Midterm Exam 2011, Econometrics, Questions from Mark Watson

Problem 1 (16 points):

(a) (4 points) X is a Bernoulli random variable with $P(X = 1) = p$. What is the moment generating function for X?

(b) (4 points) $Y = \sum_{i=1}^{n} X_i$ where X_i are iid Bernoulli with $P(X_i = 1) = p$. What is the moment generating function for Y?

(c) (8 points) <u>Use your answer in (b)</u> to compute $E(Y)$, $E(Y^2)$ and $E(Y^3)$ for $n > 3$.

Problem 2 (17 points): $X \sim N(0,1)$ and $Y|X$ is $N(X,1)$.

(a) (9 points) Find

(i) $E(Y)$

(ii) $\mathrm{Var}(Y)$

(iii) $\mathrm{Cov}(X,Y)$

(b) (8 points) Prove that the joint distribution of X and Y is normal.

Problem 3 (15 points): $X \sim N(0,\sigma^2)$. You are interested in the competing hypothesis H_o: $\sigma^2 = 1$ vs H_a: $\sigma^2 = 2$. Based on a sample of size 1, you reject the null when $X^2 > 3.84$.

(a) (5 points) Compute the size of the test.

(b) (5 points) Compute the power of the test.

(c) (5 points) Prove that this is the most powerful test.

Problem 4 (12 points): Consider the same model as in (3) (again with a sample of size 1). Let $\hat{\sigma}^2 = X^2$.

(a) (5 points) Show that $\hat{\sigma}^2$ is unbiased.

(b) (7 points) Show that $\hat{\sigma}^2$ is the minimum mean squared estimator of σ^2.

Honoré Questions

Problem 1 (12 points)

Find six of the numbers x1–x8 in the regression output below. Each correct number is worth 2 points, but you will get credit for at most six of them.

```
. reg y educ inc

      Source |       SS       df       MS              Number of obs =     160
-------------+------------------------------           F(  2,   157) =      x5
       Model |  53.7499661     2  26.8749831           Prob > F      =  0.0017
    Residual |  637.440957   157  4.06013349           R-squared     =  0.0778
-------------+------------------------------           Adj R-squared =      x8
       Total |          x6   159  4.34711273           Root MSE      =      x7

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   1.067156   .3106674      x1   0.001     .4535288    1.680783
         inc |   .5098395   .3208512    1.59   0.114           x2          x3
       _cons |         x4   .1597845    3.15   0.002     .1869384    .8181476
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Problem 2 (8 points)

In an instrumental variable regression model with one regressor, x_i, and one instrument, $z_{i,}$, the regression of x_i on z_i has $R^2 = 0.08$.

Is this sufficient for you to know whether z_i is a strong instrument? What else do you need to know? When would you conclude that it is?

Problem 3 (18 points)

Suppose that you estimate a linear regression model using individuals from a (large) sample of married couples,

$$y_{ij} = x_{ij}'\beta + \varepsilon_{ij} + v_i$$

where $i = 1, \ldots, n$ denotes a couple, and $j = 1$ for the wives and $j = 2$ for the husbands. The ε_{ij}'s are individual–specific error terms, while v_i is a couple–specific error term.

Assume that observations are independent and identically distributed across couples (so $(x_{i1}, \varepsilon_{i1}, x_{i2}, \varepsilon_{i2}, v_i)$ has the same distribution for all i, and $(x_{i1}, \varepsilon_{i1}, x_{i2}, \varepsilon_{i2}, v_i)$ is independent of $(x_{\ell i1}, \varepsilon_{\ell 1}, x_{\ell 2}, \varepsilon_{\ell 2}, v_\ell)$ provided that $i \neq \ell$). Finally assume that the model is correctly specified in the sense that $E[v_i | x_{i1}, x_{i2}] = 0$ and $E[\varepsilon_{ij} | x_{i1}, x_{i2}] = 0$ for $j = 1, 2$.

1. (9 points) For this question only, assume that $\varepsilon_{i1}, \varepsilon_{i2}$ and v_i are independent of each other. Find the asymptotic distribution of the OLS estimator in a regression of y_{ij} on x_{ij} using all the observations. State any additional assumptions that you make.

2. (9 points) Explain how you would test a hypothesis of the form

$$R'\beta = r$$

Your test should be based on the asymptotic distribution of $R'\widehat{\beta}_{OLS} - r$. State any additional assumptions that you make.

Problem 4 (22 points)

Consider the model

$$y_i = z_i\delta + \varepsilon_i, \qquad with \qquad E[\varepsilon_i z_i] = 1$$

Moreover, assume that there is another variable x_i such that $E[\varepsilon_i x_i] = 0$. Both z_i and x_i are one–dimensional. Assume that the usual assumptions for consistency and asymptotic normality of GMM estimators in linear models are satisfied (these are assumptions (3.1)–(3.5) in Hayashi) and that $E[x_i] = E[z_i] = E[\varepsilon_i] = 0$, $E[x_i^2] = 2$, $E[x_i^3] = 0$, $E[x_i^4] = 9$, $E[x_i z_i] = 1$, $E[z_i^2] = 2$ and $E[\varepsilon_i^2 | x_i, z_i] = 2$. Also assume that all other moments that show up in your answers exist and are finite.

1. (6 points) Find the probability limit of the OLS estimator in a regression of y_i on z_i and a constant.

2. (9 points) Find the asymptotic distribution of the 2SLS estimator of δ that uses 1 and x_i as instruments. Does it depend on moments other than the ones given above? (Note that a constant in not included as an explanatory variable).

3. (7 points) Find the asymptotic distribution of the 2SLS estimator of δ that uses x_i as an instrument. How does it compare to the asymptotic distribution of the 2SLS estimator of δ that uses 1 and x_i as instruments?