WATSON (90)

(20) 1. Suppose that y_t is a quarterly series, and follows the MA(4) process: $y_t = \varepsilon_t + 0.8\varepsilon_{t-4}$, where ε_t is iid(0, σ^2).

(10) (a) Derive and plot the spectrum of y. Discuss how the seasonality in the process is evident in spectrum.

(10) (b) Suppose that x_t is another quarterly series, and follows the MA(4) process $x_t = e_t - 0.8e_{t-4}$, where e_t is iid(0, σ^2), and e_t and ε_τ are uncorrelated for all t and τ. Let $z_t = x_t + y_t$. Derive and plot the spectrum of z_t.

(15) 2. Suppose y_t follows the MA(1) process $y_t = \varepsilon_t - 3\varepsilon_{t-1}$, where $\varepsilon_t \sim$ iid N(0,1). You have data $\{y\}_{t=1}^{T}$, where T is large.

(8) (a) How would you forecast Y_{T+1}?

(7) (b) What is the mean square error of your forecast of Y_{T+1}?

(20) 3. Suppose that $y_t = x_t\beta + u_t$, where $x_t = \phi x_{t-1} + e_t$, $u_t = \psi u_{t-1} + \varepsilon_{t-1}$, where ε_t and e_t are both i.i.d. with mean zero and variance σ_ε^2 and σ_e^2, and ε_t and e_τ are independent for all t and τ. Let $\hat{\beta}$ denote the OLS estimator of β based on a sample of size T, and let $\hat{u}_t$ denote the OLS residual.

(10) (a) Show that $\sqrt{T}(\hat{\beta} - \beta) \xrightarrow{d} N(0,V)$ and derive an expression for V.

(10) (b) Suppose that $T = 100$, $\hat{\beta} = 2.1$, $\frac{1}{100}\sum_{t=1}^{100} x_t^2 = 5$, $\frac{1}{99}\sum_{t=2}^{100} x_t x_{t-1} = 2.5$, $\frac{1}{98}\sum_{t=3}^{100} x_t x_{t-2} = 1.0$, $\frac{1}{100}\sum_{t=1}^{100} \hat{u}_t^{\,2} = 4$, $\frac{1}{99}\sum_{t=2}^{100} \hat{u}_t\hat{u}_{t-1} = 3.6$, $\frac{1}{98}\sum_{t=3}^{100} \hat{u}_t\hat{u}_{t-2} = 3.1$, $\frac{1}{99}\sum_{t=2}^{100} x_t\hat{u}_{t-1} = 0.8$, $\frac{1}{99}\sum_{t=2}^{100} \hat{u}_t x_{t-1} = 0.2$. Construct a 95% confidence interval for β.

HONORE (90)

Problem 6. (16 points)

Suppose that a duration, T, has the following hazard given an explanatory variable, x,

$$h(t|x) = t^{\alpha-1} \exp(\beta_0 + x\beta_1).$$

You will recognize this as a proportional hazard model

Suppose that $\alpha = 0.2$, $\beta_0 = -1$ and $\beta_1 = 1$. Find $P\left(T > 2|\, x = \frac{1}{2}\right)$.

Problem 7. (30 points)

Suppose that you have a panel of n individuals observed in T time periods, and that for each time period

$$y_{it} = x_{it}'\beta + \alpha_i + u_{it}, \tag{1}$$

where y_{it} is the variable of interest and x_{it} is some observed explanatory variable. α_i is a time–invariant, unobserved explanatory variable. u_{it} is an unobserved error–term.

(a) (7 points) Since no assumptions are made on α_i, it seems natural to consider the "difference–from–means" equation:

$$(y_{it} - y_{i\cdot}) = (x_{it} - x_{i\cdot})'\beta + (u_{it} - u_{i,\cdot})$$

where

$$y_{i\cdot} = \frac{1}{T}\sum_{t=1}^{T} y_{it}, \qquad x_{i\cdot} = \frac{1}{T}\sum_{t=1}^{T} x_{it} \qquad \text{and} \qquad u_{i\cdot} = \frac{1}{T}\sum_{t=1}^{T} u_{it}$$

and then estimate β by OLS. What do you need to assume about the relationship between the x's and u's for this to be a good idea?

(b) (8 points) Now assume that the original equation is

$$y_{it} = \gamma y_{i,t-1} + x_{it}'\beta + \alpha_i + u_{it} \tag{2}$$

and consider the "difference–from–means" equation:

$$(y_{it} - y_{i\cdot}) = \gamma(y_{i,t-1} - y_{i,\cdot-1}) + (x_{it} - x_{i\cdot})'\beta + (u_{it} - u_{i\cdot}) \tag{3}$$

where

$$y_{i\cdot} = \frac{1}{T}\sum_{t=1}^{T} y_{it}, \qquad y_{i,\cdot-1} = \frac{1}{T}\sum_{t=1}^{T} y_{i,t-1} \qquad x_{i\cdot} = \frac{1}{T}\sum_{t=1}^{T} x_{it} \qquad \text{and} \qquad u_{i\cdot} = \frac{1}{T}\sum_{t=1}^{T} u_{it}$$

Suppose you estimate γ and β by OLS on (3). (You should not worry about the fact that this implicitly assumes that you observe y_{it} in time–period 0). Is that a good idea? Why?

(c) (7 points) You could also eliminate a_i by differencing (2):

$$(y_{it} - y_{i,t-1}) = \gamma (y_{i,t-1} - y_{i,t-2}) + \beta (x_{it} - x_{i,t-1}) + u_{it} - u_{i,t-1}$$

and then estimate γ and β by OLS. Is that a good idea? Why?

(d) (8 points) Explain how your answer to (c) would change if the original equation had been

$$y_{i,t} = \gamma y_{i,t-2} + x_{it}'\beta + \alpha_i + u_{it} \tag{4}$$

Problem 8. (22 points)

Suppose that you have a random sample $\{X_i\}_{i=1}^n$ from some distribution with density, f. Consider the following estimator of f at a point x,

$$\widehat{f}(x) = \frac{1}{nh_n} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_n}\right)$$

where K is the density for a uniform random variable on the interval $(-\frac{1}{2}, \frac{1}{2})$, and h_n is some (small) bandwidth. If the true (unknown) f is

$$f(x) = \begin{cases} 2e^{-2x} & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x < 0 \end{cases}$$

find an expression for the value of h_n that minimizes the (approximate) means square error of $\widehat{f}(1)$. How does it depend on n?

Problem 9. (22 points)

This problem is concerned with bounding treatment effects. Suppose that D is a random variable that indicates whether an individual has been "treated". If $D = 1$, the individual has received treatment and we observe a random variable Y_1. If $D = 0$, the individual has not received treatment and we observe a random variable Y_0. We do not observe Y_0 if $D = 1$ and we do not observe Y_1 if $D = 0$. This is the standard notation in this literature.

Suppose that the population can be broken into two groups depending on whether an explanatory variable, X, equals 0 or 1 (X could, for example, be a dummy variable for being female). Suppose that

$$\begin{aligned} P(X=0) &= P(X=1) = \frac{1}{2}, \\ P(D=1|X=0) &= 0.3, \quad E[Y_1|D=1,X=0]=40 \quad \text{and} \quad E[Y_0|D=0,X=0]=10, \\ P(D=1|X=1) &= 0.5, \quad E[Y_1|D=1,X=1]=60 \quad \text{and} \quad E[Y_0|D=0,X=1]=20. \end{aligned}$$

Also suppose that it is known that (whether $X=0$ or $X=1$), $0 \leq Y_0 \leq 100$ and $0 \leq Y_1 \leq 100$.

Use this to construct bounds on the average treatment effect, $E[Y_1 - Y_0]$.

(20) 4. Suppose that $y_t = x_t\beta + u_t$, where $x_t = \varepsilon_{t+1}$, $u_t = u_{t-1} + \varepsilon_t$, $u_0 = 0$, $\varepsilon_t \sim$ iid $N(0,\sigma^2)$, and let $\hat{\beta}$ denote the OLS estimator of β.

(10) (a) Prove that $\hat{\beta} - \beta \xrightarrow{d} \frac{1}{2}(z^2 - 1)$, where $z \sim N(0,1)$.

(10) (b) Explain how you would you consistently estimated β.

(15) 5. Suppose that $y_t = x_t + u_t$, where $x_t = \varepsilon_t + 0.8\varepsilon_{t-1}$, and

$\begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix} \sim iidN\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 9 & 3 \\ 3 & 4 \end{bmatrix}\right)$. You are told that $y_{100} = 6$.

(8) (a) Compute the best (minimum mean square error) estimate of x_{100}.

(7) (b) Compute the best (minimum mean square error) estimate of x_{101}.