STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

Swiss Program for Beginning Doctoral Students in Economics 2002

Final Exam in Microeconomics

Wednesday, February 26, 2003, 08.30h – 11.30h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3. Please use **a pen** rather than a pencil so that your answers can be read without problems.

4. Please **write legibly**. Remember that your exams will be photocopied for grading.

5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.

6. Good luck!

ID-Number:______________________________

Problem 1:

Let $v(p,x)$ be the indirect utility function of a consumer.

(a) Show that Marshallian demand is invariant to arbitrary monotonic transformations of $v(p,x)$.

(b) Suppose that the consumer consumes two goods and that his indirect utility function is given by:

$$v(p,w) = \left(\frac{\alpha}{p_1}\right)^{\alpha}\left(\frac{1-\alpha}{p_2}\right)^{1-\alpha} w$$

Derive the Hicksian demand function for good 1.

(c) Show that the Hicksian demand function for good 1 does not violate homogeneity of degree 0 in p if $\alpha = \alpha\left(\frac{p_1}{p_2}\right)$, but that it does violate homogeneity of degree 0 in p if $\alpha = \alpha(p_1)$.

Problem 2:

Consider an exchange economy with 2 goods $(\ell = 1,2)$ and 2 consumers (i = 1,2). The utility functions of these consumers are:

$$u_1 = x_{11} + 2x_{21} - x_{21}^2,$$
$$u_2 = x_{12} + 6x_{22} - x_{22}^2,$$

where $x_{\ell i}$ represents the quantity of good ℓ consumed by consumer i. The initial endowments are $\omega_1 = \left(1, \frac{5}{2}\right)$ for consumer 1 and $\omega_2 = (1,0)$ for consumer 2.

a) Characterize the Pareto optima of this economy. Represent them in the Edgeworth box.

b) Explain why all interior Pareto optima are associated with the same allocation of good 2 among the 2 consumers.

c) Compute the competitive equilibrium of this economy.

d) Explain why it is unique.

e) **(optional)** Compute the allocation that maximizes the Rawlsian welfare function $W = \max(u_1, u_2)$ among feasible allocations.

Problem 3: *Bertrand Competition with Asymmetric Information*

Consider a Bertrand duopoly with differentiated complementary products, where demand for firm i is $q_i(p_i, p_j) = a - p_i - b_i p_j$. Costs are zero for both firms. The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L, where $b_H > b_L > 0$. For each firm, $b_i = b_H$ with probability θ and $b_i = b_L$ with probability $1 - \theta$, independent of the realization of b_j. Each firm knows its own b_i but not its competitor's. All of this is common knowledge.

a) What conditions define a symmetric pure-strategy Bayesian equilibrium in this game (assume an interior solution exists)?

b) Solve for such an equilibrium.

Problem 4: *Spence model with productive-but-costly education.*

Assume two types of workers, H and L, with probabilities a and $1-a$ in the population.
Their respective utilities are $w-c_He$ and $w-c_Le$, where e is the level of education, w the wage and we have $1<c_H<c_L$.
Worker productivity is L+e for type L and H+e for type H (with $0<L<H$).

a) Show graphically the equilibrium if worker types are publicly observable. Why will unobservability of worker types matter ?

b) Describe *the set* of fully separating Bayesian-perfect equilibria (be precise).

c) Describe *the set* of fully pooling Bayesian-perfect equilibria (be precise).

d) Which of these equilibria satisfies the Cho-Kreps intuitive criterion ? (be precise).

e) Explain the two shortcomings of the Cho-Kreps refinement when a is close to 1 (be precise).

Problem 5

An agent has preferences $U = -\frac{1}{2}(p-\theta-\delta)^2+t$ (with $\delta > 0$) where p is a policy-variable chosen by a decision-maker, θ is the state of nature ($\theta \in \Theta = \{\underline{\theta}, \bar{\theta}\}$ with respective probabilities $1-\nu$ and ν with $\Delta\theta > 0$) and t is a monetary transfer. The decision-maker has preferences $V = -\frac{1}{2}(p-\theta)^2 - t$. θ is known only by the agent.

a) Give an economic interpretation of δ.

b) Describe the set of incentive feasible contracts when p and t are the contracting variables. We suppose that the agent's participation constraint is normalized at $U_0 < 0$. Show that the monotonicity condition $\bar{p} \geq \underline{p}$ holds.

c) Optimize the principal's expected payoff and find the optimal contract. Interpret your results.

d) Suppose now that transfers are not feasible. A contract stipulates now $\{p(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ where $\hat{\theta}$ is the agent's report. Write the set of incentive feasible contracts. Give a graphical representation of this set. Show that the monotonicity condition $\bar{p} \geq \underline{p}$ still holds.

e) Optimize the principal's expected payoff neglecting the agent's participation constraints. One can distinguishe the cases $\delta \leq \frac{\Delta\theta}{2}$ and $\delta > \frac{\Delta\theta}{2}$. Interpret your results. Compare the optimal contracts with and without transfers.