Final Exam - Micro I

(20 points)

A consumer's utility function is given by

$$u(x_1, x_2) \;=\; (x_1 - \gamma_1)^{\beta_1} \cdot (x_2 - \gamma_2)^{\beta_2}$$

$(\beta_1, \beta_2 > 0)$ which is defined for $x_l \geq \gamma_l$.

(a) Can we assume (without loss of generality) that $\beta_1 + \beta_2 = 1$?

(b) Compute the consumer's Marshallian demand functions and show that his expenditures ($p_i x_i$) for goods 1 and 2 give rise to a linear expenditure system, i.e. $p_i x_i$ is equal to a constant plus a term that is linear in the income that remains after the two constants have been deducted. Assuming that $\gamma_1, \gamma_2 \geq 0$, how can this expenditure system be interpreted?

(c) Under what assumptions on γ_1 and γ_2 are the preferences of the consumer homothetic? [A consumer has *homothetic preferences* if his utility function is homothetic. A function $f(x)$ is *homothetic* if $f(x) = g(h(x))$, where $g(\cdot)$ is a strictly increasing function and $h(\cdot)$ is a function that is homogeneous of degree one in the vector x.]

ROCHET (20)

Consider a production economy with 2 goods $(\ell = 1, 2)$. Good 1 is consumed and used as an input for the production of good 2. Good 2 is produced by two competitive firms (*j*=1, 2), with cost functions:

$$C_1(q_1) = \frac{1}{2} q_1^2,$$
$$C_2(q_2) = q_2^2.$$

The initial endowment of the economy consists of ω units of good 1 and none of good 2.

1. Compute the supply functions of the two firms, denoting by *p* the price of good 2 (the price of good 1 is normalized to 1). Compute also their profit functions, and their input demand functions.

There are two consumers (*i*=1,2) with the same utility function:

$$u_i = x_{1i} + 3 \ln x_{2i}$$

2. Compute the competitive equilibrium price of this economy. Why is it unique?
3. Compute also the production plans of the two firms. What happens if ω<3?

We assume from now on that ω>3.

The utility function of the two consumers is modified:

$$u_i = \ln x_{1i} + 3 \ln x_{2i}$$

4. Find the new equilibrium price as a function of ω. Why is it that it does not depend on property rights (who owns the firms and the initial endowment)?

DEWATRIPONT (70)

Question 1: Banking

Two investors have each deposited D with a bank. The bank has invested these deposits in a long-term project. If the bank is forced to liquidate its investment before the project matures, a total of $2r$ can be recovered, where $D > r > D/2$. If the bank allows the investment to reach maturity, the project will pay out a total of $2R$, where $R > D$.

There are two dates at which the investors can make withdrawals from the bank: date 1 is before the bank's investment matures; date 2 is after. For simplicity, assume that there is no discounting. If both investors make withdrawals at date 1, then each receives r and the game ends. If only one investor makes a withdrawal at date 1, then that investor receives D, the other receives $2r - D$ and the game ends. Finally, if neither investor makes a withdrawal at date 1, then the project matures and the investors make withdrawal decisions at date 2. If both investors make withdrawals at date 2, then each receives R and the game ends. If only one investor makes a withdrawal at date 2, then that investor receives $2R - D$, the other receives D and the game ends. Finally, if neither investor makes a withdrawal at date 1, then the bank returns R to each investor and the game ends.

- Draw the extensive form of this game.
- Derive the pure-strategy subgame-perfect equilibrium/equilibria of this game.
- Is this game relevant to think about the financial crisis of the past month? Discuss.

Question 2 : Spence model with productive-but-costly education.

Assume two types of workers, H and L, with equal probabilities in the population.

Their respective utilities are $w-c_He$ and $w-c_Le$, where e is the level of education, w the wage and we have $1<c_H<c_L$.

Worker productivity is L+e for type L and H+e for type H (with $0<L<H$).

- show graphically the equilibrium if worker types are publicly observable. Why will unobservability of worker types matter ?
- describe *the set* of fully separating Bayesian-perfect equilibria (be precise).
- describe *the set* of fully pooling Bayesian-perfect equilibria (be precise).
- which of these equilibria satisfies the Cho-Kreps intuitive criterion ? (be precise).
- is the Cho-Kreps equilibrium Pareto-dominated by the pooling equilibrium which maximizes the utilities of the workers?

MOORE (70)

Please answer TWO of the following FIVE questions!

1. Consider an Akerlof model in which workers differ in two dimensions, according to parameters θ and α, each independently and uniformly distributed on [0,1]. A worker of type (θ,α) has productivity θ and has opportunity cost α/θ. Firms are risk neutral and have additive technologies.

(a) What would be a first-best efficient allocation of labour?

(b) Suppose that each worker privately knows her parameter α, but her parameter θ is publicly observable. Find the equilibrium wages, $w(\theta)$, and the allocation of labour.

(c) Now suppose that each worker privately knows her parameter θ, but her parameter α is publicly observable. Find the equilibrium wages, $w(\alpha)$, and the allocation of labour.

(d) Finally suppose that each worker privately knows both her parameters θ and α (i.e. neither is publicly observable). Find the equilibrium wages, w, and the allocation of labour.

(e) Compare the allocations of labour in parts (a) – (d).

2. Consider the following Spence signalling model where workers are of three types, $\theta_L = 1$, $\theta_M = 2$ and $\theta_H = 3$. A worker privately knows his own type θ. There is a fraction λ_L of type θ_L; a fraction λ_M of type θ_M; and a fraction λ_H of type θ_H, where $\lambda_L + \lambda_M + \lambda_H = 1$. A worker of type θ has cost e/θ of acquiring a level of university education $e \geq 0$. Competitive firms earn θ from a worker of type θ (university education is intrinsically useless). The workers vote over the proposition to shut down universities in order to stop signalling (assume that if the proposition is voted down, the least inefficient separating equilibrium ensues). Voting is one-worker-one-vote, with majority rule. (In the event of an exact tie, the proposition is voted through.) What is the outcome of the vote? (Hint: Consider the cases $\lambda_L > ½$ and $\lambda_M > ½$ separately.)

3. Consider a competitive screening model with many firms and workers. There are equal numbers of two types of worker, respectively with productivities $\theta_L = 1$ and $\theta_H = 3$. And there are two possible tasks, $t = 0$ and $t = T > 0$. A worker with productivity θ has a cost of t/θ of undertaking task t, and the return from such a worker to a firm is θ (the task is intrinsically useless). The timing of the model is as follows. First, each worker privately learns his own type θ. Then, the firms offer contracts comprising a wage $w(t)$ for undertaking task t. Finally, each worker chooses which contract to accept. Consider three possible pure-strategy equilibria: Equilibrium P_0, a pooling equilibrium in which both types undertake task $t = 0$; Equilibrium P_T, a pooling equilibrium in which both types undertake task $t = T$; and Equilibrium S, a separating equilibrium in which type θ_L workers undertake task $t = 0$ and type θ_H workers undertake task $t = T$. Discuss which, if any, of these three equilibria, P_0, P_T and S, exist in each of the following cases: (a) $0 < T < 1$; (b) $1 < T < 2$; (c) $2 < T < 3$; and (d) $3 < T < 6$.

4. A risk-neutral Principal contracts with a risk-neutral Agent, whose type θ may be either $\theta_L > 0$ or $\theta_H = 2\theta_L$. The Agent privately knows which type she is at the time of being hired. From the Principal's perspective, the Agent is of type θ_H with probability λ. The Agent of type θ has a cost of effort, e, equal to $e^2/2\theta$. The Principal's revenue is e. A contract specifies a menu comprising two wage-effort pairs, (w_L, e_L) and (w_H, e_H), from which the Agent selects. The Agent of type θ has an outside opportunity equal to $\mu\theta$, where μ is a nonnegative constant. Find, as a function of μ, what the optimal (w_L, e_L) and (w_H, e_H) would approximately equal, in each of the two cases: (a) λ close to 0; (b) λ close to 1.

5. What assumptions do Hellwig and Schmidt import to the Holmstrom and Milgrom repeated discrete-time agency model, in order to show that an incentive scheme linear in total profits is (approximately) optimal? Discuss.