Midterm Examination: Mark Watson

(12) 1. $X \sim N(1, 4)$ and $Y = e^X$.

(6) (a) What is the density of $Y$?

(6) (b) Compute $E(Y)$. (Hint: What is the MGF for $X$?)

(15) 2. The density of $Y$ is $e^Y$ for $-\infty < Y < 0$ and zero elsewhere.

(6) (a) Compute $P(Y \geq -1)$.

(9) (b) Find the value of $c$ that minimizes $E|Y - c|$.

(18) 3. $Y_i \sim$iid N($\mu_Y$, 1) and $X_i \sim$ iid N($\mu_X$, 1) for $i = 1, \ldots, n$. Suppose $X_i$ and $Y_j$ are independent for all $i$ and $j$. A researcher is interested in $\theta = \mu_Y\mu_X$.

(8) (a) Suppose that $\mu_Y = 0$ and $\mu_X = 5$. Show that $\sqrt{n}\hat{\theta} \xrightarrow{d} W \sim N(0,V)$ and derive an expression for $V$.

(10) (b) Suppose that $\mu_Y = 1$ and $\mu_X = 5$. Show that $\sqrt{n}\left(\hat{\theta} - 5\right) \xrightarrow{d} W \sim N(0,V)$ and derive an expression for $V$.

(15) 4. Suppose $Y_1$ and $Y_2$ are iid with mean $\mu$ and variance $\sigma^2$. Let $\hat{\mu} = 0.25Y_1 + 0.75Y_2$. Suppose that risk is mean squared error, so that $R(\hat{\mu}, \mu) = E[(\hat{\mu} - \mu)^2]$.

(7) (a) Compute $R(\hat{\mu}, \mu)$.

(8) (b) Show that $\hat{\mu}$ is inadmissable.

Midterm 2009, Metrics, Bo Honore's part (starts with problem 5)

**Problem 5 (22 points)**

An economist runs the regression

$$y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + x_{3i}\beta_3 + \varepsilon_i \tag{1}$$

The results are summarized in the following table:

| **Equation 1.** | | |
|---|---|---|
| | | |
| Variable | Coefficient | Std. Error |
| $\beta_0$ | 0.82 | 1.04 |
| $\beta_1$ | -1.16 | 5.74 |
| $\beta_2$ | 7.63 | 3.70 |
| $\beta_3$ | -2.57 | 0.94 |
| | | |
| R-squared | 0.102 | |
| Sum squared resid | 10297 | |
| Number of observations | 1000 | |

In answering the follow questions, assume that the assumptions needed to derive finite sample properties of the OLS estimator (Hayashi, Chapter 1) are satisfied.

**(a)** (3 points) Construct a 95% confidence interval for $\beta_3$ in Equation 1.

**(b)** (3 points) Test whether $\beta_2$ in Equation 1 equals 1 (against the alternative that it differs from 1). Test at a 5% level of significance.

**(c)** (5 points) Construct an estimate of the unconditional variance of $y_i$

Using the same data, the economist also runs the regression

$$y_i = \beta_0 + x_{2i}\beta_2 + \nu_i \tag{2}$$

Some of the results are summarized in the following table:

| **Equation 2.** | | |
|---|---|---|
| | | |
| Variable | Coefficient | Std. Error |
| $\beta_0$ | 0.83 | 1.07 |
| $\beta_2$ | 6.86 | 3.81 |
| | | |
| R-squared | 0.032 | |
| Sum squared resid | ? | |
| Number of observations | 1000 | |

**(d)** (6 points) Test the hypothesis that both $\beta_1$ and $\beta_3$ in Equation 1 equal 0 (against the alternative that at least one of them differs from 0) Test at a 5% level of significance.

**(e)** (5 points) What is the sum of squared residuals in equation 2?

**Problem 6 (20 points)**

Suppose that

$$y = X\beta + \varepsilon$$

with

$$E\left[\varepsilon \middle| X\right] = 0 \qquad \text{and} \qquad V\left[\varepsilon \middle| X\right] = \sigma^2 I$$

Assume that $X'X$ is invertible with probability 1.

Let $\widehat{\beta}$ be the OLS estimator of $\beta$ and let $\widetilde{\beta} = Ay$ be some alternative unbiased estimator of $\beta$ (i.e., $E\left[\widetilde{\beta}\middle| X\right] = \beta$). $A$ is allowed to depend on $X$, but conditional on $X$, it is a fixed (non–random) matrix.

**(a)** (5 points) Show that unbiasedness of $\widetilde{\beta}$ ($E\left[\widetilde{\beta}\middle| X\right] = \beta$ for all $\beta$) implies that $AX = I$.

**(b)** (15 points) Let $V = \widetilde{\beta} - \widehat{\beta}$. Find $\operatorname{cov}\left(\widehat{\beta}, V\middle| X\right)$.

**Problem 7 (18 points)**

Consider the model

$$\begin{aligned} y_i &= x_i^* \delta + \varepsilon_i \\ z_i &= x_i^* + v_i \\ x_i &= \rho x_i^* + u_i \end{aligned}$$

For simplicity assume that $x_i^*, \varepsilon_i, v_i$ and $u_i$ independent (one–dimensional) random variables with mean 0 and finite variances. $\delta$ is the parameter of interest and $\rho$ is some number different from 0.

**(a)** (9 points) Suppose that you have a random sample of size $n$ from the distribution of $(y_i, x_i, z_i)$, and that you calculate the 2SLS estimator for the regression of $y_i$ on $z_i$ that uses $x_i$ as instrument. Find the asymptotic distribution of this estimator.

**(b)** (9 points) Does the variance of the asymptotic distribution found in (a) depend on $\rho$. If so, what value(s) of $\rho$ will minimize it?