Studienzentrum Gerzensee Doctoral Program in Economics
Final Exam in Microeconomics

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Write your Identification Number in the space provided below. (Do not give us your name – just your ID Number.)

Identification Number____

There are 180 points on this 180 minute exam. Please answer all questions on the exam sheet. If you need additional space, use the back of the exam sheet. Feel free to use your notes and any textbooks that you may find useful.

1. Suppose that an agent's utility function has constant relative risk aversion with $r_R(x) = 1$.

(a) Show that this is the case if the agent's utility function is a positive affine transformation of $u(x) = \ln x$.

(b) Let W denote the level of wealth of the agent. The agent has to decide which fraction, a, of W to invest in a risky asset that yields a random return $\tilde{\pi}$ per unit of investment. The rest of his wealth has to be held in cash yielding no interest. Show that the optimal level of a is independent of W.

2. Consider an economy with 3 goods: gas, electricity and a composite good called money, that is taken as a numiraire. There is one consumer, with a utility function

$$U = x_g + 2x_e - \frac{1}{2}x_g^2 - \frac{1}{2}x_e^2 + m,$$

where x_g represents the consumption of gas, x_e the consumption of electricity and m the consumption of money. There are 2 firms: a gas producer with a cost function $C_g(y_g) = \frac{1}{2}y_g^2$ and an electricity producer with a cost function $C_e(y_e) = y_e$.

y_g and y_e represent respectively the productions of gas and electricity. The consumer owns the two firms and its initial endowment, which consists of M units of money. The prices of gas and electricity are denoted respectively p_g and p_e.

(a) Determine the demand functions of the consumer.

(b) Determine the supply functions of the producers.

(c) Compute the quantities of gas and electricity consumed at the competitive equilibrium.

(d) The gas producer builds an electricity generator, which uses gas to produce electricity with constant returns to scale: $\beta < 1$ units of electricity are produced for each unit of gas used as an input. Compute the new competitive equilibrium and compare it with the one derived in part (c). (Hints: Distinguish the cases $\beta < 1/2$ and $\beta > 1/2$; recall that outputs are always positive).

(e) (Optional) What would happen if U was replaced by $V = U - \gamma x_e x_g$, with $0 < \gamma < 1$?

3. Downsizing a Public Firm

We consider a public firm which is producing a public good with a continuum of workers of mass 1. Each worker produces one unit of public good. A mass $q \in [0,1]$ of workers produces an output q which has social value

$$S(q) \quad S' > 0 \quad S'' < 0.$$

New outside opportunities appear for workers calling for a downsizing of the public firm.

Let θ_i the outside utility level that worker i can obtain. θ_i can take one of two positive values $\{\underline{\theta}, \bar{\theta}\}$ with $\Delta\theta = \bar{\theta} - \underline{\theta} > 0$.

These outside opportunities are independently distributed between workers on $\{\underline{\theta}, \bar{\theta}\}$ with $\nu = \Pr(\theta_i = \underline{\theta})$ and $1 - \nu = \Pr(\theta_i = \bar{\theta})$.

A new allocation of labor is characterized by the proportions $\underline{p}$ (resp. $\bar{p}$) of workers of type $\underline{\theta}$ (resp. $\bar{\theta}$) who remain in the public firm.

Total production of the public firm is then

$$q = \nu\underline{p} + (1-\nu)\bar{p}.$$

Let us first assume that the values of outside opportunities are public knowledge. In other words the government is under complete information.

To keep workers in the firm, the government must pay them, $\underline{t}$ for type $\underline{\theta}$, $\bar{t}$ for type $\bar{\theta}$ and satisfy their participation constraints (at least for those it wants to keep)

$$\begin{aligned} \underline{t} &\geq \underline{\theta} \\ \bar{t} &\geq \bar{\theta}. \end{aligned}$$

If there is a cost $1 + \lambda$ of public funds these constraints will be binding and social welfare is equal to:

$S(\nu\underline{p} + (1-\nu)\bar{p})$	Social value of the public good
$-(1+\lambda)(\nu\underline{p}\underline{\theta} + (1-\nu)\bar{p}\bar{\theta})$	Social cost of workers remaining in the public firm
$+\nu\underline{p}\underline{\theta} + (1-\nu)\bar{p}\bar{\theta}$	Welfare of workers in the public firm
$+\nu(1-\underline{p})\underline{\theta} + (1-\nu)(1-\bar{p})\bar{\theta}$	Welfare of workers outside the public firm.

Social welfare reduces to

$$S(\nu\underline{p} + (1-\nu)\bar{p}) + \nu\underline{\theta} + (1-\nu)\bar{\theta} - (1+\lambda)\nu\underline{p}\underline{\theta} - (1+\lambda)(1-\nu)\bar{p}\bar{\theta}.$$

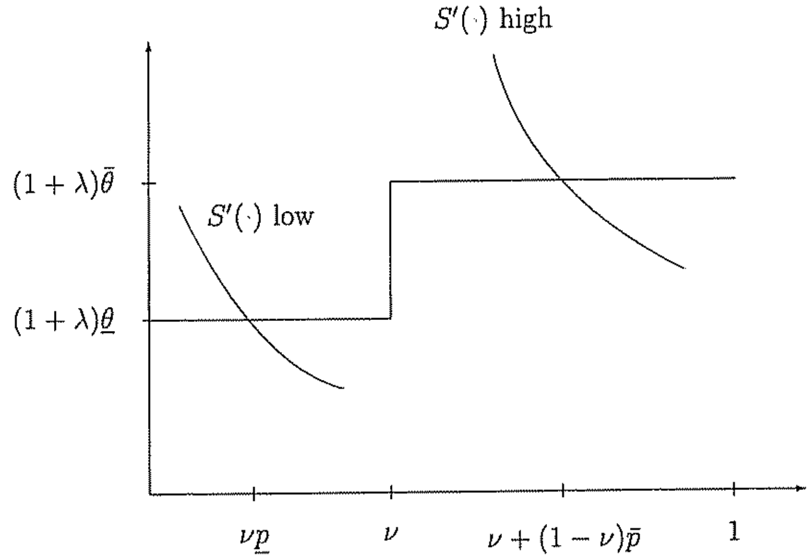
The social optimum is then characterized by:

if $S'(\cdot)$ is large

$$S'(\nu + (1-\nu)\bar{p}) = (1+\lambda)\bar{\theta} \text{ and } \underline{p} = 1,$$

if $S'(\cdot)$ is small

$$S'(\nu \underline{p}) = (1+\lambda)\underline{\theta} \text{ and } \bar{p} = 0.$$



Consider now that the θ_i are private information of the workers.

A *downsizing mechanism* $(\underline{p}, \underline{t})$ $(\bar{p}, \bar{t})$ is incentive compatible if

$$\begin{aligned} \bar{t} + (1-\bar{p})\bar{\theta} &\geq \underline{t} + (1-\underline{p})\bar{\theta} \\ \underline{t} + (1-\underline{p})\underline{\theta} &\geq \bar{t} + (1-\bar{p})\underline{\theta}. \end{aligned}$$

It satisfies the *participation constraints* if

$$\begin{aligned} \bar{t} + (1-\bar{p})\bar{\theta} &\geq 0 \\ \underline{t} + (1-\underline{p})\underline{\theta} &\geq 0. \end{aligned}$$

Discuss these constraints. Characterize the allocation of labor which maximizes expected social welfare.

4. Suppose now that workers differ by the quality of their production in the public firm. A type $\underline{\theta}$ worker produces $\varphi(\bar{\theta})$ units of public good while a type $\bar{\theta}$ worker produces $\psi(\bar{\theta})$ Characterize the allocation of labor which maximizes expected social welfare (distinguish two cases: $\bar{\theta}/\varphi(\bar{\theta}) > \underline{\theta}/\psi(\underline{\theta})$ and the opposite inequality).

5. Competition and Market Integration

Consider a country A where demand for a product is defined by:

$$P_A = \sum_i q_i$$

where q_i is the output of firm i. Each firm has a constant marginal cost equal to 2. Competition takes place in quantities, that is, a la Cournot.

(a) If each firm has a fixed cost of being active of 0.1, what is the free-entry equilibrium in terms of:

- the number of active firms (n_A);
- the quantity per firm (call it q_A);
- and the price level P_A?

Assume there is also country B, initially separated from country A, which has the same demand as country A, with domestic firms in this country that produce with the same technology, and therefore the same free-entry equilibrium, if the two markets are segmented (that is, $n_B = n_A$, $q_B = q_A$, and $P_B = P_A$).

But now assume that the two markets become integrated, in that all firms now face the same *overall* market demand.

(b) What is the equation of this demand?

(c) What is the free-entry equilibrium now (number of active firms in the unified market, quantity per firm and price level), if each firm still has the same fixed cost of being active of 0.1?

(d) Does market integration increase total welfare? How does such a policy compare with the entry of new firms on a market of given size? (10 lines max)

(e) What are the limits of this analysis? (10 lines max)

6. Bargaining under Incomplete Information

Consider a seller with zero valuation for a good who makes offers to a buyer who can have valuation b_1 or b_2 for this good, where both valuations are equiprobable and $0 < b_1 < b_2 < 2b_1$.

(a) If the seller can only make one take-it-or-leave-it offer, what is the unique perfect Bayesian equilibrium?

Assume the buyer has discount factor $\delta_B < 1$ and the seller has discount factor $\delta_S < 1$, and the seller can make one offer in period 1, and a second offer in period 2 if the first offer has been rejected:

(b) Prove that the unique perfect Bayesian equilibrium involves immediate trade with probability 1 if $\delta_B = \delta_S$.

(c) What is the unique perfect Bayesian equilibrium if $\delta_B < \delta_S = 1$ instead?