

**Swiss Program for Beginning Doctoral Students in Economics 2004**

**Final Exam in Microeconomics**

**Wednesday, February 23, 2005, 08.30h – 11.30h**

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Please use **a pen** rather than a pencil so that your answers can be read without problems.
4. Please **write legibly**. Remember that your exams will be photocopied for grading.
5. Answers should be **concise and precise!** The space provided should be sufficient to answer each question.
6. Good luck!

ID-Number: \_\_\_\_\_

**Part I (Total 40 points)****Question 1 (20 points)**

Consider a consumer 1 with an expenditure function  $e_1(p_1, p_2, u_1) = u_1 \sqrt{p_1 p_2}$  and a consumer 2 with a utility function  $u_2(x_1, x_2) = 17x_1^6 x_2^{2a}$ .

- (a) What are the Walrasian demand functions for each of the goods by each of the consumers? Denote the wealth of consumer 1 by  $w_1$  and the wealth of consumer 2 by  $w_2$ .

**(b)** For what value(s) of the parameter  $a$  will there exist an aggregate demand function that is independent of the distribution of income but only depends on  $w_1 + w_2$ ?

**Question 2 (20 points)**

Consider an exchange economy with two consumers, consumer 1 and consumer 2, and two goods, good 1 and good 2. Suppose that consumer 1's utility function is given by

$$u_1(x_{11}, x_{21}) = x_{11}x_{21}$$

and consumer 2's by

$$u_2(x_{12}, x_{22}) = x_{12} + x_{22}.$$

Suppose the initial endowment of consumer 1 is  $w_{11} = 1$  (for good 1),  $w_{21} = 2$  (for good 2) and that of consumer 2 is  $w_{12} = 3$ , and  $w_{22} = 2$ .

(a) Find the set of interior Pareto optima of this economy and represent it in the Edgeworth box.

**(b)** Are there also Pareto optima on the boundary of the Edgeworth box?

**(c)** Characterize the subset of Pareto optima that are individually rational (i.e. give to both agents a higher utility than the one they get from their initial endowment).

**(d)** Find the competitive equilibrium of this economy.

**(e)** Check that it is Pareto optimal and individually rational.

**(f)** Explain this result.

**Part II (Total 70 points)****Question 3: Bertrand vs Cournot Competition with Differentiated Products**

Consider a market with two differentiated products with the following demands:

$$\begin{aligned}q_1 &= 1 - p_1 + p_2/2 \\ q_2 &= 1 - p_2 + p_1/2\end{aligned}$$

Assume each product is supplied by a single firm at zero costs.

**(a)** Compute the Nash equilibrium of this duopoly game when prices are the strategic variables.

**(b)** Compute the Nash equilibrium of this duopoly game when quantities are the strategic variables.

**(c)** Compare the total surplus in both cases. Explain intuitively the difference.

**Question 4: Spence model with productive-but-costly education.**

Assume two types of workers, H and L, with probabilities  $a$  and  $1 - a$  in the population.

Their respective utilities are  $w - c_{He}$  and  $w - c_{Le}$ , where  $e$  is the level of education,  $w$  the wage and we have  $1 < c_H < c_L$ .

Worker productivity is  $L + e$  for type L and  $H + e$  for type H (with  $0 < L < H$ ).

**(a)** Show graphically the equilibrium if worker types are publicly observable. Why will unobservability of worker types matter?

(b) Describe *the set* of fully separating Bayesian-perfect equilibria (be precise).

(c) Describe *the set* of fully pooling Bayesian-perfect equilibria (be precise).

**(d)** Which of these equilibria satisfies the Cho-Kreps intuitive criterion? (be precise).

**(e)** Why is the Cho-Kreps criterion called “intuitive”?

**(f)** Explain the two specific shortcomings of the Cho-Kreps refinement when  $a$  is close to 1 (be precise).

**Part III (Total 70 points)**

Please answer any **TWO** of the following **FIVE** questions.

**Question 5**

Consider an Akerlof model in which firms are risk neutral and have additive technologies. Workers have individual productivities  $\theta$  that are uniformly distributed on  $(0, 18)$ . A worker of type  $\theta$  has an opportunity cost  $3\theta/5$  of being employed by a firm. The worker population is made up of men and women. It is publicly known that any man's  $\theta$  lies either between 0 and 2, or between 11 and 15; and that any woman's  $\theta$  lies either between 2 and 11, or between 15 and 18. The sex of each worker is also public knowledge, although his or her individual  $\theta$  is only privately known by the worker.

(a) Confirm that there are twice as many women as men, but that, on average, both sexes are equally productive.

(b) What wage is paid to men in equilibrium and which men will choose to work at this wage?

**(c)** What wage is paid to women in equilibrium and which women will choose to work at this wage?

**(d)** A law is passed banning sexual discrimination: all workers that choose to work must be paid the same wage, irrespective of their sex. Show that the introduction of this law leads to a Pareto deterioration.

**Question 6**

(a) Consider a Spence signalling model where workers are of two types,  $\theta = \theta_H$  and  $\theta = \theta_L$ , where  $\theta_H = 3\theta_L$  and  $\theta_L > 0$ . Half the workers are of each type. A worker privately knows his own type. A worker of type  $\theta$  has cost  $e^2/\theta$  of acquiring education  $e \geq 0$ . Competitive firms earn revenue  $\theta$  from a worker of type  $\theta$  (education is intrinsically useless). Calculate the aggregate welfare,  $W$ , measured by aggregate revenue minus aggregate cost, of the least inefficient separating equilibrium.

**(b)** Repeat your calculation for the case of three types,  $\theta = \theta_H, \theta_M$  or  $\theta_L$ , where  $\theta_M = 2\theta_L$ , and a third of the workers are of each type. How does this  $W$  compare with your answer in (a) ? Discuss briefly.

**Question 7**

Consider a competitive screening model with many firms and workers. There are two types of worker, respectively with productivities  $\theta = \theta_L > 0$  and  $\theta = \theta_H$ , where  $\theta_H = 2\theta_L$ . A worker with productivity  $\theta$  has a cost of  $t^2/\theta$  of undertaking some task  $t \geq 0$ , and the return from such a worker to a firm is  $\theta + t$  (i.e., the task is intrinsically useful). The timing of the model is as follows. First, each worker privately learns her own type  $\theta$ . Then, the firm offers contracts comprising a wage  $w(t)$  for undertaking some task  $t$ . Finally, each worker chooses which contract, if any, to accept. The fraction of type  $\theta_H$  workers is  $\lambda$ . Discuss using diagrams - - and, if you can, calculate explicitly - - the values of  $\lambda$  for which a pure strategy equilibrium exists.

**Question 8**

A risk-neutral Principal contracts to hire a risk-neutral Agent. The Agent is one of three types  $\theta_i$ ,  $i = H, M$  or  $L$ , where  $\theta_H = 3\theta_L$ ,  $\theta_M = 2\theta_L$  and  $\theta_L > 0$ . From the Principal's perspective, the Agent is of type  $\theta_i$  with probability  $\lambda_i$ . The Agent of type  $\theta_i$  has a cost of effort,  $e$ , equal to  $e^2/2\theta_i$ . The Principal's revenue is  $e$ . A contract specifies a menu comprising three wage-effort pairs,  $(w_H, e_H)$ ,  $(w_M, e_M)$  and  $(w_L, e_L)$ , from which the Agent selects. The Agent of type  $\theta_i$  has an outside opportunity  $\bar{u}_i = \theta_i/6$ .

- (a) If the Principal can observe the Agent's type  $\theta_i$ , what contract would be offered?

**(b)** From now on, assume that the Principal cannot observe the Agent's type. For  $\lambda_M$  close to 1, what would  $(w_M, e_M)$  and  $(w_H, e_H)$  approximately equal?

**(c)** For  $\lambda_M$  close to 0, what would  $(w_L, e_L)$  and  $(w_H, e_H)$  approximately equal? [Hint – consider your answer from (a).]

**Question 9**

Write a critical account of the Holmstrom-Milgrom and Hellwig-Schmidt models of linear incentives in dynamic agency theory.