STUDY CENTER
GERZENSEE

Swiss Program for Beginning Doctoral Students in Economics 2005

Final Exam in Macroeconomics

Tuesday, February 21, 2006, 08.30h – 11.30h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3. Please use **a pen** rather than a pencil so that your answers can be read without problems.

4. Please **write legibly**. Remember that your exams will be photocopied for grading.

5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.

6. ID Number should be on **every** page of the exam including the **backside** of each page.

7. When the back of a page is used, make sure that the answer is on the **same** sheet as the question itself. The exams will be separated and sent to various professors.

8. There are 180 points to this 180 minute exam. There are 5 questions. Please answer **all** questions.

9. Good luck!

ID-Number:____

Question 1 (30 points)

5 parts, 6 points per part

Consider a household that derives utility from durable (d) and nondurable (c) consumption, so that its utility function takes the form

$$E_0\{\sum_{t=0}^{\infty}\beta^t[u(c_t)+m(d_t)]\} \tag{1}$$

where c_t is a flow of nondurables consumption at date t and d_t is the stock of nondurables at date t. Suppose further that the individual has stochastic income, $y(\varsigma_t)$, where ς_t is the state of the economy which evolves in a Markovian manner. Suppose finally that the individual can borrow or lend at an interest rate r, with his financial wealth a evolving according to

$$a_{t-1}+y_t-c_t-i_t=\frac{1}{1+r}a_t \tag{2}$$

That is, a_{t-1} is the amount of wealth at the start of period t. In this expression, i_t is investment in durables and the stock of durables evolves according to

$$d_t=i_t+(1-\delta)d_{t-1} \tag{3}$$

(a) Explain why the Bellman equation for this problem can be written as

$$v(a_{t-1},d_{t-1},\varsigma_t)=\max_{d_t,a_t,c_t}\{u(c_t)+m(d_t)+\beta E_t v(a_t,d_t,\varsigma_{t+1}) \tag{4}$$

where the maximization is constrained by

$$a_{t-1}+(1-\delta)d_{t-1}+y(\varsigma_t)-d_t-c_t-\frac{1}{1+r}a_t=0 \tag{5}$$

(b) Derive first-order conditions for optimal consumption of both types

and show that optimal evolution of financial wealth over time requires

$$\lambda_t = \beta(1+r)E_t\lambda_{t+1}$$

with λ_t being the multiplier on the constraint above.

(c) Consider two households, which differ in their levels of a_{t-1} and d_{t-1}, but have the same income and same level of $a_{t-1} + (1 - \delta)d_{t-1}$. How will they differ in their chosen values of c_t, i_t, and i_t?

Assuming that $\beta(1+r) = 1$, it can be shown that a linear approximation to the conditions in (b) indicates that

$$i_t = i_{t-1} + e_t - \theta e_{t-1}$$

where e_t is a forecasting error.

(d) Discuss why this makes sense relative to Hall's work on consumption and your answer to (c) above.

(e) Show that the value of $\theta = (1 - \delta)$ is implied by linearization of the efficiency condition(s).

Question 2 (60 points)

8 parts: 7 points for parts (a)-(d) and 8 points for parts (e)-(h).

Suppose that a government seeks to maximize the revenue that it derives from inflation, within a model with purely flexible prices (P_t is the price level at date t). In real terms, the government's revenue at date t is

$$z_t = m_t - \frac{1}{\pi_t} m_{t-1}$$

where π_t is the inflation rate – defined as $\pi_t = P_t/P_{t-1}$– and m_t is the amount of real balances held by agents in period t. [1]

The demand for real money balances is given by

$$m_t = \beta f(\pi_{t+1})$$

which is assumed to be positive, but declining in the inflation rate ($f(\pi) \geq 0$ for all $\pi \geq 0$ and $f_\pi < 0$). The parameter β satisfies $0 < \beta < 1$. In some of the analysis below, it is also assumed that the money demand function takes the particular functional form

$$m_t = \kappa \pi_{t+1}^{-\alpha}$$

with κ and α being positive parameters.

(a) What inflation rate maximizes steady-state rate revenue,

$$m(1 - \frac{1}{\pi})$$

subject to the particular money demand function $m = \kappa\pi^{-\alpha}$? How does this revenue-maximizing inflation rate depend on κ and α?

[1] This revenue may be derived as follows. First, the nominal money stock in period t is M_t and the newly printed money in period t is $M_t - M_{t-1}$. The real value of this newly printed money is $(M_t - M_{t-1})/P_t$, with P_t being the price level. Hence, the real revenue is as specified in the body of the question, if $\pi_t = P_t/P_{t-1}$.

(b) Consider next a government that maximizes the present discounted value of its revenue,

$$\sum_{t=0}^{\infty} \beta^t z_t$$

and assume that the government can commit to a series of inflation rates at dates $t = 0, 1, 2, ...$ Form a dynamic Lagrangian for the government's revenue maximization problem, treating the money demand function as an inequality of the form

$$m_t \leq \beta f(\pi_{t+1})$$

$t = 0, 1, 2, ...$ That is: assume that the government can pick a "tax base" for the inflation tax which is no larger than the real balances that individuals are willing to hold. Call the multiplier on this constraint ϕ_t.

(c) Find the first-order conditions for optimal choice of m_t, π_t, and ϕ_t for all dates $t = 0, 1, 2, \ldots$. Record these as follows

$$\pi_0 \quad :$$
$$m_0 \quad :$$
$$\phi_0 \quad :$$

for $t > 0$:

$$\pi_t \quad :$$
$$m_t \quad :$$
$$\phi_t \quad :$$

Using the economics of the problem and the first-order conditions, explain why the government has sharply different inflation incentives at date $t = 0$ and date $t > 0$.

(d) Show how to make these first order conditions stationary over time by adding a term to the Lagrangian at date 0 which involves the multiplier ϕ_{-1}. Show that this augmented Lagrangian, $\widetilde{L}$, can be written in a recursive form.

(e) Find equations that restrict the long-run optimal rate of inflation under commitment. Is this stationary value higher or lower than that in part (a)? Why?

(f) Show that this steady-state solution is immediately applicable, for all dates $t \geq 1$. Explain why this is a reasonable outcome given the nature of the environment.

(g) Suppose that the model were modified to make the parameter κ time-varying in a deterministic manner. How would the above first-order conditions be modified?

(h) Suppose now that the system is one in which κ varies according to a Markov process. Determine a recursive specification – a shadow price functional equation in the terminology of Marcet and Marmon – that can be used to determine the optimal inflation policy.

Question 3: Monetary Policy and the Effects of Technology Shocks (40 points)

Consider an economy with Calvo-type staggered price setting with equilibrium conditions:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - \rho) \tag{6}$$

$$\pi_t = \beta\; E_t\{\pi_{t+1}\} + \kappa\,(y_t - y_t^n) \tag{7}$$

where y is log output, $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation, i_t is the short-term nominal rate (in deviation from steady state), and y_t^n is the (log) natural level of output.

Monetary policy is described by a simple rule of the form

$$i_t = \rho + \phi\; \pi_t$$

where $\phi > 1$. Labor productivity is given by

$$y_t - n_t = a_t$$

where a_t is an exogenous technology parameter which evolves according to

$$a_t = \rho_a\; a_{t-1} + \varepsilon_t$$

where $\rho_a \in [0, 1)$ and $\{\varepsilon_t\}$ is an i.i.d. process.

The RBC model in the background is assumed to imply a natural level of output proportional to technology

$$y_t^n = \psi_y\; a_t$$

where $\psi_y > 1$.

(a) Describe in words where (6) and (7) come from.

(b) Determine the equilibrium response of output, employment, and inflation to a technology shock. (*hint*: guess that each endogenous variable will be proportional to the contemporaneous value of technology).

(c) Describe how those responses depend on the value of ϕ and κ. Provide some intuition. What happens when $\phi \longrightarrow \infty$? What happens as we change the degree of price rigidities?

(d) Compare the joint response of employment and output to a technology shocks with their flexible price counterpart, and discuss briefly the implications for our assessment of the role of technology as a source of business cycles.

Question 4: Money Supply Rules (30 points)

Consider an economy described by the equilibrium conditions:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right)$$

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$$

$$m_t - p_t = y_t - \eta\ i_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ is the output gap, $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation, i_t is the short-term nominal rate, m_t is the (log) money supply, and r_t^n is the natural interest rate. Both y_t^n and r_t^n evolve exogenously, independently of monetary policy.

The central bank seeks to minimize a loss function of the form

$$\alpha\ var(\widetilde{y}_t) + var(\pi_t)$$

(a) Show how the optimal policy could be implemented by means of an interest rate rule.

(b) Show that a rule requiring a constant money supply will generally be suboptimal. Explain. (*Hint*: derive the path of money under the optimal policy)

(c) Derive a money supply rule that would implement the optimal policy.

Question 5 (20 points)

Consider the following small open economy model. The life-time utility of the representative agent (U) is given by:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma},$$

where C_t represents consumption. Output (Y_t) is produced with labor (N) and capital services, which are the product of the capital stock (K_t) and the rate of capital utilization (u_t):

$$Y_t = A(u_t K_t)^{1-\alpha} N^{\alpha}.$$

The supply of labor is constant at N. The rate of depreciation (δ_t) is increasing in the rate of capital utilization:

$$\delta_t = \psi u_t^{\gamma}.$$

where $\psi > 0$ and $\gamma \geq 1$. The law of motion for the stock of capital is given by:

$$K_{t+1} = I_t + (1 - \delta_t) K_t,$$

where I_t is the level of investment. The representative agent can borrow and lend in international capital markets at a fixed real interest rate, r. Assume that $\beta = 1/(1+r)$. The agent's flow budget constraint is:

$$a_{t+1} = (1+r) a_t + Y_t - C_t - I_t,$$

where a_t denotes net foreign assets at time t. Agents are subject to the no-Ponzi game condition:

$$\lim_{t \to \infty} \frac{a_{t+1}}{(1+r)^t} = 0.$$

The initial conditions are K_0 and a_0.

(a) Why do we need to impose the no-Ponzi game condition?

(b) Show that the model exhibits no transitional dynamics, i.e. all variables remain constant from time zero on (you can appeal without proof to the result that the only solution that satisfies the no-Ponzi game condition is one where net foreign assets are constant over time). Compute the levels of capital, consumption, and capital utilization.