

Final Exam BDP 2010, Econometrics, Questions from Mark Watson (90 points)

Problem 1 (25 points): Suppose $\{y_t\}$ is stationary and ergodic with finite second moment. Let $f_{t+3/t}$ denote the minimum mean square error of y_{t+3} , and $e_{t+3} = y_{t+3} - f_{t+3/t}$ denote the resulting forecast error. Let

$$\hat{\beta} = \frac{\sum_{t=1}^T y_{t+3} f_{t+3/t}}{\sum_{t=1}^T f_{t+3/t}^2}$$

denote the OLS regression coefficient from the regression of y_{t+3} onto $f_{t+3/t}$.

- (a) Show that $\hat{\beta} \xrightarrow{p} 1$ (where the limit uses $T \rightarrow \infty$).
- (b) Show that $\sqrt{T}(\hat{\beta} - 1) \xrightarrow{d} N(0, V)$ and derive an expression for V . (Again, the limit is as $T \rightarrow \infty$).
- (c) A forecasting company has been publishing 3-period ahead forecasts of y for $T = 400$ time periods, and claims their forecast is the best forecast in the market. Let $\tilde{f}_{t+3/t}$ denote the company's forecast. You regress y_{t+3} onto $\tilde{f}_{t+3/t}$ and find $\hat{\beta} = 0.93$.

Let $\hat{e}_{t+3} = y_{t+3} - \hat{\beta} \tilde{f}_{t+3/t}$. Suppose $T^{-1} \sum_{t=1}^T \tilde{f}_{t+3/t}^2 = 0.81$, $T^{-1} \sum_{t=1}^T \hat{e}_{t+3}^2 = 2.31$,

$T^{-1} \sum_{t=1}^T \hat{e}_{t+3}^2 \tilde{f}_{t+3/t}^2 = 1.84$, $T^{-1} \sum_{t=1}^T \hat{e}_{t+3} \hat{e}_{t+2} \tilde{f}_{t+3/t} \tilde{f}_{t+2/t-1} = 1.15$,

$T^{-1} \sum_{t=1}^T \hat{e}_{t+3} \hat{e}_{t+1} \tilde{f}_{t+3/t} \tilde{f}_{t+1/t-2} = 0.70$, $T^{-1} \sum_{t=1}^T \hat{e}_{t+3} \hat{e}_t \tilde{f}_{t+3/t} \tilde{f}_{t/t-3} = 0.47$. Test the null hypothesis that $\tilde{f}_{t+3/t}$ is the minimum mean squared error forecast.

Problem 2 (20 points): Consider the filter $(1-L)$.

- (a) Show that the gain of the filter is equal to 0 at frequency $\omega = 0$.
- (b) Show that the gain of the filter is equal to 2 at frequency $\omega = \pi$.
- (c) Suppose that $y_t = 0.9y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim \text{iid}(0,1)$ and let $x_t = (1-L)y_t$. A researcher plots a realization of y_t and x_t . He comments that the plot of x_t looks a lot more “jagged or rough or choppy” than the plot of y_t . Use the gain of the filter $(1-L)$ to explain why.

Problem 3 (15 points): Suppose that $y_t = \frac{1}{T} u_t + \varepsilon_t$, where $u_t = u_{t-1} + e_t$, $u_0 = 0$, and where $\{\varepsilon_t\}$ and $\{e_t\}$ are mutually independent sequences of standard normal random variables.

(a) Show that $\frac{1}{T} \sum_{t=1}^T y_t^2 \xrightarrow{p} 1$.

(b) Let $x_t = x_{t-1} + y_t$, with $x_0 = 0$. Derive a limiting representation for $\frac{1}{T} x_T^2$.

Problem 4 (10 points): Let $x_t = \Delta y_t$. Consider the structural VAR model:

$$x_t = \gamma_0 z_t + \phi x_{t-1} + \gamma_1 z_{t-1} + \varepsilon_{xt}$$

$$z_t = \rho z_{t-1} + \varepsilon_{zt}$$

show that $\lim_{k \rightarrow \infty} \frac{\partial y_{t+k}}{\partial \varepsilon_{zt}} = \frac{(\gamma_0 - \gamma_1)}{(1 - \phi)(1 - \rho)}$

Problem 5 (20 points): y_t follows the stationary AR(1) model $y_t = \phi y_{t-1} + \varepsilon_t$. A researcher wants to estimate ϕ . He cannot find data on y_t , but can find estimates of y_t based on survey. Let x_t denote the survey estimates of y_t , and suppose that $x_t = y_t + u_t$, where u_t is iid(0, σ_u^2) and u_t is independent of ε_j for all t and j . The researcher estimates ϕ by regressing x_t onto x_{t-1} using x_{t-2} as an instrument.

(a) Suppose that $\phi \neq 0$. Derive the asymptotic distribution of the IV estimator

(b) Suppose $\phi = 0$, show that the IV estimator is not consistent. Derive limiting distribution.

(c) How would you test $\phi = 0$?

Problem 6. (12 points)

Consider the logit estimation output below.

- (a) (2 point) Construct a 95% confidence interval for the coefficient on x_1 .
- (b) (3 points) What is the estimated marginal effect of x_1 for an individual with $x_1 = 1$ and $x_2 = \frac{1}{2}$?
- (c) (7 points) How would you construct a 95% confidence interval for the marginal effect of x_1 for an individual with $x_1 = 1$ and $x_2 = 0$? Do you have the necessary information for calculating it? If not, what else do you need?

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. logit y x1 x2, robust
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Iteration 0: log pseudolikelihood = -622.21149
Iteration 1: log pseudolikelihood = -589.39235
Iteration 2: log pseudolikelihood = -589.30272
Iteration 3: log pseudolikelihood = -589.30271
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Logistic regression                                Number of obs =          900
                                                    Wald chi2(2)      =          61.98
                                                    Prob > chi2       =          0.0000
Log pseudolikelihood = -589.30271                Pseudo R2        =          0.0529
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	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
x1	.9054771	.13924	6.50	0.000	?	?
x2	.6983965	.1393683	5.01	0.000	.4252397	.9715533
_cons	.1252023	.0693965	1.80	0.071	-.0108123	.2612168

Problem 7. (36 points)

Suppose that you have a random sample of n observations of (x_i, y_i) from

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \tag{1}$$

where ε_i is independent of x_i , and is symmetrically distributed with mean 0. Assume that all relevant moments exist and are finite. Moreover, assume that $E[x_i] = 0$ and $V[x_i] > 0$.

In (a) to (d), argue why each of the five estimators of β_1 is consistent and find its asymptotic variance (note that we are not concerned with the estimator of β_0):

(a) (6 points) The OLS estimator obtained in a regression of y_i on a constant and x_i .

(b) (6 points) The extremum estimator that minimizes

$$\sum_{i=1}^n (y_i - (b_0 + b_1 x_i))^4$$

over b_0 and b_1 .

(c) (6 points) The median regression estimator defined by minimizing

$$\sum_{i=1}^n |y_i - (b_0 + b_1 x_i)|$$

over b_0 and b_1 .

(d) (6 points) The quantile regression estimator defined by minimizing

$$\sum_{i=1}^n \rho_{0.25}(y_i - (b_0 + b_1 x_i))$$

over b_0 and b_1 . Here $\rho_\alpha(\cdot)$ is the “check-function” defined by

$$\rho_\alpha(\eta) = \begin{cases} -(1 - \alpha)\eta & \text{if } \eta < 0 \\ \alpha\eta & \text{if } \eta \geq 0 \end{cases}$$

for $\alpha \in (0, 1)$

(e) (6 points) Provide an example of a distribution of ε_i for which the estimator in (c) is more efficient than the estimator in (a).

- (f) (6 points) Provide an example of a distribution of ε_i for which the estimator in (d) is more efficient than the estimator in (c).

Problem 8. (14 points)

This problem is concerned with bounding treatment effects. Suppose that D is a random variable that indicates whether an individual has been “treated”. If $D = 1$, the individual has received treatment and we observe a random variable Y_1 . If $D = 0$, the individual has not received treatment and we observe a random variable Y_0 . We do not observe Y_0 if $D = 1$, and we do not observe Y_1 if $D = 0$. This is the standard notation in this literature.

- (a) (7 points) Suppose that Y_0 and Y_1 can only take the values 0 and 1, and that we observe that $P(D = 1) = 0.5$, $E[Y_1|D = 1] = 0.7$ and $E[Y_0|D = 0] = 0.3$. Use this to construct bounds on the average treatment effect, $E[Y_1 - Y_0]$.
- (b) (7 points) How would your answer to (a) change if you also knew that $Y_0 \leq Y_1$ with probability 1?

Problem 9. (10 points)

Suppose that a duration, T , can be written as

$$\ln(T) = x'\beta + \varepsilon \tag{2}$$

where ε is independent of x and is uniformly distributed on the interval $(0, 1)$. Find the condition hazard function for T given x .

Problem 10. (18 points)

Suppose that you have a random sample $\{X_i\}_{i=1}^n$ from some distribution with density, f . Consider the following estimator of f at a point x ,

$$\hat{f}(x) = \frac{1}{nh_n} \sum_{i=1}^n K\left(\frac{x - X_i}{h_n}\right)$$

where K is the density for a uniform random variable on the interval $(-a, a)$ for some $a > 0$.

- (a) (7 points) If the true (unknown) f is standard normal,

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2},$$

find an expression for the (approximate) means square error of $\hat{f}(0)$ (as a function of n and h).

- (b) (7 points) For given n , what is the lowest (approximate) mean square error that you can get?
- (c) (4 points) How does it depend on a ? Explain in words, why this is not surprising.