

Swiss Program for Beginning Doctoral Students in Economics 2003

Final Exam in Microeconomics

Wednesday, February 25, 2004, 08.30h – 11.30h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Please use a **pen** rather than a pencil so that your answers can be read without problems.
4. Please **write legibly**. Remember that your exams will be photocopied for grading.
5. Answers should be **concise and precise!** The space provided should be sufficient to answer each question.
6. Good luck!

ID-Number: _____

Part I: [110 points] *Answer all questions*

Question 1 [20 points]

A price taking firm produces output q from inputs z_1 and z_2 according to a differentiable concave production function $f(z_1, z_2)$. The price of its output is $p > 0$, and the prices of its inputs are $(w_1, w_2) \gg 0$. However, there are two unusual things about this firm. First, rather than maximizing profit, the firm maximizes revenue. Second, the firm is cash constrained. In particular, it has only C dollars on hand before production and, as a result, its total expenditures on inputs cannot exceed C .

Suppose that one of your econometrician friends tells you that she has used repeated observations of the firm's revenues under various output prices, input prices, and levels of the financial constraint and has determined that the firm's revenue level R can be expressed as the following function of the variables (p, w_1, w_2, C) :

$$R(p, w_1, w_2, C) = p \cdot (g + \ln C - a \ln w_1 - (1-a) \ln w_2)$$

where g and a are known parameters.

- (a) What is the firm's input demand for input z_1 when prices are (p, w_1, w_2) and it has C dollars of cash on hand?

Question 1 continued

- (b) Suppose the firm wants to achieve a given level of revenues R . What is the minimum cost that the firm has to incur given prices (p, w_1, w_2) ?

Question 1 continued

- (c) What is the firm's input demand input z_I when prices are (p, w_1, w_2) and it has to achieve revenue level R at minimum cost?

Question 1 continued

- (d) Under what condition is the input demand of question (a) identical to the input demand of question (c)?

Question 2

Consider a pure Bertrand duopoly problem where demand is given by: $Q=3-p$, if p is the lower of the two prices charged by the firms, which have constant marginal costs $c_1=c_2=1$.

Assume firm 1 can spend a fixed cost F to raise firm 2's marginal cost to 1.5 (example: lobby to impose a tariff if firm 2 is a foreign firm).

- what is the level of F above which it is not profitable for firm 1 to spend this fixed cost (be precise) ?
- explain in words how firm 1's reasoning would change if firms competed in quantities instead of prices. Is the difference the same as when F serves to decrease firm 1's marginal cost ? (max. 15 lines).

Question 3: Competition and market integration

Consider a country A where demand for a product is defined by:

$$P_A = 4 - (\sum_i q_i),$$

where $\sum_i q_i$ is the total output of all active firms (q_i being the output of firm i). Each firm has a constant marginal cost equal to 3. Competition takes place in quantities, that is, à la Cournot. If each firm has a fixed cost of being active of 0.1, what is the free-entry equilibrium, in terms of:

- the number of active firms (n_A);
- the quantity per firm (call it q_A);
- and the price level P_A ?

Question 3 continued

Assume there is also country B, initially separated from country A, which has the same demand as country A, with domestic firms in this country that produce with the same technology, and therefore the same free-entry equilibrium, if the two markets are segmented (that is, $n_B=n_A$, $q_B=q_A$, and $P_B=P_A$).

But assume now that the two markets become *integrated*, in that all firms now face the same *overall* market demand:

- What is the equation of this demand?
- What is the free-entry equilibrium now (number of active firms in the unified market, quantity per firm, and price level), if each firm still has the same fixed cost of being active of 0.1?
- Does market integration increase total welfare? How does such a policy compare with the entry of new firms on a market of given size? (10 lines max).

Question 4 : Bargaining under Incomplete Information

Consider a seller with zero valuation for a good who makes offers to a buyer who can have valuation b_1 or b_2 for this good, where both valuations are equiprobable and $0 < b_1 < b_2 < 2b_1$.

- If the seller can only make one take-it-or-leave-it offer, what is the unique perfect Bayesian equilibrium?
- Assume both players have discount factor $\delta < 1$, and the seller can make one offer in period 1, and a second offer in period 2 if the first offer has been rejected: Prove that the unique perfect Bayesian equilibrium involves immediate trade with probability 1.

Question 5

Consider an exchange economy with 2 goods ($\mathbf{I} = 1, 2$) and 2 consumers ($i = 1, 2$). The utility functions of these consumers are:

$$u_1 = x_{11} + \ln(1 + x_{21}),$$

$$u_2 = \ln x_{12} + \ln x_{22},$$

where $x_{i\mathbf{I}}$ represents the quantity of good \mathbf{I} consumed by consumer i and \ln denotes the Neperian logarithm. The initial endowments are $w_1 = (2, 1)$ for consumer 1 and $w_2 = (2, 3)$ for consumer 2. Good 1 is taken as a numéraire.

a) Compute the demands of both consumers as functions of the price p of good 2.

Question 5 continued

- b)** Show that there is a unique competitive equilibrium and compute its characteristics (price, allocations, utility levels).

Question 5 continued

- c) A firm is introduced that behaves competitively and has a linear technology, transforming one unit of good one into two units of good two. Determine the new competitive equilibrium.

Question 5 continued

- d) Show that the utility of consumer two has increased but that of consumer one has decreased . Explain why.

Part II: [70 points]

Please answer any **TWO** of the following **FIVE** questions.

Question 6

Consider an Akerlof model in which workers' individual productivities θ are distributed on $(1, \infty)$ with probability density function $2/\theta^3$. Each worker privately knows his own θ . A worker with productivity θ has an opportunity cost $\alpha(1 + \theta)$ of being employed, where α is a constant. Firms are risk neutral, and have additive technologies.

(a) If $\alpha = 0$, what would be an efficient allocation of labour? And, in equilibrium, what is the wage and allocation of labour?

Hint: Here, and later in the question, you may care to use the fact that for $n > 1$,

$$\int_1^x \frac{1}{\theta^n} d\theta = \frac{1}{n-1} \left[1 - \frac{1}{x^{n-1}} \right].$$

Question 6 continued

(b) If $\alpha = 1/2$, what would be an efficient allocation of labour? And, in equilibrium, what is the wage and allocation of labour?

Question 6 continued

(c) If $\alpha = 1$, what would be an efficient allocation of labour? And, in equilibrium, what is the wage and allocation of labour?

Question 7

Consider a Spence signalling model with equal numbers of two types of worker, $\theta = \theta_H$ and $\theta = \theta_L$, where $\theta_H > \theta_L > 0$. Each worker privately knows her own type, and chooses whether to be educated ($e = 1$) or not ($e = 0$); no other level of education is possible. A worker of type θ has cost ke/θ of acquiring the education level e , where the cost parameter $k > 0$ is a constant. Competitive firms earn θ from a worker of type θ who has acquired education e (ie. education is intrinsically useless).

(a) Suppose k is such that only the type θ_H workers choose to be educated (a pure-strategy separating equilibrium). In what range, $[\underline{k}, \bar{k}]$, must k lie?

Question 7 continued

(b) Now suppose k is a policy instrument of government. The old government set k at the highest level, k^* , compatible with a pooling equilibrium in which all workers choose to be educated. Find k^* .

Question 7 continued

(c) In order to separate the two types of worker, a new government raises k above k^* by a tiny amount $\varepsilon > 0$. Will this policy achieve the new government's aim of full separation? For $k = k^* + \varepsilon$, what is the unique pure-strategy equilibrium?

Question 8

Two risk-neutral Principals, $i = 1, 2$, compete to hire a risk-neutral Agent, whose privately-observed type θ may be either θ_L or θ_H , where $0 < \theta_L < \theta_H$. The Agent's cost of effort, e , is e/θ . A Principal's gross revenue is $2\sqrt{e}$ if he succeeds in hiring the Agent. Principal i offers a contract that specifies a menu of two wage-effort pairs, $\{[w^i(\theta_L), e^i(\theta_L)], [w^i(\theta_H), e^i(\theta_H)]\}$. The Agent chooses which Principal with whom to contract, and selects from that Principal's menu.

- (a) Show that the equilibrium is first-best efficient.

Question 8 continued

(b) Show that the same is true even if the Agent has a small fixed cost k (added to the cost of effort) of working for Principal 2 — ie. even if the Agent has a small bias in favour of Principal 1.

Question 9

A risk neutral Principal hires a risk averse Agent, whose utility equals $u(w, e)$, where w is the wage and $e \geq 0$ is the effort. The Agent has an ex ante reservation utility of \bar{u} . In the contract, w cannot be conditioned on e directly, but can be conditioned on the Principal's gross profit π . π is normally distributed with mean e and (constant) variance σ^2 .

(a) Suppose that $u(w, e) \equiv -\exp[-r(w - e^2)]$, where r is a positive constant. Calculate the optimal *linear* wage schedule $w(\pi) = \beta + \alpha\pi$. Interpret your solution for the slope α . [Hint. You may use the fact that, for a given e , the certainty equivalent of this contract equals $\beta + \alpha e - e^2 - \frac{1}{2}r\alpha^2\sigma^2$.]

Question 9 continued

(b) Now suppose $u(w, e) \equiv -\exp[-rw] - e^2$, and that e can take only two values, e_L and e_H , where $e_L < e_H$. Show how and why the first best can be approximated using a *nonlinear* wage schedule. Discuss.

Question 10

(a) What assumptions do Holmstrom and Milgrom require to show that an optimal incentive scheme in a repeated discrete-time agency model is linear in accounting variables?

Question 10 continued

(b) What additional assumptions do Hellwig and Schmidt make to show that a scheme linear in total profits is (approximately) optimal? Briefly discuss.