Swiss Program for Beginning Doctoral Students in Economics 2004

Final Exam in Macroeconomics

Tuesday, February 22, 2005, 08.30h – 11.30h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Please use **a pen** rather than a pencil so that your answers can be read without problems.
4. Please **write legibly**. Remember that your exams will be photocopied for grading.
5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.
6. Good luck!

ID-Number:____

1. (*20 points*) ***Rational expectations, forecasting, and simulation.***

A prominent economic theory that the long-term interest rate R_{Lt} is linked to the short-term interest rate R_t by the formula

$$R_{Lt} - \theta = R_t + \beta E_t[R_{L,t+1} - \theta]$$

with $0 < \beta < 1$.

(a) (*7 points*) What is the unique, stable RE solution for R_{Lt}?

Now suppose that there is an empirical model for the short-term rate which takes the form,

$$R_t = \pi s_t$$

$$s_t = M s_{t-1} + G e_t$$

where π is 1 by n; M is n by n; and G is n by m (with $n < m$).

(b) (*7 points*) There is a solution for the long rate of the form,

$$R_t = \theta + \phi s_t$$

where ϕ is 1 by n. What is this solution? How does it depend on π, M, G?

(c) (*6 points*) During the macro course, this type of approach was applied to another pair of macroeconomic variables, so as to construct a "simulated theoretical solution" for a variable governed by an equation like

$$R_{Lt} - \theta = R_t + \beta E_t[R_{L,t+1} - \theta]$$

with $s_t = Ms_{t-1} + Ge_t$ being a vector autoregression in state space form. What were these variables? What data was used in s_t?

2. (*70 points*) ***Optimal dictatorial policy.***

Suppose that there is an economy in which there is a dictator, who seeks to maximize the following measure of the present discounted utility value – to him – of the resources that he extracts from society,

$$D = \sum_{t=0}^{\infty} \beta^t \log(T_t)$$

where T_t is an amount of resources transferred from each individual in the society at date t.

Individuals in the society value consumption according to

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

and they can accumulate capital according to

$$k_{t+1} = R[k_t - c_t - T_t]$$

with $R\beta > 1$ Assume that the dictator can commit to a series of lump-sum taxes for all dates, $\{T_t\}_{t=0}^{\infty}$.

Note: It is NOT necessary to use the Marcet-Marimon approach to solve this problem and it is, in fact, easier to do so without it.

(a) (***20 points***) Consider first an arbitrary sequence of "lump-sum taxes". Assuming that $\sum_{t=0}^{\infty} R^{-t}T_t < k_0$, what is the optimal consumption and capital accumulation behavior for a private individual who takes the series of lump sum taxes as given.

(b) (*20 points*) What is the individual's welfare, $V(k_0, \{T_t\}_{t=0}^{\infty})$ under the pattern of decisions described in part (a)?

(c) (*30 points*) Suppose now that private individuals may choose *once*, at date 0, to operate the production technology described above OR to simply receive a given amount of output $y = 1$ each period. (In the event that they do not operate the production technology, the capital evaporates and no output is produced by it). Under this constraint, describe the dictator's optimal policy and provide an economic interpretation of it.

3. ***Price Stabilization in a Classical Economy.*** **(*35 points*)**
Consider a classical monetary economy with equilibrium conditions:

$$r_t = \rho + E_t\{\pi_{t+1}\} + \sigma \, E_t\{\Delta y_{t+1}\} \tag{1}$$

$$m_t - p_t = y_t - \eta \, (r_t - \rho) + \varepsilon_t^m \tag{2}$$

$$y_t = a_t \tag{3}$$

where r_t is the short-term nominal rate, $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation, y_t is (log) output, m_t is the (log) money, and ε_t^m is a white noise shock to money demand. Productivity a_t is assumed to follow the exogenous process:

$$a_t = \rho_a \, a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0, 1]$ and ε_t^a is white noise.

(a) Discuss where the equilibrium conditions above come from.

(b) Consider first the interest rate rule

$$r_t = \rho + \phi_p \left(p_t - p^* \right)$$

where $\phi_p > 0$, and p^* is a (constant) target for the (log) price level. Determine the equilibrium behavior of the price level under the previous rule.

(c) Consider instead the money targeting rule

$$m_t = p^*$$

Determine the equilibrium behavior of the price level under the previous rule.

(d) Show that the money targeting rule considered in (c) can be combined with (2) and rewritten as a price-level targeting rule of the form

$$r_t = \rho + \psi \, (p_t - p^*) + u_t$$

where ψ is a coefficient and u_t is a stochastic process, both to be determined.

(e) Suppose that the central bank wants to minimize the volatility of the price level (as measured by its variance). Discuss the advantages and disadvantages of the interest rate rule in (b) versus the money targeting rule in (c), in light of your findings above.

4. *The Roots of Inflation Persistence.* (*35 points*)

Aggregate inflation measures for most industrialized countries are characterized by high persistence, as reflected (among other statistics) in estimates for α close to one in the OLS regression

$$\pi_t = \alpha \ \pi_{t-1} + u_t$$

Explain how that evidence may be reconciled (if at all) with the theory underlying the New Keynesian Phillips Curve (NKPC), as well as the evidence in support of the latter. (Note: Provide as many potential explanations as you can, while trying to assess their relative merits. Feel free to use algebra to illustrate/reinforce your verbal arguments).

5. (*20 points*) ***Growth Model.***

Consider an economy that differs in two ways from the standard neoclassical model. First, it takes q units of output to create an investment good. Second, the rate of capital utilization (ϕ_t) is endogenous. A higher rate of utilization increases the flow of capital services ($\phi_t K_t$) and the level of output but it also increases the rate of capital depreciation.

$$
\begin{aligned}
U &= \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma} \\
Y_t &= A(\phi_t K_t)^{1-a} N^{\alpha} \\
Y_t &= C_t + qI_t \\
K_{t+1} &= I_t + (1-\delta_t)K_t \\
\delta_t &= d\phi_t^{\eta},\ 0 < d < 1,\ \eta > 1.
\end{aligned}
$$

(a) Characterize the steady state of this economy.

(b) Suppose that there is a permanent reduction in q, so that investment goods become permanently cheaper. What is the immediate impact of this reduction on the rate of capital utilization, ϕ_t?

Swiss Program for Beginning Doctoral Students in Economics 2004

Final Exam in Econometrics

Monday, February 21, 2005, 14.00h - 17.00h

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Problem 1. (*25 points*)

Suppose y follows the AR(1) model $y_t = \phi y_{t-1} + \varepsilon_t$, where ε_t is iid (0,1). Based a sample of size $T = 100$, $\hat{\phi} = 0.65$.

(a) (*8 points*) Plot the estimated spectrum of y.

(b) (*5 points*) Suppose $x_t = d(\mathrm{L})y_t$, where $d(\mathrm{L})$ is the ideal bandpass filter that passes components with periods between 6 and 32 quarters. Draw the spectrum of x.

ID Number

(b) (***12 points***) Construct a 95% confidence interval for the spectrum at frequency $\omega = \pi/2$.

ID Number

Problem 2. (*35 points*)

Suppose that

$$y_t = x_t'\beta + u_t$$

$$u_t = \phi u_{t-1} + \varepsilon_t$$

and $x_t = \varepsilon_{t+1}$, where ε_t is iid(0,1). Let $\hat{\beta}$ denote the OLS estimator of β and $\hat{u}_t$ denote the OLS residual

(a) (*10 points*) Suppose that $|\phi| < 1$. Show that $\sqrt{T}(\hat{\beta} - \beta) \xrightarrow{d} N(0, V)$, and derive an expression for V.

(b) ***(5 points)*** Based on $T = 100$ observations I find $\hat{\beta}=1.23$, and regressing $\hat{u}_{t-1}$ onto $\hat{u}_{t-1}$ I find $\hat{\phi}=0.91$. Construct a 95% confidence interval for β.

(c) (***10 points***) A colleague suggests that the OLS estimator can be improved by correcting the regression for the autocorellation, that is by constructing the GLS estimator from the regression of $y_t - \hat{\phi}\, y_{t-1}$ onto $x_t - \hat{\phi}\, x_{t-1}$, where $\hat{\phi}$ is estimator of ϕ from part (b). Derive the probability limit of this GLS estimator. Is the estimator consistent? Is the estimator more efficient than the OLS estimator?

(d) (***10 points***) Now suppose that the true value of $\phi = 1$. Derive the asymptotic distribution of $\hat{\beta} - \beta$. Is $\hat{\beta}$ consistent?

Problem 3 (*10 points*).

Suppose that

$$y_t = \xi_t + w_t$$
$$\xi_t = F\xi_{t-1} + v_t$$

where

$$\begin{pmatrix} w_t \\ v_t \end{pmatrix} \sim iidN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R & 0 \\ 0 & Q \end{bmatrix}\right)$$

and all variables are scalars.

Suppose that $R = 2$, $Q = 1$ and $F = 0.9$. Find $\mathrm{E}(\xi_t | y_t = 2.0)$ and $\mathrm{var}(\xi_t | y_t = 2.0)$.

Problem 4. (*20 points*)

Consider the IV model

$$y_t = x_t\beta + \varepsilon_t$$

$$x_t = z_t\pi + v_t$$

where all variables are scalars and

$$\begin{bmatrix} z \\ \varepsilon \\ v \end{bmatrix}_t \sim iidN\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_z^2 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & \sigma_{\varepsilon v} \\ 0 & \sigma_{ve} & \sigma_v^2 \end{bmatrix}\right)$$

You have data for $t = 1, \ldots, 300$. Suppose that you suspect that β has undergone a shift at period $t = 151$; you estimate the model using $t = 1, \ldots, 150$ and again using $t = 151, \ldots, 300$. Let F_1 denote the F statistic for testing $\pi = 0$ using the first sample period ($t \leq 150$), and let F_2 denote the corresponding F statistic using the second sample period ($t > 150$).

(a). (*5 points*) Suppose that $F_1 = 18.2$ and $F_2 = 12.4$. Explain how you would test for a break in β?

(b) (*15 points*). Suppose that F_1 = 18.2 and F_2 = 1.4. Explain how you would test for a break in β?

Problem 5. (28 points)

You have a sample of n independent observations. For each observation, you observe a discrete variable, y, which takes values 0 and 1. You also observe a regressor, x, and you estimate a logit model in order to characterize the relationship between y and x (you use x and a constant as explanatory variables). Let α be the constant and let β be the coefficient on x.

Suppose you estimate α to be 0.1 and β to be -1. Suppose further that you have estimated the covariance matrix of $\widehat{\alpha}$ and $\widehat{\beta}$ to be $\begin{pmatrix} 0.04 & 0.01 \\ 0.01 & 0.09 \end{pmatrix}$

(a) (4 points) What would be your point estimate of $P(y = 1|x = 1)$?

(b) (8 points) Construct a 95% confidence interval for $P(y = 1|x = 1)$.

(c) (8 points) What is the estimated marginal effect of x for an observation with $x = 2$? (Hint: you are asked to estimate $\frac{\partial P(y=1|x)}{\partial x}$ evaluated at $x = 2$)

(d) (8 points) Describe how you would construct a 95% confidence interval for the marginal effect of x for an observation with $x = 2$ (but you do not need to do the calculations).

Problem 6. (30 points)

Suppose that you have a random sample of (y_i, x_i) of size n and that

$$E[y_i|x_i] = \exp(x_i'\beta + 1) + x_i'\beta$$

Assume that all relevant moments exist.

(a) (10 points) Find the asymptotic distribution of the nonlinear least squares estimator that minimizes

$$\sum_{i=1}^{n} (y_i - (\exp(x_i'b + 1) + x_i'b))^2$$

(b) (10 points) Now suppose that you have a different, *independent* random sample of $(\tilde{y}_i, \tilde{x}_i)$ of size n such that

$$E\left[\tilde{y}_i|\tilde{x}_i\right] = \exp\left(\tilde{x}_i'\alpha + 1\right) + \tilde{x}_i'\alpha$$

In other words, the model for the conditional mean of y given x is the same for two samples, but the coefficients are not necessarily the same. Discuss how you would test whether $\beta = \alpha$. Be as specific as possible.

(c) (10 points) Now suppose that the two random samples are not independent of each other. Specifically, assume that the first sample consists of married men and the second of married women, and that observation number i in the first sample is married to observation number i in the second. It that case it may not be reasonable to assume that (y_i, x_i) is independent of $(\tilde{y}_i, \tilde{x}_i)$, but it would typically still be reasonable to assume that you have a random sample of $(y_i, x_i, \tilde{y}_i, \tilde{x}_i)$ (in other words, you have independence across households). How would this change your answer to (b)?

Problem 7. (17 points)

Consider observations of (y_{it}, x_{it}) from the panel data model

$$y_{i,t} = x'_{it}\beta + \gamma \exp\left(y_{i,t-1}\right) + \alpha_i + \varepsilon_{it}, \quad t = 1, ..., T, \quad i = 1, ..., N$$

where α_i is an unobserved individual–specific effect. No assumption is made on the relationship between α_i and x_{it}. (Note that you do not observe $y_{i,0}$).

Suppose that the dimensionality of x_{it} is one and

$$E\left[\varepsilon_{is} x_{it}\right] = 0 \qquad \text{for all } t \leq s$$

It is assumed that N is much bigger than T, so the relevant asymptotic arguments should rely on N increasing to infinity with T fixed.

(a) (5 points) How would you estimate β and γ?

(b) (5 points) What is the minimum T such that β and γ are identified? Is the model over–identified for that T? Explain and explicitly state any additional "regularity conditions" that you assume.

(c) (7 points) How would your answers change if

$$E\left[\varepsilon_{is} \middle| x_{it}\right] = 0 \qquad \text{for all } t \leq s$$

Problem 8. (15 points)

Suppose that you have a random sample $\{X_i\}_{i=1}^{n}$ from some distribution with density, f. Consider the following estimator of f at a point x,

$$\widehat{f}(x) = \frac{1}{nh_n} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_n}\right)$$

where K is the density for a uniform random variable on the interval $(-1, 1)$, and $h_n = 7 \cdot n^{-1/5}$. If the true (unknown) f is

$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

find an expression for the (approximate) means square error of $\widehat{f}(1)$ (as a function of n)