Final Exam BDP 2010, Microeconomics, Questions Klaus Schmidt (20 points)

A price taking firm produces output q from inputs z_1 and z_2 according to a differentiable concave production function $f(z_1, z_2)$. The price of its output is $p > 0$, and the prices of its inputs are $(w_1, w_2) \gg 0$. However, there are two unusual things about this firm. First, rather than maximizing profit, the firm maximizes revenue. Second, the firm is cash constrained. In particular, it has only C dollars on hand before production and, as a result, its total expenditures on inputs cannot exceed C.

Suppose that one of your econometrician friends tells you that she has used repeated observations of the firm's revenues under various output prices, input prices, and levels of the financial constraint and has determined that the firm's revenue level R can be expressed as the following function of the variables (p, w_1, w_2, C):

$$R(p, w_1, w_2, C) \; = \; p[\gamma + \ln C - \alpha \ln w_1 - (1 - \alpha) \ln w_2] \; ,$$

where γ and α are known parameters.

(a) What is the firm's input demand for input z_1 when prices are (p, w_1, w_2) and it has C dollars of cash on hand?

(b) Suppose the firm wants to achieve a given level of revenues R. What is the minimum cost that the firm has to incur given prices (p, w_1, w_2)?

(c) What is the firm's input demand for input z_1 when prices are (p, w_1, w_2) and it has to achieve revenue level R at minimum cost?

(d) Under what condition is the input demand of question (a) identical to the input demand of question (c)?

Final Exam BDP 2010, Microeconomics, Questions Jean-Charles Rochet (20 points)

Consider an exchange economy with two consumers and two goods, good 1 and good 2. Consumer 1's utility function is given by

$$u_1(x_{11}, x_{21}) = \sqrt{x_{11} x_{21}}$$

and consumer 2's by

$$u_2(x_{12}, x_{22}) = \log x_{12} + x_{22}.$$

The initial endowment of consumer 1 is (2,2) and that of consumer 2 is (4,2).

1.) Find the set of interior Pareto optima of this economy and represent it in the Edgeworth box.

2.) Are there also Pareto optima on the boundary of the Edgeworth box?

3.) Find the competitive equilibrium of this economy.

4.) Check that it is Pareto optimal. Explain why.

(5 points for each question)

Final Exam BDP 2010, Microeconomics, Questions John Moore (70 points)

Answer two out of the following five questions

1. Consider an Akerlof model in which workers' individual productivities θ are uniformly distributed on $[1, 6]$. Each worker privately knows her own θ. A worker with productivity θ has an opportunity cost $r(\theta) = 6 - \theta$. Firms are risk neutral and have additive technologies.

   (a) What would be a first-best allocation of labour?

   (b) In equilibrium, what is the wage and allocation of labour?

   (c) The government levies a tax t on every worker who works. The tax revenue is used to finance a subsidy s, paid to every worker who does not work. Bearing in mind that the government must break even, find the values s^* and t^* that implement first best.

   (d) If the choice of s and t, respecting the government's budget constraint, were put to a vote of the entire workforce (working or not), speculate on whether the outcome of the vote would be higher or lower values than s^* and t^* respectively.

2. Consider the following Spence signalling model where workers are of three types, $\theta_L = 1$, $\theta_M = 2$ and $\theta_H = 4$. A worker privately knows his own type θ. There is a fraction λ_L of type θ_L; a fraction λ_M of type θ_M; and a fraction λ_H of type θ_H, where $\lambda_L + \lambda_M + \lambda_H = 1$. A worker of type θ has cost $\frac{e}{\theta}$ of acquiring a level of university education $e \geq 0$. Competitive firms earn θ from a worker of type θ (university education is intrinsically useless). The workers vote over the proposition to shut universities in order to stop signalling (assume that if the proposition is voted down, the least inefficient separating equilibrium ensues). Voting is one-worker-one-vote, with majority rule. (In the event of an exact tie, the proposition is voted through.) What is the outcome of the vote, as a function of λ_L, λ_M and λ_H?

3. Consider a competitive screening model with many firms and workers. There are two types of workers, with productivities $\theta = \theta_L$ and $\theta = \theta_H$, where $\theta_H > \theta_L > 0$. A worker with productivity θ has a cost $\frac{t^2}{\theta}$ of undertaking some task $t \geq 0$; and the return from such a worker to a firm is $\theta + t$ (i.e., the task is intrinsically useful). The timing of the model is as follows. First, each worker privately learns her own type θ. Then, the firms offer contracts comprising a wage $w(t)$ for undertaking some task t. Finally, each worker chooses which contract, if any, to accept. The fraction of type θ_H workers is λ. Discuss the effect increasing λ has on equilibrium behaviour if

   (a) $\theta_H/\theta_L = 5$;

   (b) $\theta_H/\theta_L = 2$.

4. A risk-neutral Principal contracts with a risk-neutral Agent, whose type θ may be either $\theta_L = 1$ or $\theta_H = 2$. The Agent privately knows which type she is at the time of being hired. From the Principal's perspective, the Agent is of type θ_H with probability λ. The Agent of type θ has a cost of effort, e, equal to $\frac{e}{\theta}$. The Principal's revenue is $2\sqrt{e}$. A contract specifies a menu comprising two wage-effort pairs, (w_L, e_L) and (w_H, e_H), from which the Agent selects. The Agent may refuse the contract: if she is of type θ_H she has an outside opportunity equal to 1; whereas if she is of type θ_L she has an outside opportunity equal to μ, where $0 < \mu < 1$. As λ tends to 1, in an optimal contract:

   (a) To what does (w_H, e_H) tend?

   (b) To what does (w_L, e_L) tend, depending on the value of μ?

5. A risk-neutral Principal contracts to hire an Agent on a project. The project either succeeds and yields $\pi > 0$ dollars revenue, or fails and yields nothing. The Agent is risk-neutral with respect to non-negative income, but has no money of his own with which to pay anything out (i.e. negative wages aren't feasible). The Agent has a reservation utility (an outside opportunity) equivalent to k dollars. If the Agent accepts the Principal's contract, he then chooses an effort level which determines the probability f that the project succeeds: by exerting f^2 dollars worth of effort (a privately-observed non-cash expenditure, netted from his wage to calculate his utility), the Agent can choose any f between 0 and 1. For all possible values of π and k, find the optimal contract - that is, the wage w that the Principal pays the Agent if the project succeeds.

Final Exam BDP 2010, Microeconomics, Questions Mathias Dewatripont (70 points)

Question 1 (35 points): Industry wage-setting by a union.

Suppose a union is the sole supplier of labor to all n firms in an oligopoly. The timing is as follows: (i) the union makes a single wage demand, w, that applies to *all* the firms; (ii) the firms observe (and accept) w and then simultaneously choose their own employment levels, L_i for firm i. The payoff for the union is $(w - w_a)L$, where w_a is the wage union members can earn in alternative employment and $L = L_1 + L_2 + \ldots + L_n$ is total employment in these unionized firms. The payoff for firm i is its profit $\pi(w, L_i)$. In this respect, note that demand is assumed to be $p(Q) = a - Q$, where Q is the sum of individual outputs (q_i for firm i), and that those equal the employment levels (L_i for firm i). Assume also for simplicity that firms have no costs other than wages.

- What is the subgame-perfect equilibrium of this game?
- Compute the utility of the union in equilibrium. How is it affected by the number of firms n? Discuss.
- Are there other equilibria which are Nash but not subgame-perfect? Which ones? Discuss.

Question 2 (35 points): Bertrand Competition with Asymmetric Information about Costs.

Consider a Bertrand duopoly with differentiated products, where demand for firm i is $q_i(p_i , p_j) = 2 - p_i + p_j /2$. The marginal cost for each firm, c_i, is known to firm i only. Each firm believes its competitor's marginal cost is uniformly distributed between 0 and $\theta > 0$. This is common knowledge, and so is the belief that marginal cost realizations are drawn independently.

- What conditions define a symmetric pure-strategy Bayesian equilibrium in this game (assume an interior solution exists)?
- Solve for such an equilibrium.
- What happens when θ increases? Discuss.