STUDY CENTER
GERZENSEE

Swiss Program for Beginning Doctoral Students in Economics 2005

Final Exam in Econometrics

Monday, February 20, 2006, 14.00h - 17.00h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Please use **a pen** rather than a pencil so that your answers can be read without problems.
4. Please **write legibly**. Remember that your exams will be photocopied for grading.
5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.
6. ID Number should be on **every** page of the exam including the **backside** of each page.
7. When the back of a page is used, make sure that the answer is on the same sheet as the question itself. The exams will be separated and sent to various professors.
8. There are 180 points to this 180 minute exam. There are 8 questions. Please answer **all** questions.
9. Good luck!

ID-Number:____

(15) 1. $Y \sim N(\mu, \sigma^2)$ and X is a binary random variable that is equal to 1 if $Y > 2$ and equal to 0 otherwise. Let p = Prob(X = 1). You have n i.i.d observations on Y_i, and suppose the sample mean is $\overline{Y} = 4$ and the sample variance is $n^{-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 9$.

(7) (a) Construct an estimate of p.

(8) (b) Is the estimator that you used in (a) consistent? Explain why or why not.

(45) 2. Suppose that Y_t follows the stationary process $Y_t = \phi Y_{t-2} + \varepsilon_t$,

where $\varepsilon_t \sim iid(0,1)$, and $|\phi| < 1$. Let $\hat{\phi}$ denote the OLS estimator of ϕ.

(15) (a) Show that $\sqrt{T}(\hat{\phi} - \phi) \xrightarrow{d} N(0, V_{\hat{\phi}})$, and derive a formula for $V_{\hat{\phi}}$.

(5) (b) Let $\lambda_4 = E(Y_t Y_{t-4})$. Derive an expression for λ_4 as a function of ϕ. (Note that σ_ε^2 = 1.0).

(10) (c) Let $S(\omega)$ denote the spectrum of Y. Derive an exression for $S(\pi/2)$ as a function of ϕ.

(15) (d) In a sample of size $T = 400$, it is found that $\hat{\phi} = 0.7$.

(di) Construct a 95% confidence interval for ϕ.

(dii) Construct a 95% confidence interval for λ_4.

(diii) Construct a 95% confidence interval for $S(\pi/2)$.

(15) 3. Suppose that Y_t follows a stationary AR(1), and you have data Y_t, t = 1, …, 100, and find $\bar{Y} = T^{-1}\sum_{t=1}^{T} Y_t = 18.1$, $\hat{\lambda}_0 = T^{-1}\sum_{t=1}^{T}(Y_t - \bar{Y})^2$ =2.4, and $\hat{\lambda}_1 = (T-1)^{-1}\sum_{t=1}^{T}(Y_t - \bar{Y})(Y_{t-1} - \bar{Y})$= 1.0. Construct a 95% confidence interval for μ= $E(Y)$. (Hint: Proceed in three steps. Step 1: Think about the appropriate asymptotic covariance matrix for $\sqrt{T}(\bar{Y} - \mu)$. Step 2: Think about how you would estimate the covariance matrix from the data given in the problem. Step 3: Construct the confidence interval.)

(15) 4. Consider the model $Y_t = \tau_t + \varepsilon_t$, where $\tau_t = \tau_{t-1} + e_t$, and where $\{\varepsilon_t\}$ and $\{e_t\}$ are mutually uncorrelated white noise processes with variances σ_ε^2 and σ_e^2. You are interested in the null hypothesis $\sigma_\varepsilon^2 = 0$. How would you test this hypothesis? (Be as complete as you can in the limited time.)

Problem 5. (22 points)

Suppose you are interested in the effect of education and income on internet use. Using a random sample of individuals, you model a binary variable I_i, which takes the value 1 if person i has access to the internet and 0 otherwise, as a function of person i's income Inc_i (in \$1,000) and education Ed_i (in years). Specifically, you estimate a linear regression model and a logit model with I_i as the dependent variable and Inc_i and Ed_i as the explanatory variables.

You get the following output

Linear Regression	$b_0 = 0.2$;	$b_1 = 0.012$;	$b_2 = 0.009$;
Logit	$b_0 = -1.2$;	$b_1 = 0.046$;	$b_2 = 0.038$;

where b_0, b_1 and b_2 are estimates of the constant term, the coefficient on income and the coefficient on education, respectively.

(a) *(7 points)* For each of the two models, calculate the predicted probability that a person will use the internet if she has a high school diploma (i.e. 12 years of education) and an income of \$15,000 per year. Comment on the results.

(b) *(7 points)* What is this predicted probability for an individual with a college degree (i.e. 16 years of education) who is making \$60,000 per year in each of the two models? Comment on the results.

(c) *(8 points)* Suppose you estimate a probit model to investigate the same question. What would be a good guess of the estimates that you obtain.

Problem 6. (35 points)

Suppose that you have a random sample of (y_i, x_i) of size n and that

$$E[y_i|x_i] = \exp(x_i\beta + 4)$$

and

$$V[y_i|x_i] = \sigma^2$$

Assume that all relevant moments exist. Note that both y_i and x_i are scalars.

(a) (10 points) Find the asymptotic distribution of the nonlinear least squares estimator that minimizes

$$\sum_{i=1}^{n} (y_i - \exp(x_i'b + 4))^2$$

(b) (10 points) Find the asymptotic distribution of the methods of moments estimator that uses

$$E\left[\left(y_i - \exp\left(x_i'b + 4\right)\right) x_i\right] = 0$$

as a moment condition.

(c) (15 points) Find the asymptotic distribution of the efficient GMM that uses

$$\begin{aligned} E\left[\left(y_i - \exp\left(x_i'b + 4\right)\right) x_i\right] &= 0 \\ E\left[\left(y_i - \exp\left(x_i'b + 4\right)\right) x_i^2\right] &= 0 \end{aligned}$$

as moment conditions.

Problem 7. (16 points)

Consider the following panel data model

$$y_{it} = \alpha_1 y_{i(t-1)} + \alpha_2 y_{i(t-2)} + \beta_0 x_{it} + \beta_1 x_{i(t-1)} + \eta_i + v_{it} \qquad (i = 1, ..., N; t = 2, ..., T) \quad (1)$$

where one wants to estimate the four parameters $\alpha_1, \alpha_2, \beta_0$ and β_1, η_i is an unobserved individual–specific effect and v_{it} is an unobserved error term.

(a) (8 points) Explain how you would estimate $(\alpha_1, \alpha_2, \beta_0, \beta_1)$ under the assumption that

$$E^*(v_{it}|x_i^T) = 0 \qquad (t = 1, ..., T).$$

Note that E^* denotes "linear projection", so $E^*(z|w_1, w_2, .., w_k) = 0$ means that $E[zw_j] = 0$ for $j = 1, 2, ..., k$. Also recall that x_i^T is defined as $(x_{i1}, x_{i2}, ..., x_{iT})$.

(b) (8 points) Explain how you would estimate $(\alpha_1, \alpha_2, \beta_0, \beta_1)$ under the assumption that

$$E^*(v_{it}|x_i^t, y_i^{t-1}) = 0 \qquad (t = 2, ..., T).$$

Problem 8. (17 points)

Consider the model

$$\begin{aligned} y_{1i} &= x'_{1i}\beta_1 + \varepsilon_{1i} \\ y_{2i} &= x'_{2i}\beta_1 + \varepsilon_{2i} \end{aligned}$$

where $\begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \end{pmatrix}$ is normally distributed and independent of (x_{1i}, x_{2i}). Suppose that you always observe (y_{1i}, x_{1i}) , but you observe (y_{2i}, x_{2i}) only when $y_{i1} > 0$.

(a) (8 points) Find the likelihood function (conditional on (x_{1i}, x_{2i})).

(b) (9 points) Suggest a two-step estimator for β_2?