STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Swiss Program for Beginning Doctoral Students in Economics 2002

Final Exam in Macroeconomics

Tuesday, February 25, 2003, 08.30h – 11.30h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Please use **a pen** rather than a pencil so that your answers can be read without problems.
4. Please **write legibly**. Remember that your exams will be photocopied for grading.
5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.
6. Good luck!

ID-Number:____

Problem 1 [20 points] *Rational expectations models.*

Suppose that a variable p_t is governed by the equation

$$p_t = \beta E_{t-1} p_t + \theta x_t$$

(a) Suppose that $\beta \neq 1$ and that p_t is not predetermined. Calculate the rational expectations solution. Under what condition on β is it possible to solve the model for p_t?

(b) Now define a new variable $w_t = E_{t-1} p_t$, which is predetermined. Writing the model as the pair of equations

$$\begin{aligned} p_t &= \beta w_t + \theta x_t \\ w_{t+1} &= E_t p_{t+1} \end{aligned}$$

show how to cast it in the first order form $AE_t Y_{t+1} = BY_t + CX_t$.

(c) Find the roots of $|Az - B| = 0$ and $|Bz - A| = 0$. Discuss whether these suggest that the dynamic system in (b) is uniquely solvable. In particular, what is the content of the condition that there must be some z such that $|Az - B| \neq 0$ within this model in terms of solvability?

Problem 2 [40 points] *Optimal Tax Policy*

Consider an economy in which there is a exogenous rule that makes government purchases move proportionately with ouput, $g_t = \theta y_t$, but these government purchases do not yield utility to the economy's citizens. Suppose that the production technology is $y_t = an_t$ where n_t is work effort and that individuals have preferences over consumption and leisure of the form $U = \sum_{t=0}^{\infty} \beta^t u_t$ with $u_t = \log(c_t) - \frac{\chi}{1+\gamma} n_t^{1+\gamma}$. Suppose finally that markets for goods and factors are competitive and that the economy is closed, so that $c_t + g_t = y_t$ each period.

(a) Suppose intially that government purchases are financed via lump-sum taxation. Calculate the equilibrium levels of consumption, work effort and production that will prevail in this economy.

(b) Next, calculate the levels of consumption, work effort and production that are first-best in this economy taking as given the rule for government purchases, i.e., are the result of maximizing utility subject to the constraints in the economy.

(c) How and why are the answers to (a) and (b) different?

(d) Suppose that you were asked to evaluate whether a switch from lump-sum taxation to labor income taxation at rate $\tau = \theta$ was a good idea. Based on your answers to (a),(b) and (c), what is your intuition? Why?

(e) How would you formally determine the optimal rate of labor taxation?

Problem 3 [30 points] *Small Open Economy Macroeconomics*

Consider a small open economy populated by a representative agent who has utility defined over the consumption of tradable (C_t^T) and non-tradable goods (C_t^N):

$$U = \int_0^\infty e^{-\rho t}[\theta \log(C_t^T) + (1-\theta)\log(C_t^N)]dt,$$
$$0 < \theta < 1, \rho > 0,$$

where ρ is the discount factor. Agents in this economy can borrow and lend in international capital markets at a fixed interest rate r. Assume that $r = \rho$.

The economy has in every period constant endowments of tradables (Y^T) and nontradables (Y^N). The economy's resource constraints are given by:

$$\dot{a}_t = ra_t - C_t^T + Y^T,$$
$$C_t^N = Y^N$$
$$\lim_{t\to\infty} e^{-rt}a_t = 0,$$
$$a_0 > 0, \text{ given,}$$

where a_t denotes the economy's net foreign asset position at time t.

(a) Derive the first-order conditions for the planner's problem for this economy.

(b) Characterize the current account ($\dot{a}_t$) and the trade balance ($TB = Y_t^T - C_t^T$).

(c) Derive the first order conditions for the household's problem below, where p_t denotes the relative price of nontradables.

$$\max U = \int_0^\infty e^{-\rho t}[\theta \log(C_t^T) + (1-\theta)\log(C_t^N)]dt \quad ,$$

subject to:

$$\dot{a}_t = r a_t - C_t^T - p_t C_t^N + Y^T + p_t Y^N$$
$$\lim_{t\to\infty} e^{-r t} a_t = 0,$$
$$a_0 > 0, \text{ given}$$

Use these conditions to compute the equilibrium path for p_t.

(d) Suppose that the economy receives an unanticipated transfer of net foreign assets that raises the value of a_0 to $\bar{a}_0 > a_0$. What is the impact of this transfer on the trade balance and on the relative price of nontradables.

(e) Suppose that there is a *permanent* increase in the nontradable endowment, Y^N. What is the impact of this increase on the trade balance and on the relative price of non-tradables?

(f) Suppose that the flow tradable endowment increases *temporarily* from Y^T to $Y^T + \phi$ between $t = 0$ and $t = 1$:

$$Y_t^T = \begin{cases} Y^T + \phi, & 0 \leq t \leq 1, \\ Y^T, & t > 1. \end{cases}$$

What is the impact of this increase on the trade balance and on the relative price of non-tradables for $0 \leq t \leq 1$ and for $t > 1$?

(g) **[EXTRA CREDIT]** Suppose that the flow tradable endowment increases *temporarily* from Y^T to $Y^T + \phi$ between $t = 0$ and $t = 1$. What is the impact of this increase on the current account for $0 \leq t \leq 1$ and for $t > 1$?

Problem 4 [60 points] *Inflation and Output Gap Dynamics under a Simple Interest Rate Rule*

Consider the Calvo staggered price setting model developed in class. The consumer's log-linearized Euler equation takes the form:

$$c_t = -\frac{1}{\sigma}\,(r_t - E_t\{\pi_{t+1}\} + \rho) + E_t\{c_{t+1}\}$$

where c_t is (log) consumption, r_t is the nominal rate, and $\pi_{t+1} \equiv p_{t+1} - p_t$ is the rate of inflation between t and $t+1$. The (log) real wage is given by::

$$w_t - p_t = \sigma\; c_t + \varphi\; n_t + v_t$$

where w_t denotes the (log) nominal wage, p_t is the (log) price level, n_t is (log) employment, and v_t is an exogenous, time-varying wage markup.

Firms' technology is given by the production function:

$$y_t = a_t + n_t$$

where y_t denotes (log) output, and a_t represents an exogenous productivity process.

Let $\overline{p}_t = \mu + mc_t^n$ denote the price (in logs) that a firm would choose in period t in the absence of constraints on price adjustment (i.e., under flexible prices), where $\mu > 0$ is a constant markup, and mc_t^n is the (log) nominal marginal cost. When the time between price adjustments is random, the optimal price setting rule for a firm setting a new price p_t^* in period t is assumed to be given by the weighted average:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\; E_t\{\overline{p}_{t+k}\}$$

where θ denotes the probability that a firm has to keep the price unchanged in any given period. Finally, the aggregate price level evolves according to the following law of motion:

$$p_t = \theta\; p_{t-1} + (1-\theta)\; p_t^*$$

In equilibrium all output is consumed, i.e., $y_t = c_t$, all t.

(a) Let $\mu_t \equiv p_t - mc_t^n$ denote the economy's average markup in period t. Show how the price setting behavior described above gives rise to the inflation dynamics equation:

$$\pi_t = \beta\; E_t\{\pi_{t+1}\} - \lambda\; \widehat{\mu}_t$$

where $\widehat{\mu}_t \equiv \mu_t - \mu$, and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

(b) Explain in words why stabilization of markups at the "frictionless" level μ is both necessary and sufficient in order to stabilize prices fully ($\pi_t = 0$, all t) in the above economy.

(c) Derive an expression for the *natural* level of output $\overline{y}_t$ (i.e., defined as the equilibrium level of output under flexible prices *and* no distortions in the labor market), as a function of exogenous variables.

(d) Use the previous results to derive a new Keynesian Philips curve of the form

$$\pi_t = \beta \, E_t\{\pi_{t+1}\} + \kappa \, \widetilde{y}_t + \xi \, v_t$$

where $\widetilde{y}_t \equiv y_t - \overline{y}_t$ is the output gap. Determine κ and ξ as a function of the underlying structural parameters.

(e) Assume that both productivity growth $\{\Delta a_t\}$ and the labor market distortion $\{v_t\}$ follow independent white noise processes with zero means, and variances σ_a^2 and σ_v^2, respectively. Determine the equilibrium behavior of the output gap $\widetilde{y}_t$ and inflation π_t when the monetary authority follows the simple interest rate rule $r_t = \rho + \phi_\pi \ \pi_t$ where $\phi_\pi > 1$.(hint: guess and verify that both $\widetilde{y}_t$ and π_t are themselves zero mean white noise processes in equilibrium).

(f) Suppose that the central bank seeks to minimize the loss function $var(\pi_t) + \alpha \ var(\widetilde{y}_t)$, where α captures the relative weight attached to stabilization of the output gap. Determine the optimal choice for the inflation coefficient ϕ_π. Discuss how its size depends on α and other parameters.

Problem 5 [30 points] *Consumption and Asset Pricing*

Suppose that there is a consumer who has the expected utility function

$$E_t\{\sum_{s=0}^{\infty}\beta^s u(c_{t+s})\}$$

and the budget constraint

$$y(\delta_t) + \sum_{j=1}^{J}(p_j(\delta_t) + d_j(\delta_t))x_{jt} = c_t + \sum_{j=1}^{J}p_j(\delta_t)x_{j,t+1}$$

This budget constraint reflects the idea that the consumer has labor income $y(\delta_t)$ and has financial wealth $\sum_{j=1}^{J}(p_j(\delta_t) + d_j(\delta_t))x_{jt}$ as a result of having previously purchased amount x_{jt} of each of J risky securities that currently has an ex-dividend price $p_j(\delta_t)$ and a dividend $d_j(\delta_t)$. The consumer can spend his resources on consumption c_t and on purchases of securities for next period (the amount of security j bought for holding into next period is $x_{j,t+1}$). δ_t is a markov process on which income and security prices depend.

(a) Write down the Bellman equation appropriate for a dynamic programming formulation on this consumption and asset allocation problem. What are the "state variables" on which the value function depends?

(b) Calculate a set of efficiciency conditions for consumption and security purchases. Provide an economic explanation of the form of these conditions.

(c) Show how to use the envelope theorem to determine the derivatives of the value function with respect to the controlled state variables.

(d) Assuming that all agents in the economy have the objective shown above, derive a restriction on the form of security returns similar to that studied in work on (i) the equity premium puzzle; and (ii) consumption-based asset pricing.

(e) Discuss how you could use the properties of "rational expectations forecast errors" as the basis for (i) estimation of the parameters of the utility function; and (ii) testing of this asset pricing theory.