Midterm Exam - Micro I

1. (20 points) Hicks (1956) offered the following example to demonstrate how the Weak Axiom of Revealed Preference (WARP) can fail to result in transitive revealed preferences when there are more than two goods. The consumer always chooses bundle x^i at prices p^i, $i = 0, 1, 2$, where

$$p^0 = (1,\ 1,\ 2) \qquad x^0 = \begin{pmatrix} 5 \\ 19 \\ 9 \end{pmatrix}$$

$$p^1 = (1,\ 1,\ 1) \qquad x^1 = \begin{pmatrix} 12 \\ 12 \\ 12 \end{pmatrix}$$

$$p^2 = (1,\ 2,\ 1) \qquad x^2 = \begin{pmatrix} 27 \\ 11 \\ 1 \end{pmatrix}$$

   (i) Show that these data satisfy WARP.

   (ii) Find the intransitivity in the revealed preferences.

2. (20 points) A consumer's utility function is given by

$$u(x_1, x_2) = (x_1 - \gamma_1)^{\beta_1} \cdot (x_2 - \gamma_2)^{\beta_2}$$

   ($\beta_1, \beta_2 > 0$) which is defined for $x_l \geq \gamma_l$.

   (a) Can we assume (without loss of generality) that $\beta_1 + \beta_2 = 1$?

   (b) Compute the consumer's Walrasian demand functions and show that his expenditures ($p_i x_i$) for goods 1 and 2 give rise to a linear expenditure system, i.e. $p_i x_i$ is equal to a constant plus a term that is linear in the income that remains after the two constants have been deducted. Assuming that $\gamma_1, \gamma_2 \geq 0$, how can this expenditure system be interpreted?

   (c) Under what assumptions on γ_1 and γ_2 are the preferences of the consumer homothetic? [A consumer has *homothetic preferences* if his utility function is homothetic. A function $f(x)$ is *homothetic* if $f(x) = g(h(x))$, where $g(\cdot)$ is a strictly increasing function and $h(\cdot)$ is a function that is homogeneous of degree one in the vector x.]

3. (20 points) Consider the following utility function

$$u(x) = a + bx + cx^2$$

   where $x \in I\!R$ is the wealth of the decision maker.

(a) Which restrictions, if any, must be imposed on the parameters a, b and c for this function to be a von Neumann-Morgenstern utility function of a risk averse decision maker?

(b) Over which domain is this vNM utility function well defined?

(c) Show that with this utility function the expected utility from a distribution is determined by the mean and variance of the distribution and, in fact, by these moments alone. Furthermore, show that his utility is increasing in the mean and decreasing in the variance of the distribution.

Midterm Exam 2010 – Micro II (Rochet's part)

Consider a production economy with two consumers (i=1, 2), two goods (l=1,2), one firm and two dates (t=1,2). At date 1, c_{11} units of good 1 are consumed by consumer 1 and x_1 units of good 1 are used by the firm to produce $y_2 = 2\sqrt{x_1}$ units of good 2. At date 2 good 2 is consumed by both consumers: c_{21} units are consumed by consumer 1 and c_{22} units are consumed by consumer 2. Good 1 is taken as a numeraire (its price is normalized to one). The price of good 2 is denoted *p*. Consumer 1 initially owns the total endowment of the economy (one unit of good 1) and consumer 2 owns the firm. Their preferences are given by:

$$u_1(c_{11}, c_{21}) = \sqrt{c_{11}c_{21}}, u_2(c_{22}) = c_{22}.$$

1. Compute the profit function of the firm and the demand functions of the consumers. Characterize the competitive equilibrium of this economy (prices and allocations). **(20 points)**
2. Show that the Pareto Optima of this economy can be parameterized as follows: for each c_{11} in (0, 2/3) there is a unique Pareto optimum such that consumer 1 consumes exactly c_{11} at date 1**.** Compute the associated values of c_{12} and c_{22}. **(20 points)**

Uncertainty is introduced in the model. The production function becomes $y_{2s} = 2\alpha_s\sqrt{x_1}$ where s =H, L represents the state of the world at date 2 and $\alpha_H > \alpha_L$. The probability of state s is denoted q_s. We assume that $q_H\alpha_H + q_L\alpha_L = 1.$ The quantity of good 2 consumed by consumer i in state s is denoted c_{si}. When consumption plans and production plans are chosen at date 1, s is still unknown, but there are contingent markets: for a price p_s (good one is still taken as the numeraire) each consumer can obtain the delivery of one unit of good 2 if state s occurs. Each consumer maximizes his expected utility under his budget constraint. The firm chooses x_1 so as to maximise its value at date 1, namely $\pi = \sum_{s=H,L} p_s y_{2s} - x_1$.

3. Compute the demand functions of consumer 2: Show that they are interior only when contingent prices are proportional to probabilities: $p_s = pq_s$ for some *p>0.* Show that in this case consumer 1 chooses $c_{H1} = c_{L1}$. Show that the equilibrium value of *p* is the same as in the deterministic case. Compute the equilibrium allocations. (**20 points)**