Swiss Program for Beginning Doctoral Students in Economics 2003

Final Exam in Macroeconomics

Tuesday, February 24, 2004, 08.30h – 11.30h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Please use **a pen** rather than a pencil so that your answers can be read without problems.
4. Please **write legibly.** Remember that your exams will be photocopied for grading.
5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.
6. Good luck!

ID-Number:____

Each of the first two questions below is worth 20 points. If there are components to a question, then these are weighted equally within each question..

1. *Solving a rational expectations model.* Consider the following simple rational expectations model,

$$\begin{aligned} y_t &= \beta E_t y_{t+1} + \gamma x_t \\ x_t &= \rho x_{t-1} + e_t \end{aligned}$$

where e_t is an independently and identically distributed zero mean random variable.

(a) Critically evaluate the following statement: "The necessary and sufficient condition for a unique, stable solution is that $0 < \beta < 1$ if y_t is not predetermined. In this case, the solution takes the form

$$y_t = \gamma \sum_{j=0}^{\infty} \beta^j E_t x_{t+j}$$

If this condition is not fulfilled, then there are a multiplicity of solutions, which can be described as

$$y_{t+1} = \frac{1}{\beta} y_t - \frac{\gamma}{\beta} x_{t+1} + \xi_{t+1}$$

where ξ_t is a series of iid zero mean random variables."

(b) What is the value of the coefficient π in the rational expectations solution for

$$y_t = \pi x_t.$$

2. *Approximating a model.* Consider the discrete time version of the Solow model.

$$k_{t+1} - k_t = sa_t k_t^{\alpha} - \delta k_t$$

(a) Find the stationary point of this model and discuss its properties.

(b) Find a loglinear approximation to this model and discuss the stability of the capital formation process. .

(c) Suppose that there is a productivity shock of the form $a_t = \exp(\mathrm{z}_t)$,with $z_t = \rho z_{t-1} + e_t$ being a stationary stochastic process. Provide the appropriate loglinear approximation for the model and show how to calculate $var(k_t)$.

3. *Consumption, Asset Retuns and the Equity Premium Puzzle*
Consider a household which maximizes expected utility,

$$E_t\{\sum_{j=0}^{\infty} b^j u(c_{t+j})\}$$

with $u(c) = \frac{1}{1-\gamma}c^{1-\gamma}$ and with $\gamma > 0$.
The household can invest in a variety of assets $j = 1, 2, ...J$ that have ex dividend prices $p_j(\varsigma_t)$ and payouts random $d_j(\varsigma_t)$, which are functions of an exogenous state vector ς_t. Thus, the budget constraint takes the form:

$$\sum_{j=1}^{J}(p_j(\varsigma_t) + d_j(\varsigma_t))x_{jt} \geq c_t + \sum_{j=1}^{J} p_j(\varsigma_t)x_{j,t+1}$$

where x_{jt} is the predetermined number of shares that the individual brings into date t.

(a) [6 points] Write the Bellman equation for a dynamic programming formulation of this problem.

(b) [6 points] Determine the first order conditions

(c) [4 points] Use the envelope theorem to show that a necessary condition for the jointly optimal consumption and portfolio problem is

$$0 = \beta E_t[u_c(c_{t+1})R_{j,t+1}] - u_c(c_t) = 0$$

where

$$R_{j,t+1} = \frac{p_{j,t+1} + d_{j,t+1}}{p_{jt}}$$

is the gross return on asset j.

(d) [12 points] Assume that there are only two assets, one with a certain rate of return $R_{f,t+1}$ and another with a random return, $R_{s,t+1}$, which we call "the stock market" for short. Assume that consumption and stock returns are jointly lognormally distributed, with means and variances that do not depend on time. Use this setup to discuss "the equity premium puzzle", defining carefully what the puzzle is and describing what the model says about it.

(e) [12 points] Continuing to assume that there are just two assets, consider the condition

$$0 = \beta E_t[u_c(c_{t+1})R_{j,t+1}] - u_c(c_t)$$

under the assumption that $u(c_t) = \frac{1}{1-\sigma}c_t^{1-\sigma}$, with β and σ being unknown parameters. Discuss how to isolate an expectation errror in this expression, so as to (i) estimate β and σ; and (ii) test implications of this model.

4. *Growth Model* [30 points]

Consider an economy populated by N identical individuals. Each individual supplies one unit of labor. There is a single good in this economy which is produced by combining labor with a continuum of intermediate goods, $X_t(i)$ with $i \in [0, A_t]$, according to the production function:

$$Y_t = BN^{1-\gamma} \left\{ \int_0^{A_t} [X_t(i)]^\gamma di \right\},$$

where $0 < \gamma < 1$. All intermediate goods $X_t(i)$ are imported at the same price, p.

1. **(a)** For a given level of A_t characterize the level of utilization of intermediate input $X_t(i)$ that maximizes net output, defined as output net of the costs of purchasing the intermediate inputs ($\int_0^{A_t} pX_t(i)di$):

$$\text{Net Output} = BN^{1-\gamma} \left\{ \int_0^{A_t} [X_t(i)]^\gamma di \right\} - \int_0^{A_t} pX_t(i)di$$

.

(b) Show that, when the level of intermediate inputs is chosen optimally, net output can be written as:

$$\text{Net Output} = DNA_t p^{-\gamma/(1-\gamma)}$$

where D is a constant that depends on B and γ.

(c) Net output can either be consumed or invested to teach workers to expand the range of intermediate inputs that they can use. Suppose that a fraction s of the net output is saved, so investment is given by:

$$I_t = s(\text{Net Output})$$

The increase in the range of goods that workers can use is given by:

$$\dot{A}_t = I_t/N$$

Show that the growth rate of net output, g, is given by:$g = sDp^{-\gamma/(1-\gamma)}$

(d) What is the impact of an increase in the price of intermediate goods on the growth rate of this economy?

5. [15 points] Consider the inflation equation:

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \lambda\ \widehat{mc}_t \tag{1}$$

where c_t is (log) consumption, π_t denotes inflation, r_t is the nominal rate, $\widehat{mc}_t = mc_t - mc$ is the (log) deviation of real marginal cost from steady state. Describe briefly (in words but as rigorously as possible)

(a) where (1) comes from,

(b) what is the intuition behind it.

6. [10 points] Assume that the representative household's utility is given $E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t)$ with $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$, and where C_t denotes consumption and N_t denotes work hours. Let firms' technology be given by the production function $Y_t = A_t N_t$, where Y_t denotes output and A_t is an exogenous technology parameter. All output is consumed. Derive an expression for the (log) of the *efficient* level of output (which we will denote by y_t^*) as a function of (log) productivity a_t (i.e., the level of output that a benevolent social planner would choose, given preferences and constraints).

7. [20 points] Assume that the (log) nominal wage w_t is set each period according to the schedule $w_t = p_t + \sigma c_t + \delta n_t$, where $0 < \delta < \varphi$.

   (a) Compare the behavior of the equilibrium real wage under that schedule with the one that would be observed under competitive labor markets (in what sense $\delta < \varphi$ can be interpreted as a "real rigidity"?).

(b) Derive the implied *natural* level of output (denoted by $\overline{y}_t$, in logs), defined as the equilibrium level of output under flexible prices (when all firms keep a constant (log) markup μ).

(c) Derive an expression for the real marginal cost $\widehat{mc}_t$ as a function of the output gap $\widetilde{y}_t \equiv y_t - \overline{y}_t$.

8. [25 points] Supose that the monetary authority has a loss function given by $E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha(y_t - y_t^*)^2 + \pi_t^2\right]$. Analyze the solution to the optimal monetary policy under discretion (time consistent solution) and explain the difference with the (standard) case with $\delta = \varphi$ considered in class.(note: for simplicity assume that the frictionless markup μ is infinitesimally small when answering this question).