January 2010 Final Exam Questions: Mark W. Watson
(Points/Minutes are given in Parentheses).

(15) 1. Suppose that y_t is iid$(0,\sigma^2)$ with $E(y_t^4) = \kappa$. Let $\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T} y_t^2$.

(10) (a) Show that $\sqrt{T}(\hat{\sigma}^2 - \sigma^2) \xrightarrow{d} N(0, V_{\hat{\sigma}^2})$ and derive an expression for $V_{\hat{\sigma}^2}$.

(5) (b) Suppose that $T = 100$, $\hat{\sigma}^2 = 2.1$ and $\frac{1}{T}\sum_{t=1}^{T} y_t^4 = 16$. Construct a 95% confidence interval for σ, where σ is the standard deviation of y.

(25) 2. Suppose that y_t follows the AR(1) process $y_t = \phi y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim$ iidN$(0,\sigma_\varepsilon^2)$. Let $\hat{\phi}$ denote the OLS estimator, $\hat{\varepsilon}_t = y_t - \hat{\phi} y_{t-1}$ denote the OLS residual, and $\hat{\sigma}_\varepsilon^2 = \frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t^2$ denote the estimator of σ_ε^2.

(10) (a) Suppose that $|\phi| < 1$ and $\{y_t\}$ is stationary. Show that $\hat{\sigma}_\varepsilon^2 \xrightarrow{p} \sigma_\varepsilon^2$

(15) (b) Suppose that $\phi = 1$ and $y_0 = 0$. Show that $\hat{\sigma}_\varepsilon^2 \xrightarrow{p} \sigma_\varepsilon^2$.

(30) 3. Suppose that y_t follows the stationary process $y_t = \beta y_{t-2} + u_t$, where $u_t = (1-\theta L)\varepsilon_t$, $|\beta| < 1$, and $\varepsilon_t \sim$ iid$(0, \sigma_\varepsilon^2)$. Let $\hat{\beta}$ denote the OLS estimator of β and $\hat{u}_t$ denote the OLS residual.

(8) (a) Show that $\hat{\beta} \xrightarrow{p} \beta$.

(12) (b) Show that $\sqrt{T}(\hat{\beta} - \beta) \xrightarrow{d} N(0, V_{\hat{\beta}})$, and derive an expression for $V_{\hat{\beta}}$.

(10) (c) Suppose that $T = 400$, $\hat{\beta} = 0.59$; $T^{-1}\sum_{t=1}^{T} \hat{u}_t^2 y_{t-2}^2 = 1.67$, $T^{-1}\sum_{t=1}^{T} \hat{u}_t y_{t-2} \hat{u}_{t-1} y_{t-3} = 0.14$, $T^{-1}\sum_{t=1}^{T} \hat{u}_t y_{t-2} \hat{u}_{t-2} y_{t-4} = -0.10$, $T^{-1}\sum_{t=1}^{T} \hat{u}_t^2 = 1.06$, $T^{-1}\sum_{t=1}^{T} \hat{u}_t \hat{u}_{t-1} = 0.30$, $T^{-1}\sum_{t=1}^{T} \hat{u}_t \hat{u}_{t-2} = 0.02$, $T^{-1}\sum_{t=1}^{T} y_{t-2}^2 = 1.63$, $T^{-1}\sum_{t=1}^{T} y_{t-2} y_{t-3} = 0.65$, $T^{-1}\sum_{t=1}^{T} y_{t-2} y_{t-4} = 0.95$. Test H_0: $\beta = 0.55$ vs H_a: $\beta \neq 0.55$ using a test with size of 5%.

(20) 4. Suppose that y_{1t} and y_{2t} are scalar random variables with

$$y_{1t} = x_t + \varepsilon_{1t}$$

$$y_{2t} = x_t + \varepsilon_{2t}$$

where x_t follows the stationary AR(1) model $x_t = 0.8x_{t-1} + e_t$, and where $\{e_t\}$, $\{\varepsilon_{1t}\}$, and $\{\varepsilon_{2t}\}$ are are mutually independent i.i.d. sequences of $N(0,1)$ random variables. A researcher has data on y_{1t} and y_{2t} and would like to use these data to estimate the value of x_t. He proposes the estimator $\hat{x}_t = \frac{1}{2}(y_{1t} + y_{2t})$.

(10) (a) Compute the mean squared error of $\hat{x}_t$.

(10) (b) A more general estimator is $\tilde{x}_t = \lambda_1 y_{1t} + \lambda_{2t} y_{2t}$, where λ_1 and λ_2 are two constants. What values of λ_1 and λ_2 yield the estimator with the smallest mean squared error?

Problem 5. (16 points)

Suppose that a duration, T, has the following hazard given an explanatory variable, x,

$$h(t|x) = t^{\alpha-1} \exp(\beta_0 + x\beta_1).$$

You will recognize this as a proportional hazard model

Suppose that $\alpha = 0.8$, $\beta_0 = 0$ and $\beta_1 = 0.5$. Find the density of T conditional on $x = -1$.

Problem 6. (25 points)

Suppose you are interested in the effect of education and income on internet use. Using a random sample of individuals, you observe a binary variable, I_i, which takes the value 1 if person i has access to internet and 0 otherwise. You model I_i as a function of person i's income Inc_i (in \$1,000) and education Ed_i (in years). Specifically, you estimate a linear regression model and a logit model with I_i as the dependent variable and Inc_i and Ed_i as the explanatory variables.

You get the following output

Linear Regression	$b_0 = 0.2;$	$b_1 = 0.012;$	$b_2 = 0.009;$
Logit	$b_0 = -1.2;$	$b_1 = 0.046;$	$b_2 = 0.038;$

where b_0, b_1 and b_2 are estimates of the constant term, the coefficient on income and the coefficient on education, respectively.

(a) (8 points) For each of the two models, calculate the predicted probability that a person will use the internet if she has a high school diploma (i.e. 12 years of education) and an income of \$15,000 per year.

(b) (8 points) What is this predicted probability for an individual with a college degree (i.e. 16 years of education) who is making \$60,000 per year in each of the two models?

(c) (9 points) For each of the two models, calculate the predicted marginal effect of income (measured in \$1,000) on the probability that a person will use the internet if she has a high school diploma (i.e. 12 years of education) and an income of \$15,000 per year.

Problem 7. (30 points)

Suppose that you have a random sample $\{X_i\}_{i=1}^n$ from some distribution with density, f. Consider the following estimator of f at a point x,

$$\widehat{f}(x) = \frac{1}{nh_n} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_n}\right)$$

where K is the density for a uniform random variable on the interval $(-\frac{1}{2}, \frac{1}{2})$, and h_n is some (small) bandwidth. If the true (unknown) f is is the density of a uniform distribution

$$f(x) = \begin{cases} 1 & \text{if} \quad 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

find the (approximate) means square error of $\widehat{f}(x)$. How does it depend on n? How does it depend on x?

Problem 8. (19 points)

Consider the linear panel data model

$$y_{it} = \alpha_i + x_{it}'\beta + \varepsilon_{it}, \qquad t = 1, \ldots T_i \qquad i = 1, \ldots n.$$

(a) (9 points) Discuss the relative meritst of the within–estimator and the between–estimator of β.

(b) (10 points) Which other estimators of β might you consider?