Midterm Exam 2011, Macro I (Bob King's part)

Problem 1: Linear rational expectations models (12 points)
Consider a rational expectations model in which the demand d_t for a durable good depends negatively on its expected implicit rental price, which is the difference between its current market price p_t and its discounted expected future price $\beta E_t p_{t+1}$ with $0 < \beta < 1$ being the discount factor:

$$d_t = d - \theta[p_t - \beta E_t p_{t+1}]$$

Suppose further that the supply s_t of the durable good is taken as exogenous and governed by the stochastic process,

$$s_t = s + m(s_{t-1} - s) + e_t,$$

where $|m| \leq 1$ and e_t is a serially independent zero mean shock.

Show that the rational expectations solution for the durables price takes the form

$$p_t = a + bs_t$$

and provide a formula that determines the parameters a and b.

Problem 2. Optimal purchases of consumer durables (48 points)
Consider a consumer who derives a utility flow from two goods, a standard consumption good c and a durable consumption good according to

$$u(c_t, d_t + z_t),$$

where d_t is the stock of consumer durables held at the start of period t and z_t is the flow of newly purchased durable goods in period t.

Suppose that the individual has lifetime preferences

$$E_t\{\sum_{j=0}^{\infty} \beta^j u(c_{t+j}, d_{t+j} + z_{t+j})\}.$$

Suppose that the individual can save or borrow at the interest rate r with his asset balance evolving according to

$$\frac{1}{1+r} a_{t+1} = a_t + y - c_t - p_t z_t,$$

where y is the individual's fixed income each period. His durables stock evolves according to

$$d_{t+1} = d_t + z_t.$$

Suppose finally that the price follows the Markov process

$$\begin{aligned} p_t &= \xi \exp(x_t) \\ x_t &= \rho x_{t-1} + e_t \end{aligned}$$

with e_t being a serially independent random variable.

Consider the optimal consumption/saving/durables problem. Since individuals derive utility from $d_t + z_t$, they can be viewed as having a utility function defined as $u(c_t, d_{t+1})$. The two accumulation equations can be combined to yield

$$c_t + \frac{1}{1+r} a_{t+1} + p_t d_{t+1} = y + a_t + p_t d_t = w_t,$$

where the right hand side captures the combined value of income, financial assets, and durables that we will call "wealth" in this problem.

(a) (12 points) Write a Bellman equation with state variables w_t, x_t and choice variables c_t, d_{t+1} and a_{t+1}. How is wealth at $t+1$ affected by period t choices?

(b) (12 points) What is the current model's version of the standard intertemporal efficiency condition for consumption? How is it similar to and different from that used in the analysis of the life-cycle/permanent income model considered in class?

(c) (12 points) Show that an efficient demand for durables satisfies

$$u_d(c_t, d_{t+1}) = u_c(c_t, d_{t+1}) p(x_t) - \beta E_t\{u_c(c_{t+1}, d_{t+2}) p(x_{t+1})\}$$

and provide an economic interpretation of this condition.

(d) (12 points) Consider the following statement: "If the prices of durables fall when other consumption falls, then the effective cost of owning a durable good is higher because represents a risky investment. Optimal durables purchases would therefore take into account this risk."

Critically evaluate this statement using the results of the earlier parts of this question and provide an analytical derivation of the extent of price risk on the demand for durables.

Midterm Exam 2011, Macro II (Sergio Rebelo's part)

1. Short question: a two sector neoclassical growth model (16 points) Consider a model in which consumption and investment goods are produced in different sectors. Households maximize their life-time utility

$$U = \sum_{t=0}^{\infty} \beta^t \log(C_t)$$
$$0 < \beta < 1,$$

where β is the discount factor. Households supply a fixed amount of labor, N.

Investment (I_t) and consumption (C_t) goods are produced as follows:

$$I_t = A(K_t^i)^{1-\alpha}(N_t^i)^{\alpha},$$
$$C_t = B(K_t^c)^{1-\alpha}(N_t^c)^{\alpha},$$

where K_t^j and N_t^j denote the capital and labor used in sector j, respectively.

Capital is accumulated as follows (the rate of depreciation is 100 percent):

$$K_{t+1} = I_t.$$

The following adding-up constraints for the two factors of production complete the description of the model:

$$K_t^c + K_t^i = K_t,$$

$$N_t^c + N_t^i = N.$$

a) Compute the first-order conditions for the planner's problem for this economy.

b) Show that

$$\frac{K_t^i}{N_t^i} = \frac{K_t^c}{N_t^c} = \frac{K_t}{N_t}.$$

c) Show that a solution to the planner's problem takes the form

$$C_t = \mu(K_t)^{1-\alpha}(N_t)^{\alpha},$$

and compute the value of μ. (Hint: the result derived in b) is very useful in computing the value of μ).

2. Short question: small open economy (16 points) Consider a small open economy in which the representative agent has the following utility function:

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma} \\ \sigma &> 0,\ 0 < \beta < 1, \end{aligned}$$

where β is the discount factor.

The level of output is exogenous and denoted by Y_t. This output level is higher in odd periods than in even periods:

$$Y_t = \begin{cases} y + \varepsilon & t = 1, 3, 5, ... \\ y & t = 0, 2, 4, 6, ... \end{cases}$$

where $\varepsilon > 0$.

The budget constraint for the economy is given by the flow budget constraint

$$a_{t+1} = (1+r)a_t + Y_t - C_t,$$

and the no-Ponzi game condition

$$\lim_{t \to \infty} \frac{a_{t+1}}{(1+r)^t} = 0.$$

The initial level of net foreign assets is zero:

$$a_0 = 0.$$

Assume that

$$\beta = \frac{1}{1+r}.$$

a) Derive the first-order conditions for the planner's problem for this economy.

b) Compute the path for C_t and the current account in period one, $a_1 - a_0$.

3. Long question: monopolistic competition (28 points). Consider an economy with n monopolistically competitive firms. Monopolist i faces the following demand curve for his product:

$$p_i = P x_i^{(1-\nu)/\nu} C^{(\nu-1)/\nu},\ \nu > 1,$$

where p_i and x_i are the price and quantity of good i, respectively. The variables P and C denote the aggregate price and the aggregate level consumption, respectively. These variables are given by

$$C = \left[\int_0^n x_i^{1/\nu} di\right]^{\nu},$$

$$P = \left[\int_0^n p_i^{1/(1-\nu)} di\right]^{1-\nu}.$$

All monopolists take P and C as given. Output is produced according to

$$x_i = AK^{\alpha}(N_i - \bar{N})^{\gamma},$$

where $\bar{N}$ is the amount of overhead labor needed to run the plant. We assume that $\alpha + \gamma < 1$, so there are decreasing returns to scale. Capital and labor are hired in competitive factor markets at rates R and w, respectively.

a) Compute the cost function for the monopolist and show that the marginal cost is increasing in x_i.

b) Compute the price that maximizes the profits of monopolist i as a function of the quantity produced.

c) Assume that there is free entry, so profits are zero in equilibrium. Treating the value of C as given, derive the amount produced by each producer (x_i) and the equilibrium number of firms (n).