KING (90)
**Shorter question (30 minutes)**

*Dynamic programming.* Consider a unemployed individual that must decide whether to search for a job ($a = 1$) or not ($a = 0$). If the individual finds a job, then he has a future wage rate of $w$ forever and works one time unit per period. If not, then he can search again next period. The general momentary utility function is of the form $u(c, l)$, where $c$ is the amount of consumption and $l$ is the amount of leisure ($0 \leq l \leq L$, with $L > 1$).

A individual that does not search or work has a utility flow of

$$u(b, L)$$

based on an unemployment benefit of $b$.

An individual that searches receives a flow utility of

$$u(b, L - s)$$

where $s$ is the (assumed constant) amount of time spent searching.

An individual that works has a utility flow of

$$u(w, L - 1)$$

The job finding probability is

$$\begin{aligned} f \text{ if a } &= 1 \\ d \text{ if a } &= 0 \end{aligned}$$

That is, there is some probability $d < f$ that the individual will find a job via "dumb luck" even if he does not search.

Individuals maximize lifetime utility,

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

via their search decisions. There is no asset management.

(a) Formulate a dynamic programming problem for the unemployed worker.

(b) Determine when $a = 1$ is optimal today, under the assumption that one is behaving optimally in the future.

(c) If search is optimal, then what is the welfare level of an unemployed individual?

**Longer question (60 minutes)**

*Optimal taxation and public expenditure* Consider an economy in which individuals value a public good "g". All individuals in the economy have preferences of the form

$$u(c, l, g) = \log(c) + \theta \log(g) - \frac{\chi}{1+\eta}(1-l)^{1+\eta}$$

and the resource constraint takes the form

$$c + g = a(1-l)$$

(a) Determine the "first best" levels of $c, g, l$.

(b) Show that the first best levels of $c, g, l$ can be supported in a decentralized competitive equilibrium by a lump sum tax system, in which the private budget constraint is

$$c = a(1-l) - T$$

with a specified level of lump-sum tax $T$. Such an equilibrium involves households behaving optimally, taking as given $g$ and $T$. The public budget constraint, $g = T$, must also be satisfied.

(c) Now suppose that there are no lump sum taxes, but only income taxes, so that household budget constraint takes the form

$$c = (1-\tau)a(1-l)$$

where $\tau$ is the tax rate. What is the optimal pattern of consumption and labor supply for a household facing this constraint?

(d) Substitute these decision rules and the government budget constraint into the objective above, to get a welfare function that depends directly on $\tau$. Find the optimal tax level. How does the optimal tax rate depend on $\theta$?

(e) Looking at the level of work effort (leisure), are the outcomes under optimal taxation first best? Why?

(f) How would you formulate this expenditure and tax problem as a Ramsey problem, along the lines discussed in class? (It is not necessary to derive the optimal policy, but simply to set it up: do so generally, i.e., without using the specific form of the utility function).

REBELO (20)

**A decline in the demand for housing in the neoclassical growth model**
Consider a version of the neoclassical growth model in which the planner's problem is given by:

$$\max U = \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \theta \log(H_t)\right]$$

subject to:

$$AK_t^{1-\alpha} N^{\alpha} = C_t + I_t + I_t^H,$$

$$K_{t+1} = I_t + (1-\delta)K_t,$$

$$H_{t+1} = I_t^H + (1-\delta)H_t,$$

where $H_t$ represents the housing stock, $I_t^H$ denotes residential investment, and $N$ is the exogenous supply of labor.

a) Compute the steady state levels of consumption ($C_t$), capital ($K_t$), investment in physical capital ($I_t$), and investment in housing ($I_t^H$).

b) Suppose that $\theta$ falls making houses less desirable from the standpoint of the consumer. What is the impact of this fall on the steady state levels of consumption, capital, investment in physical capital, and investment in housing?

Gali (70)

**Real Wage Rigidities and the New Keynesian Phillips Curve (50 points).**

Consider an economy with staggered price-setting à la Calvo. Firms have access to a simple technology $Y_t = A_t N_t$ , where $Y_t$ is output $N_t$ denotes hours of work and $A_t$ is an (exogenous) technology parameter. The representative consumer has a period utility given by

$$U(C_t, N_t) = \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where $C_t$ is a CES function of the quantities consumed of different types of goods. All output is consumed. (Note: henceforth lower case letter denote logs of the original variables).

(a) Show that the efficient level of (log) output is given by $y_t^* = a_t$ (hint: solve the social planner's problem).

(b) Assume employment is subsidized at a rate $\tau$ that exactly offsets the monopolistic competition distortion. Show that inflation $\pi_t$ will be given by equation

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \lambda \; (\omega_t - a_t) \tag{1}$$

where $\omega_t \equiv w_t - p_t$ denotes the (log) real wage.

(c) Assume that the real wage is determined according to:

$$\omega_t = \gamma \; \omega_{t-1} + (1-\gamma) \; mrs_t$$

where $mrs_t \equiv c_t + \varphi n_t$ Parameter $\gamma \in [0,1]$ can be thought of as an index of real wage rigidities. Determine the evolution of the natural rate of output $y_t^n$ (i.e. the flexible price equilibrium output).

(d) Let $x_t \equiv y_t - y_t^*$ denote the welfare-relevant output gap. Show that the implied New Keynesian Phillips curve under real wage rigidities takes the form:

$$\pi_t = \phi_b \; \pi_{t-1} + \phi_f \; E_t\{\pi_{t+1}\} + \kappa \; x_t + \chi \; \Delta a_t + \varepsilon_t \tag{2}$$

(e) Describe the difference between (2) and the corresponding equation in the basic New Keynesian model. Discuss some of the empirical and policy implications of those differences. How would you go about estimating (2)?

**Essay (20 points).**

Discuss the main empirical shortcomings of classical monetary models, i.e. monetary models that assume perfect competition in all markets and flexible prices and wages (400 words max.)