

**Studienzentrum Gerzensee Doctoral Program in Economics
Final Exam in Econometrics**

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Write your Identification Number in the space provided below. (Do not give us your name – just your ID Number.)

Identification Number: _____

There are 180 points on this 180-minute exam. The number of possible points for each question is shown in parentheses preceding the question. Please answer all questions on the exam sheet. If you need additional space use the back of the exam sheet. Feel free to use your notes and any textbooks that you may find useful.

(20) 1. Suppose that Y_t is generated recursively by

$$Y_t = \sigma_t \varepsilon_t$$

where $\varepsilon_t \sim iidN(0,1)$ and $\sigma_t^2 = 1 + (1/2)Y_{t-1}^2$. The initial value of $\sigma_1^2 = 2$.

Prove that Y_t is covariance stationary.

(20) 2. Suppose that X_t is generated by the stationary MA(1) model

$$X_t = \varepsilon_t - 0.8\varepsilon_{t-1}$$

where $\varepsilon_t \sim iid(0,1)$.

(10) (a) Derive and plot the spectrum of X .

(10) (b) Let $Z_t = (I+L)X_t + v_t$ where $v_t \sim iid(0,4)$ and $E(v_t X_{t-k}) = 0$ for all t and k .

Derive and plot the spectrum of Z .

(25) 3. Suppose that X_t is generated by one of two possible stationary stochastic processes:

$$MA(1): X_t = \varepsilon_t + 0.5\varepsilon_{t-1} \quad \text{or} \quad AR(1): X_t = 0.5X_{t-1} + \varepsilon_t$$

where, in either case $\varepsilon_t \sim iidN(0, \sigma^2)$. Given a sample of size T , a researcher needs to decide which of these processes generated the data. To make the decision, he plans to run the $AR(2)$ regression

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + e_t \quad (2.1)$$

where e_t is the regression error. He proposes to do a t -test for the null hypothesis $H_0: \phi_2=0$. If he fails to reject this null, then he will decide the process is an $AR(1)$, otherwise he will decide that the process is an $MA(1)$.

(15) (a) Suppose the data are really generated by the $MA(1)$ model. Derive population values of the least squares regression coefficients in equation (2.1). (Equivalently, derive the probability limit of the OLS estimators of ϕ_1 and ϕ_2 – if you attack the problem this way, there is no need to prove convergence – just state the value of the plim.) Is the value of ϕ_2 non-zero?

(10) (b) Suppose that the data are generated the $AR(1)$ model. Will the researcher's procedure correctly choose the $AR(1)$ model if the sample size is very, very large? Explain?

(25) 4. Suppose that Y_t is generated by a stationary $AR(1)$ model $Y_t = \rho Y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iidN(0, \sigma^2)$.

(10) (a) Derive the probability density of the $T \times 1$ vector $\mathbf{Y} = (Y_1, Y_2, \dots, Y_T)'$.

(7) (b) Let $X_1=Y_T, X_2=Y_{T-1}, \dots, X_T=Y_1$, and let the $T \times 1$ vector $\mathbf{X}=(X_1, X_2, \dots, X_T)'$. Show that \mathbf{X} has the same probability density as \mathbf{Y} .

(4) (c) Suppose that you have data on $(Y_1, Y_2, \dots, Y_{T-1})$ and you know that value of ρ and σ^2 . How would you forecast the value of Y_T ? Explain.

(4) (d) Suppose that you have data on (Y_2, Y_3, \dots, Y_T) and you know that value of ρ and σ^2 . How would you backcast the value of Y_1 ? Explain.

5. (31 points) You have a sample of n independent observations. For each observation, you have a discrete variable, y , which takes values 0 and 1. You also have a regressor, x . Suppose that you estimate a Probit model in order to characterize the relationship between y and x (you use x and a constant as explanatory variables). Let α be the constant and let β be the coefficient on x . Suppose that you estimate α to be 1 and β to be -1 . Suppose further that you have estimated the covariance matrix of $\hat{\alpha}$ and $\hat{\beta}$ to be $\begin{pmatrix} 0.04 & 0.01 \\ 0.01 & 0.04 \end{pmatrix}$

(a) (4) What would be your estimate of $P(y = 1|x = 1)$?

(b) (6) Test whether $P(y = 1|x = 1) = 0.45$ at a 5% level of significance.

(c) (7) Construct a 95% confidence interval for $\left. \frac{dP(y=1|x)}{dx} \right|_{x=0}$.

(d) (5) How would you estimate $E \left[\frac{dP(y=1|x)}{dx} \right]$, where the expectation is over the distribution of x ?

(e) (8) Explain how you would find the asymptotic distribution of the estimator that you proposed in the previous question.

6. (21 points) Suppose that you have n independent and identically distributed observations of (y, x, z) from the nonlinear regression model

$$y = z\beta + \exp(x\beta) + \varepsilon, \quad E[\varepsilon | x, z] = 0 \quad \text{and} \quad V[\varepsilon | x, z] = \exp(z + x)$$

where y , x and z are all one-dimensional and β is the parameter of interest. Assume that all relevant moments exists.

- (a) (7) Find the asymptotic distribution of the non-linear least squares estimator, $\hat{\beta}_1$, defined by minimizing

$$\sum_{i=1}^n (y_i - z_i b - \exp(x_i b))^2$$

over b .

(b) (7) Find the asymptotic distribution of the estimator, $\hat{\beta}_2$, defined by minimizing

$$\sum_{i=1}^n \frac{1}{\exp(z_i)} (y_i - z_i b - \exp(x_i b))^2$$

over b .

(c) (7) Find the asymptotic distribution of the estimator, $\hat{\beta}_3$, defined by minimizing

$$\sum_{i=1}^n \frac{1}{\exp(z_i + x_i)} (y_i - z_i b - \exp(x_i b))^2$$

over b .

7. (10 points) Consider observations (y_{it}, x_{it}) from the linear panel data model

$$y_{i,t} = x_{it}\beta_1 + x_{i,t-1}\beta_2 + y_{i,t-1}\gamma + \alpha_i + \varepsilon_{it}, \quad t = 2, \dots, T, \quad i = 1, \dots, N$$

where α_i is an unobserved individual-specific effect.

Consider the two competing models corresponding to the assumptions

$$E^* [\varepsilon_{it} | x_{i1}, \dots, x_{iT}] = 0$$

and

$$E^* [\varepsilon_{it} | y_{i1}, \dots, y_{i,t-1}, x_{i1}, \dots, x_{it}] = 0.$$

How would you distinguish between the two models if $T = 4$? What if $T = 3$?

8. (15 points) Consider the model

$$\begin{aligned}w_{1i}^* &= x'_{1i}\beta_1 + \varepsilon_{1i} \\w_{2i}^* &= x'_{2i}\beta_2 + \varepsilon_{2i}\end{aligned}$$

where $\begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \end{pmatrix}$ is normally distributed and independent of (x_{1i}, x_{2i}) . Let $V[\varepsilon_{1i}] = \sigma_1^2$, $V[\varepsilon_{2i}] = \sigma_2^2$ and $cov(\varepsilon_{1i}, \varepsilon_{2i}) = \sigma_{12}$. Suppose that you observe $(y_{i1}, y_{i2}, x_{1i}, x_{2i})$ where $y_{i1} = 1\{w_{1i}^* > w_{2i}^*\}$ and $y_{i2} = \max\{w_{1i}^*, w_{2i}^*\}$.

(a) (12) Find the likelihood function.

(b) (3) Are all the parameters of the model identified? If not, which ones are not?

9. (13 points) Suppose that you have a random sample of n observations from the distribution of a continuously distributed, non-negative random variable. Suppose you estimate the density by the usual kernel estimator,

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - x_i}{h_n}\right)$$

Find an approximate expression for $E[\hat{f}(0)]$ under the usual assumptions that

- $K(u)$ bounded density
- $K(u) = K(-u)$
- $K(u) \rightarrow 0$ as $|u| \rightarrow \infty$
- $\int u^2 K(u) du = 1$.

(Hint: We are looking for an approximations which is good when h is small)