Swiss Program for Beginning Doctoral Students in Economics 2003

Final Exam in Econometrics

Monday, February 23, 2004, 14.00h - 17.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3. Please use **a pen** rather than a pencil so that your answers can be read without problems.

4. Please **write legibly**. Remember that your exams will be photocopied for grading.

5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.

6. Good luck!

ID-Number:____

Problem 1. (20 points)
Suppose that y_t is a series that is available semiannually (that is, twice per year), once in the winter and once in the summer. Suppose that the semiannual seasonal process for the series is

$$y_t = 0.9 y_{t-1} + \varepsilon_t$$

where ε_t is iid(0, 1).

(a) (10 points) Derive and plot the spectrum of y. Discuss how the seasonality in the process is evident in spectrum.

(b) (10 points) A researcher proposes to use $x_t = 0.5\ (1+L)y_t$, as a "seasonally adjusted" version of y. Compute the gain of the filter 0.5(1+L). Does this filter eliminate/and or attenuate the seasonality in y? Explain.

Problem 2. (15 points)
Suppose

$$y_t = \xi_t + w_t$$
$$\xi_t = 0.8\xi_{t-1} + v_t$$

where w_t is iid N(0,2), v_t is iid N(0,3) and $\{w_t\}$ and $\{v_t\}$ are independent. Suppose that $\xi_{t-1} = 3.4$ and $y_t = 4.1$. Find $E(\xi_t | y_t = 4.1, \xi_{t-1} = 3.4)$ and $var(\xi_t | y_t = 4.1, \xi_{t-1} = 3.4)$.

Problem 3. (15 points)
Suppose that $y_t = \mu_1 + \varepsilon_t$, ε_t iid$(0, \sigma_1^2)$ for $t = 1, \ldots, \tau$, that $y_t = \mu_2 + \varepsilon_t$, ε_t iid$(0, \sigma_2^2)$ for $t = \tau + 1, \ldots, T$, and $E(\varepsilon_t^4) < \infty$ for all t.

(a) (7 points) Suppose that you knew τ, and that both τ and $T-\tau$ are large. How would you test H_o: $\mu_1 = \mu_2$ versus H_a: $\mu_1 \neq \mu_2$, using a test that allowed $\sigma_1^2 \neq \sigma_2^2$?

(b) (8 points) Suppose that you knew τ, and that both τ and $T{-}\tau$ are large. How would you test H_o: $\sigma_1^2 = \sigma_2^2$ versus H_a: $\sigma_1^2 \neq \sigma_2^2$ using a test that allowed $\mu_1 \neq \mu_2$?

Problem 4. (20 points)
Suppose that y and x follow the process

$$y_t = \alpha x_{t-1} + \varepsilon_t$$

$$x_t = \phi x_{t-1} + v_t$$

where $v_t = \varepsilon_t + e_t$ where ε_t and e_t are mutually independent iid(0,1) processes. Suppose that $\phi < 1$. Let $\hat{\alpha}$ and $\hat{\phi}$ denote the OLS estimators of α and ϕ.

(a) (12 points) Derive the asymptotic distribution of the 2×1 vector $(\hat{\alpha}, \hat{\phi})$.

(b) (8 points) In a sample of 100 observations, $\hat{\alpha}$ = 1.3 and $\hat{\phi}$ = 0.72. Derive a 95% confidence interval for α.

Problem 5. (20 points)
Consider the same model as in question (2), but now assume that $\phi = 1$.

(a) (10 points) Show that x and y are cointegrated. What is the cointegrating vector?

(b) (10 points) Derive the asymptotic distribution of $\hat{\alpha}$.

Problem 6. (20 points)

This problem is concerned with bounding treatment effects. Suppose that D is a random variable that indicates whether an individual has been "treated". If $D = 1$, the individual has received treatment and we observe a random variable Y_1. If $D = 0$, the individual has not received treatment and we observe a random variable Y_0. We do not observe Y_0 if $D = 1$ and we do not observe Y_1 if $D = 0$. This is the standard notation in this literature.

The parameter of interest is the average treatment effect on the treated, $E[Y_1 - Y_0 | D = 1]$.

(a) (10 points) Suppose that it is known that $0 \leq Y_0 \leq 100$ and $0 \leq Y_1 \leq 100$. Use this to construct bounds on $E[Y_1 - Y_0 | D = 1]$.

(b) (10 points) Suppose that it is known that $0 \leq Y_0 \leq Y_1 \leq 100$. Use this to construct bounds on $E\left[Y_1 - Y_0 \middle| D = 1\right]$.

Problem 7. (30 points)

Suppose that you have a random sample of (y_i, x_i) of size n from

$$y_i = \log\left((x_i'\beta)^2 + 1\right) + \varepsilon_i$$

where $E[\varepsilon_i | x_i] = 0$. Assume that all relevant moments exist.

(a) (10 points) Find the asymptotic distribution of the nonlinear least squares estimator that minimizes

$$\sum_{i=1}^{n}\left(y_i - \log\left((x_i'b)^2 + 1\right)\right)^2$$

(b) (10 points) How would you estimate the asymptotic variance of the nonlinear least squares estimator?

(c) (10 points) Now assume that you know that $V[\varepsilon_i | x_i] = \exp(x_i'\alpha)$ and that you estimate α by minimizing

$$\sum_{i=1}^{n} (e_i - \exp(x_i'a))^2$$

where $e_i = y_i - \log\left(\left(x_i'\widehat{\beta}_{NLS}\right)^2 + 1\right)$. Briefly discuss how you would find the asymptotic distribution of this estimator.

Problem 8. (20 points)

Consider observations of (y_{it}, x_{it}) from the linear panel data model

$$y_{i,t} = x_{it}'\beta + \gamma y_{i,t-1} + \alpha_i + \varepsilon_{it}, \quad t = 1, ..., T, \quad i = 1, ..., N$$

where α_i is an unobserved individual–specific effect. No assumption is made on the relationship between α_i and x_{it}. (Note that you do not observe $y_{i,0}$).

(a) (10 points) Suppose that the dimensionality of x_{it} is 1 and

$$E\left[\varepsilon_{is} x_{it}\right] = 0 \qquad \text{for all } t \leq s \tag{1}$$

What is the minimum T such that β are identified? Is the model over–identified for that T? Explain and explicitly state any additional "regularity conditions" that you assume.

(b) (10 points) Suppose that the dimensionality of x_{it} is 2 and

$$E\left[\varepsilon_{is}x_{it}\right] = 0 \qquad \text{for all } t \leq s+1 \tag{2}$$

(loosely speaking, we allow feedback from todays ε to future values of x, but we are willing to assume that it takes two periods for this to happen). What is the minimum T such that β are identified? Is the model over–identifies for that T? Explain and explicitly state any additional "regularity conditions" that you assume.

Problem 9. (20 points)

You have a sample of n independent observations. For each observation, you have a discrete variable, y, which takes values 0 and 1. You also have a regressor, x, and you estimate a logit model in order to characterize the relationship between y and x (you use using x and a constant as explanatory variables). Let α be the constant and let β be the coefficient on x.

Suppose you estimate α to be 1 and β to be -1. Suppose further that you have estimated the covariance matrix of $\widehat{\alpha}$ and $\widehat{\beta}$ to be $\begin{pmatrix} 0.04 & 0.02 \\ 0.02 & 0.04 \end{pmatrix}$

(a) (4 points) What would be your estimate of $P(y = 1|x = 1)$?

(b) (6 points) Test whether $P(y = 1|x = 1) = 0.45$ at a 5% level of significance.

(c) (10 points) Explain how you would construct a 95% confidence interval for the marginal effect $\frac{dP(y=1|x=a)}{da}$. (The confidence interval will depend on a. You are not expected to calculate it numerically).