Midterm 2009, Macro, Bob King's part

Q1: Short answer question: optimal consumption with an uncertain lifetime [15 points]

Consider an individual that faces a constant positive risk of death at the end of each period in a discrete time model. More specifically, his probability of dying is δ and his probability of living is θ ($\delta + \theta = 1$). If the individual is alive for τ periods, then his utility is

$$U_\tau = \sum_{t=0}^{\tau-1} \beta^t u(c_t)$$

with $u(c)$ being positive, increasing, and concave.

If he is alive, then he receives a constant stream of income

$$y_t = y$$

If he is dead, then he receives neither flow utility ($u(c)$) or income (y): both of these are zero.

The individual can lend at an interest rate r and he can borrow only if he can repay with certainty. His assets therefore evolve according to

$$a_{t+1} = R(a_t + y_t - c_t)$$

Assets are 0 in the initial period of life (t=0).

The individual can live at most T periods. Formulate the individual's decision problem in dynamic programming terms and determine his optimal borrowing/lending policy if $R\beta = 1$, where $R = 1 + r$.

Q2: Short answer question: Rational expectations models [15 points]

Consider the following rational expectations model, with λ_t nonpredetermined and k_t being predetermined. This model is written in matrix form as

$$E_t \begin{bmatrix} \lambda_{t+1} \\ k_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} \lambda_t \\ k_t \end{bmatrix} + \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} x_t$$

where $x_t = \rho x_{t-1} + e_t$ with e_t mean zero and serially independent. The parameter ρ satisfies $|\rho| < 1$.

(a) If $|\mu_1| > 1$ and $|\mu_2| < 1$, is there a unique stable RE solution? If so, what is it? If not, then are there multiple stable solutions or no stable solution?

(b) If $|\mu_1| < 1$ and $|\mu_2| > 1$, is there a unique stable RE solution? If so, what is it? If not, then are there multiple stable solutions or no stable solution?

Q3: Long question: Investment adjustment costs [30 points]

Consider a firm which has a production technology

$$y_t = a(\varsigma_t) k_{t-1}$$

where a is a time-varying level of productivity that depends on an exogenous state variable ς_t; y_t is output; and k_{t-1} is the stock of capital which is determined in the prior period.

The firm can alter its capital stock via investment expenditure. The cost of a unit of the investment good is $p(\varsigma_t)$ and the amount of investment is i_t. Given that investment, the capital stock evolves according to

$$k_t - k_{t-1} = \phi(\frac{i_t}{k_{t-1}})k_{t-1} - \delta k_{t-1}$$

The function ϕ has the following properties. First it is zero when i is zero, so that capital then falls with depreciation (at rate δ). Second, it is increasing and strictly concave, so that successive units of investment have positive and diminishing effects on capital accumulation. Third, it has the characteristic that $\phi(\delta) = \delta$. That is: investment of $i_t = \delta k_{t-1}$ is sufficient to maintain the capital stock.

(a) Suppose that the firm seeks to maximize the present value of its profits,

$$E_t\{\sum_{j=0}^{\infty} \beta^j [y_{t+j} - p_{t+j}i_{t+j}]\}$$

Write a Bellman equation for the firm's problem.

(b) What is the efficiency condition for investment?

(c) A celebrated implication of this model is that there is an investment specification that depends on Tobin's Q, i.e., the stock market value of the firm relative to the replacment cost of its capital. Assuming that the firm pays out its profits, $y - pi$ as dividends, define the stock market value as the ex dividend value and derive the Tobin's Q relation.

(d) Now suppose that each period, the firm receives a draw of a random fixed cost ξ, from a continuously distributed iid random variable with support (0,B). That is, the firm must pay the fixed cost to invest, but this cost might be low (near zero) or high (near B). The volume of fixed costs which the firm must pay is assumed proportional to its previous capital stock, so that these are ξk. Write a Bellman equation for the firm that has such adjustment costs.

(e) Suppose that positive investment is desirable when there is a zero random draw for the adjustment cost and that it is not when the highest cost is realized. Show that there is a critical level of adjustment costs, below which the firm should invest and above which it should not.

(f) Now suppose that there are many firms, with each firm receiving an independent adjustment cost draw but with all facing the same productivity shock and investment good price. Would this model have the following implications?

(f-1) In each period, some firms would have positive investment, while others would not;

(f-2) Large firms would be more likely to adjust than small firms;

Midterm 2009, Macro, Sergio Rebelo's part

1. Short question: a neoclassical model with two consumption goods (16 points). Consider the following version of the neoclassical growth model in which households consume both nondurable goods (C_t) and services (S_t). Life-time utility is given by:

$$\max U = \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \theta \log(S_t)\right].$$

The economy's resource constraints are as follows:

$$Y_t = AK_t^{\alpha} N^{1-\alpha},$$

$$Y_t = C_t + S_t + I_t,$$

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where Y_t, N, I_t and K_t denote the values of output, labor, investment, and capital, respectively.

a) Compute the steady state level of the capital stock.

b) Compute the steady state levels of S_t and C_t.

2. Short question: monopolistic competition (16 points) Consider an economy in which final output, y_t, is produced by perfectly competitive firms that combine labor with a continuum $j \in [0,1]$ of sectoral goods, $q_t(j)$. There is a continuum of workers of measure n. To simplify we assume that each worker operates her own firm. She supplies H hours of work per period and produces according to the production function:

$$y_t = \left[\int_0^1 q_t(j)^{\alpha} dj\right] H^{1-\alpha},\ 0 < \alpha < 1.$$

We choose the price of final output as the numeraire. The problem of the worker can be written as maximizing output net of the cost of purchasing sectoral goods:

$$\max \text{ Net Output Per Worker} = \left[\int_0^1 q_t(j)^{\alpha} dj\right] H^{1-\alpha} - \int_0^1 p_t(j) q_t(j) dj,$$

where $p_t(j)$ is the price of sectoral good j.

a) Compute the first order conditions for this problem.

b) Compute the total demand for good j, $Q_t(j)$, defined as:

$$Q_t(j) = q_t(j)n.$$

c) Show that in a symmetric equilibrium in which all sectoral goods have the same price, p_t, the net output per worker is given by:

$$\text{Net Output Per Worker} = (1-\alpha)\left(\frac{\alpha}{p_t}\right)^{\alpha/(1-\alpha)} H.$$

d) Suppose that each sectoral good j is produced with ϕ units of final output by a different monopolist. Write down the monopolist's profit maximization problem and compute the optimal price that she should charge.

3. Long question: small open economy (28 points) Consider a small open economy populated by a representative household who maximize its utility defined as:

$$U = \int_0^\infty e^{-\rho t}\left[\log\left(C_t^T\right) + \log(C_t^N)\right] dt,$$

where C_t^T and C_t^N represent the consumption of tradable and non-tradable goods, respectively. The economy has a constant endowment of tradable and nontradable goods denoted by Y^T and Y^N, respectively. The budget constraint of the household is given by:

$$\dot{a}_t = ra_t + Y^T + p_t Y^N - C_t^T - p_t C_t^N,$$

and

$$\lim_{t\to\infty} e^{-rt} a_t = 0.$$

where a_t is the level of net foreign assets and p_t denotes the relative price of nontradable goods in terms of tradable goods. The value of $a_0 > 0$ is given. Suppose that $r > \rho$.

a) Derive the first-order conditions for the household's problem.

b) Impose the equilibrium condition:

$$C_t^N = Y^N.$$

which results from the fact that nontradable goods cannot be imported, so they have to be locally produced. Compute the equilibrium path for the relative price of nontradable goods, p_t.

c) Characterize the path for this economy's current account.

d) Suppose that at time zero, the economy learns that the endownment of nontradable goods rises *permanently* from Y^N to $Y^N + \phi$ from time zero on. What is the impact of this shock on the path for the relative price of nontradables?

e) Suppose that at time zero, the economy learns that the endowment of nontradable goods rises *temporarily* from Y^N to $Y^N + \phi$ between time zero and time $T > 0$. What is the impact of this shock on the path for the relative price of nontradables?