3) of Arrow contingent prices that is compatible with the prices T, B, S of these financial assets. Recall that an Arrow contingent price is the price of a security that delivers one in one state of the world and zero otherwise.
2. Compute this vector.
3. Compute the price C at date 0 of a call option on the stock at date 1, with a strike (exercise price) of 1. Why is it independent of T and B?