Midterm Exam - Micro I

1. (20 points) In an L-commodity world, a consumer's demand function is

$$x_k(p,w) \;=\; \frac{w}{\left(\sum_{l=1}^{L} p_l\right)} \quad \text{for } k = 1,\ldots,L.$$

   (a) Is the demand function homogeneous of degree zero in (p,w)?

   (b) Does it satisfy Walras' law?

   (c) Does it satisfy the weak axiom?

   (d) Compute the Slutsky substitution matrix for this demand function. Is it negative semidefinite? Symmetric?

2. (20 points) A consumer has *homothetic preferences* if his utility function is homothetic. A function $f(x)$ is *homothetic* if $f(x) = g(h(x))$, where $g(\cdot)$ is a strictly increasing function and $h(\cdot)$ is a function that is homogeneous of degree 1 in the vector x. Show:

   (a) If preferences are homothetic, then they can be represented by a utility function that is homogenous of degree 1.

   (b) If preferences are homothetic, then the expenditure function can be written as $e(p,u) \;=\; e(p) \cdot u$.

   (c) If preferences are homothetic, then the indirect utility function can be written as $v(p,w) = v(p) \cdot w$.

   (d) If preferences are homothetic, then the Marshallian demand functions take the form $x_i(p,w) = x_i(p) \cdot w$, i.e., they are linear functions of income.

   Hint: You may use result (b) to answer parts (c) and (d) of the problem.

3. (20 points) Patrick's utility function is $u(x_1,x_2) = x_1 \cdot x_2$, where good 1 is food and good 2 is housing. Patrick gets a monthly salary of \$ 3000. The price of good 1 and the price of good 2 are $p_1 = p_2 = 1$. Patrick's boss is thinking of sending him to another town where the price of food is the same, but the price of housing is 2.25. The boss offers no raise in pay. Patrick, who understands compensating and equivalent variation perfectly, complains bitterly. He says that although he doesn't mind moving for its own sake and the new town is just as pleasant as the old, having to move is as bad as a cut in pay of \$ A. He also says he wouldn't mind moving if - when he moved - he got a raise of \$B. What are A and B equal to?

Midterm Exam - Microeconomics II, July 2011

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Problem 1 (40 points)

Consider a pure exchange economy with two goods, $l = 1, 2$, and two (types of) agents, $h = 1, 2$, with utility functions $U^A(x_1^A, x_2^A) = \log x_1^A + 3 \log x_2^A$, $U^B(x_1^B, x_2^B) = \log x_1^B + \log x_2^B$ and endowments $\omega^A = (13, 0)$ and $\omega^B = (0, 14)$.

(a) (23 points) Find a competitive equilibrium of this economy (equilibrium prices and allocation).

(b) (4 points) Is the equilibrium allocation you found in part (a) Pareto efficient? Explain.

(c) (13 points) Consider the feasible allocation: $(x_1^A, x_2^A) = (4, 8)$, $(x_1^B, x_2^B) = (9, 6)$. Verify that this allocation is Pareto efficient and show that the allocation can be decentralized as a competitive equilibrium with lump sum transfers [in particular, find the equilibrium prices and level of the transfers to A and B supporting this allocation as a competitive equilibrium].

Problem 2 (20 points)

Consider a pure exchange economy under uncertainty with a single type of consumer ($H = 1$), a single commodity ($L = 1$) and $S = 2$ states of nature. The single (representative) consumer has endowments equal to 10 units in state 1 and 4 units in state 2 and preferences $\mathbb{E} \log(x) = \pi \log x_1 + (1 - \pi) \log x_2$, with $\pi = 1/2$. There are $J = 2$ assets traded at the beginning of the period (before the uncertainty is realized). The vector of the payoffs of asset 1 in the two states (to be delivered to whoever holds one unit of the asset at the end of the period) is given by $r_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ while that of asset 2 is $r_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Let q_1, q_2 denote the prices of the two assets.

(a) (5 points) Write the budget constraints the consumer faces in this economy.

(b) (10 points) Consider the prices $q_1 = 1, q_2 = 1$. Find the value of the consumer's demand for asset 1 at these prices. Show that $q_1 = 1, q_2 = 1$ are not equilibrium prices (i.e. such that the consumer's demand equals the supply of the two assets, equal to zero since assets are in zero net supply)? Can you give an economic intuition for why they are not?

(c) (5 points) On the basis of the analysis and discussion in part (b) do you conjecture the equilibrium prices are such that $q_1 > q_2$ or instead $q_1 < q_2$? [*You do not need to find the value of the equilibrium prices to answer this question*]. Motivate your answer.