

Exam Questions

1. True, False or Uncertain ? (60 points)

Discuss the following statements. Credit given will be a function of the quality and coherence of your explanation. Feel free to combine verbal, graphical, or algebraic languages in your answer

- a. "Suppose that technological shocks were a dominant source of aggregate fluctuations. In that case the concept of output gap that arises in recent dynamic optimizing models (with staggered price setting) should be well approximated by measures of detrended log GDP (i.e., the residual from eliminating from log GDP a fitted trend that is a smooth function of time)."
- b. "The Friedman rule is optimal, always and everywhere."
- c. "A straightforward application of the Ramsey approach to optimal taxation indicates that it is always optimal to fully stabilize the tax rate on labor income, so long as there are exactly offsetting income and substitution effects of productivity shifts on labor supply."
- d. "Modern equilibrium asset pricing theories predict that the expected return on long bonds must always exceed that on short bonds."

2. Inflation Targeting with and without Noisy Data (60 points)

Consider a model economy whose output gap (x) and inflation (π) dynamics are described by the system:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t \quad (0.1)$$

$$x_t = -\frac{1}{\sigma} (r_t - E_t \{\pi_{t+1}\} - \bar{r}_t) + E_t \{x_{t+1}\} \quad (0.2)$$

where \bar{r}_t denotes the natural rate of interest. The latter is assumed to follow the exogenous process

$$\bar{r}_t = \rho_r \bar{r}_{t-1} + \varepsilon_t$$

where $\{\varepsilon_t\}$ is white noise and $\rho_r \in [0, 1)$

- a. Derive the interest rule that guarantees that inflation will be fully stabilized at a target level π^* . Determine the output gap level x^* consistent with that constant inflation target.
- b. Consider the simple Taylor-type rule:

$$r_t = \pi^* + \phi_\pi (\pi_t - \pi^*) \quad (0.3)$$

where $\phi_\pi > 1$. Show that the equilibrium dynamics under the previous rule can be represented by the system:

$$\begin{bmatrix} \hat{x}_t \\ \hat{\pi}_t \end{bmatrix} = \mathbf{A}_\Pi \begin{bmatrix} E_t \{\hat{x}_{t+1}\} \\ E_t \{\hat{\pi}_{t+1}\} \end{bmatrix} + \mathbf{B}_\Pi \bar{r}_t$$

where $\hat{x}_t = x_t - x^*$, $\hat{\pi}_t = \pi_t - \pi^*$,

$$\mathbf{A}_\Pi = \Theta \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta\sigma \end{bmatrix} \quad ; \quad \mathbf{B}_\Pi = \Theta \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Theta = \frac{1}{\sigma + \kappa\phi_\pi}$.

- c. Assume for simplicity that $\rho_r = 0$. Solve for the equilibrium processes for inflation and the output gap under the rule (0.3). Describe the behavior of inflation, the output gap, and the nominal rate when ϕ_π approaches infinity.

d. Suppose that inflation is measured with some i i d error ξ_t , i e, $\tilde{\pi}_t = \pi_t + \xi_t$. Assume that the central bank now follows the rule

$$r_t = r + \phi_\pi (\tilde{\pi}_t - \pi^*) \quad (0.4)$$

Solve for the equilibrium processes for inflation and the output gap under the rule (0.4) (again, assume $\rho_r = 0$). Describe the behavior of inflation, the output gap, and the nominal rate when ϕ_π approaches infinity. Determine the size of the inflation coefficient that minimizes the variance of inflation.

3. Linear Rational Expectations Models (22.5 points)

Consider the linear rational expectations model comprised of the following two equations,

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa x_t$$

defined as in question 2 above and a strong inflation target rule of the form

$$\begin{aligned} \pi_t &= u_t \\ u_t &= \rho u_{t-1} + e_t \end{aligned}$$

where $|\rho| \leq 1$ and e_t is a white noise random variable.

a. Write the model the form $AE_t Y_{t+1} = BY_t + CX_t$ with Y_t being a vector of endogenous variables and X_t being a vector of exogenous variables (note that X_t is not identical to x_t).

b. Is the matrix A singular? Is the matrix B singular? Is the model solvable?

c. What is the rational expectations solution that links π_t to u_t ?

4. Output, input and business cycles (37.5 points)

Suppose that a representative firm has a production function,

$$y_t = a_t f(n_t, z_t k)$$

where a_t is a productivity shock; n_t is labor input, z_t is the utilization rate of capital and k is the stock of capital, which is taken to be constant throughout the discussion.

a. Explain why the introduction of variable capital utilization would work, in a real business cycle context, to somewhat amplify the effects of productivity shocks on output;

b. Explain why the combination of highly elastic labor supply and variable capital utilization would work to greatly amplify the effects of productivity shocks on output;

c. Now suppose that there are no productivity shocks; that the production function is Cobb-Douglas and that the capital stock is normalized to one for convenience. Then, the production function is

$$y_t = a(n_t)^\alpha (z_t)^{1-\alpha}$$

Suppose also that the firm can buy its factor inputs at prices w_t per unit of labor and price q_t per unit of capital services

Derive the demands for labor and capital services, at given output level y_t , associated with cost minimization. Find the associated marginal cost, relating it to w_t and q_t .

d. Additionally, suppose that the supply of labor is given by

$$n_t = n w_t^\gamma$$

and the supply of capital services is given by

$$z_t = z q_t^\delta$$

Find how marginal cost is related to output in general equilibrium. Describe the consequences of highly elastic supply behavior for the general equilibrium relationship between marginal cost and output. What are some potential implications of such supply behavior for sticky price business cycle theory?