Swiss Program for Beginning Doctoral Students in Economics 2004

Final Exam in Econometrics

Monday, February 21, 2005, 14.00h - 17.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Please use **a pen** rather than a pencil so that your answers can be read without problems.
4. Please **write legibly**. Remember that your exams will be photocopied for grading.
5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.
6. Good luck!

ID-Number:____

ID Number

Problem 1. (*25 points*)

Suppose y follows the AR(1) model $y_t = \phi y_{t-1} + \varepsilon_t$, where ε_t is iid (0,1). Based a sample of size $T = 100$, $\hat{\phi} = 0.65$.

(a) (***8 points***) Plot the estimated spectrum of y.

(b) (***5 points***) Suppose $x_t = d(\mathrm{L})y_t$, where $d(\mathrm{L})$ is the ideal bandpass filter that passes components with periods between 6 and 32 quarters. Draw the spectrum of x.

(b) (***12 points***) Construct a 95% confidence interval for the spectrum at frequency $\omega = \pi/2$.

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Problem 2. (*35 points*)

Suppose that

$$y_t = x_t'\beta + u_t$$

$$u_t = \phi u_{t-1} + \varepsilon_t$$

and $x_t = \varepsilon_{t+1}$, where ε_t is iid(0,1). Let $\hat{\beta}$ denote the OLS estimator of β and $\hat{u}_t$ denote the OLS residual

(a) (*10 points*) Suppose that $|\phi| < 1$. Show that $\sqrt{T}(\hat{\beta} - \beta) \xrightarrow{d} N(0, V)$, and derive an expression for V.

(b) (***5 points***) Based on $T = 100$ observations I find $\hat{\beta}$ =1.23, and regressing $\hat{u}_{t-1}$ onto $\hat{u}_{t-1}$ I find $\hat{\phi}$ =0.91. Construct a 95% confidence interval for β.

(c) (***10 points***) A colleague suggests that the OLS estimator can be improved by correcting the regression for the autocorellation, that is by constructing the GLS estimator from the regression of $y_t - \hat{\phi} y_{t-1}$ onto $x_t - \hat{\phi} x_{t-1}$, where $\hat{\phi}$ is estimator of ϕ from part (b). Derive the probability limit of this GLS estimator. Is the estimator consistent? Is the estimator more efficient than the OLS estimator?

(d) (*10 points*) Now suppose that the true value of $\phi = 1$. Derive the asymptotic distribution of $\hat{\beta} - \beta$. Is $\hat{\beta}$ consistent?

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Problem 3 (*10 points*).

Suppose that

$$y_t = \xi_t + w_t$$
$$\xi_t = F\xi_{t-1} + v_t$$

where

$$\begin{pmatrix} w_t \\ v_t \end{pmatrix} \sim iidN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R & 0 \\ 0 & Q \end{bmatrix} \right)$$

and all variables are scalars.

Suppose that $R = 2$, $Q = 1$ and $F = 0.9$. Find $\mathrm{E}(\xi_t | y_t = 2.0)$ and $\mathrm{var}(\xi_t | y_t = 2.0)$.

Problem 4. (*20 points*)

Consider the IV model

$$y_t = x_t\beta + \varepsilon_t$$

$$x_t = z_t\pi + v_t$$

where all variables are scalars and

$$\begin{bmatrix} z \\ \varepsilon \\ v \end{bmatrix}_t \sim iidN\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_z^2 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & \sigma_{\varepsilon v} \\ 0 & \sigma_{ve} & \sigma_v^2 \end{bmatrix}\right)$$

You have data for $t = 1, \ldots, 300$. Suppose that you suspect that β has undergone a shift at period $t = 151$; you estimate the model using $t = 1, \ldots, 150$ and again using $t = 151, \ldots, 300$. Let F_1 denote the F statistic for testing $\pi = 0$ using the first sample period ($t \leq 150$), and let F_2 denote the corresponding F statistic using the second sample period ($t > 150$).

(a). (*5 points*) Suppose that $F_1 = 18.2$ and $F_2 = 12.4$. Explain how you would test for a break in β?

(b) (*15 points*). Suppose that $F_1 = 18.2$ and $F_2 = 1.4$. Explain how you would test for a break in β?

Problem 5. (28 points)

You have a sample of n independent observations. For each observation, you observe a discrete variable, y, which takes values 0 and 1. You also observe a regressor, x, and you estimate a logit model in order to characterize the relationship between y and x (you use x and a constant as explanatory variables). Let α be the constant and let β be the coefficient on x.

Suppose you estimate α to be 0.1 and β to be -1. Suppose further that you have estimated the covariance matrix of $\hat{\alpha}$ and $\widehat{\beta}$ to be $\begin{pmatrix} 0.04 & 0.01 \\ 0.01 & 0.09 \end{pmatrix}$

(a) (4 points) What would be your point estimate of $P(y = 1|x = 1)$?

(b) (8 points) Construct a 95% confidence interval for $P(y = 1|x = 1)$.

(c) (8 points) What is the estimated marginal effect of x for an observation with $x = 2$?
(Hint: you are asked to estimate $\frac{\partial P(y=1|x)}{\partial x}$ evaluated at $x = 2$)

(d) (8 points) Describe how you would construct a 95% confidence interval for the marginal effect of x for an observation with $x = 2$ (but you do not need to do the calculations).

Problem 6. (30 points)

Suppose that you have a random sample of (y_i, x_i) of size n and that

$$E[y_i|x_i] = \exp(x_i'\beta + 1) + x_i'\beta$$

Assume that all relevant moments exist.

(a) (10 points) Find the asymptotic distribution of the nonlinear least squares estimator that minimizes

$$\sum_{i=1}^{n} (y_i - (\exp(x_i'b + 1) + x_i'b))^2$$

(b) (10 points) Now suppose that you have a different, *independent* random sample of $(\tilde{y}_i, \tilde{x}_i)$ of size n such that

$$E[\tilde{y}_i | \tilde{x}_i] = \exp(\tilde{x}_i'\alpha + 1) + \tilde{x}_i'\alpha$$

In other words, the model for the conditional mean of y given x is the same for two samples, but the coefficients are not necessarily the same. Discuss how you would test whether $\beta = \alpha$. Be as specific as possible.

(c) (10 points) Now suppose that the two random samples are not independent of each other. Specifically, assume that the first sample consists of married men and the second of married women, and that observation number i in the first sample is married to observation number i in the second. It that case it may not be reasonable to assume that (y_i, x_i) is independent of $(\widetilde{y}_i, \widetilde{x}_i)$, but it would typically still be reasonable to assume that you have a random sample of $(y_i, x_i, \widetilde{y}_i, \widetilde{x}_i)$ (in other words, you have independence across households). How would this change your answer to (b)?

Problem 7. (17 points)

Consider observations of (y_{it}, x_{it}) from the panel data model

$$y_{i,t} = x_{it}'\beta + \gamma \exp\left(y_{i,t-1}\right) + \alpha_i + \varepsilon_{it}, \quad t = 1, ..., T, \quad i = 1, ..., N$$

where α_i is an unobserved individual–specific effect. No assumption is made on the relationship between α_i and x_{it}. (Note that you do not observe $y_{i,0}$).

Suppose that the dimensionality of x_{it} is one and

$$E\left[\varepsilon_{is} x_{it}\right] = 0 \qquad \text{for all } t \leq s$$

It is assumed that N is much bigger than T, so the relevant asymptotic arguments should rely on N increasing to infinity with T fixed.

(a) (5 points) How would you estimate β and γ?

(b) (5 points) What is the minimum T such that β and γ are identified? Is the model over–identified for that T? Explain and explicitly state any additional "regularity conditions" that you assume.

(c) (7 points) How would your answers change if

$$E\left[\varepsilon_{is} \middle| x_{it}\right] = 0 \qquad \text{for all } t \leq s$$

Problem 8. (15 points)

Suppose that you have a random sample $\{X_i\}_{i=1}^n$ from some distribution with density, f. Consider the following estimator of f at a point x,

$$\widehat{f}(x) = \frac{1}{nh_n} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_n}\right)$$

where K is the density for a uniform random variable on the interval $(-1, 1)$, and $h_n = 7 \cdot n^{-1/5}$. If the true (unknown) f is

$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

find an expression for the (approximate) means square error of $\widehat{f}(1)$ (as a function of n)