STUDY CENTER
GERZENSEE

Swiss Program for Beginning Doctoral Students in Economics 2005

Final Exam in Microeconomics

Wednesday, February 22, 2006, 08.30h – 11.30h

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.

2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.

3. Please use **a pen** rather than a pencil so that your answers can be read without problems.

4. Please **write legibly**. Remember that your exams will be photocopied for grading.

5. Answers should be **concise and precise**! The space provided should be sufficient to answer each question.

6. ID Number should be on **every** page of the exam including the **backside** of each page.

7. When the back of a page is used, make sure that the answer is on the **same** sheet as the question itself. The exams will be separated and sent to various professors.

8. There are 3 parts of this 180 minute exam. Answer **all** the questions in parts 1 and 2. However, answer only **two out of five** questions in part 3.

9. Good luck!

ID-Number:_____

Part I (*Total 40 points*)

Question 1 (*20 points*)

Consider an individual with initial wealth w which can be invested either in a safe asset with a gross return of one Dollar per Dollar of investment or in a risky asset with random return $\tilde{z}$ Dollar per Dollar of investment. The random return $\tilde{z}$ has a distribution function $F(z)$ which satisfies $\int z dF(z) > 1$, i.e., its mean return exceeds that of the safe asset. Let a denote the amount of money that the individual invests in the risky asset. Assume that the vNM utility function $u(\cdot)$ exhibits constant absolute risk aversion.

(a) Show that a vNM utility maximizer has constant absolute risk aversion with

$$r(x) = -\frac{u''(x)}{u'(x)} = c > 0$$

if and only if $u(x)$ is a positive affine transformation of $v(x) = -e^{-cx}$ (if $c > 0$).

(b) Show that the optimal allocation between the safe and the risky asset places a constant amount $\overline{a}$ in the risky asset as w rises. You may assume an interior solution of the investor's decision problem.

Question 2 (*20 points*)

Consider an economy with 3 goods ($\ell = 1,2,3$), 2 consumers ($i = 1, 2$), and 2 firms ($j = 1, 2$). The utility functions of both consumers are Cobb Douglas:

$$u_i = \ln x_{1i} + \ln x_{2i},$$

where $x_{\ell i}$ represents the quantity of good ℓ consumed by consumer i and ln denotes the Neperian logarithm.

Firm 1 transforms good 3 into good 1 with a constant marginal cost 1. Firm 2 transforms good 3 into good 2 with a cost function $C(q_2) = \frac{1}{2}q_2^2$ (q_2 denotes the quantity of good 2 produced by firm 2). The initial endowments are $\omega_1 = (0,0,5)$ for consumer 1, who also owns firm 1, and $\omega_2 = (1,0,0)$ for consumer 2, who also owns firm 2. Good 3, which is not consumed, is taken as a numéraire.

(a) Characterize the Pareto optima of this economy.

(b) Show that there is a unique competitive equilibrium and compute its characteristics (price, consumption bundles, production plans, utility levels).

(c) Compute the competitive equilibrium when the property rights of the firms are swapped: consumer 1 owns firm 2 and vice versa (the endowments of goods are unchanged).

Part II (*Total 70 points*)

Question 3: Bertrand Competition with Asymmetric Information

Consider a Bertrand duopoly with differentiated complementary products, where demand for firm i is $q_i(p_i, p_j) = a - p_i - b_i p_j$. Costs are zero for both firms.
The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L, where $1 > b_H > b_L > 0$. For each firm, $b_i = b_H$ with probability θ and $b_i = b_L$ with probability $1 - \theta$, independent of the realization of b_j.
Each firm knows its own b_i but not its competitor's. All of this is common knowledge.

(a) What conditions define a symmetric pure-strategy Bayesian equilibrium in this game (assume an interior solution exists)?

(b) Discuss the economics of the reaction functions of this game.

(c) Solve for the equilibrium.

(d) Assume now that $\theta = 0.5$, and that, for each firm, Prob $(b_i = b_H) = 1$ when $b_j = b_L$ and Prob $(b_i = b_L) = 1$ when $b_j = b_H$. Define the symmetric pure-strategy Bayesian equilibrium of this game (assume an interior solution exists) and solve for the equilibrium.

Question 4: Spence model with productive-but-costly education.

Assume two types of workers, H and L, with probabilities a and 1- a in the population. Their respective utilities are $w-c_H e$ and $w-c_L e$, where e is the level of education, w the wage and we have $1<c_H<c_L$.
Worker productivity is L+e for type L and H+e for type H (with $0<L<H$).

(a) Show graphically the equilibrium if worker types are publicly observable. Why will unobservability of worker types matter?

(b) Describe *the set* of fully separating Bayesian-perfect equilibria (be precise).

(c) Describe *the set* of fully pooling Bayesian-perfect equilibria (be precise).

(d) Which of these equilibria satisfies the Cho-Kreps intuitive criterion? (be precise).

(e) Explain the two shortcomings of the Cho-Kreps refinement when a is close to 1 (be precise).

Part III (*Total 70 points*)

Answer TWO out of the following FIVE questions.

Question 5

On an island off the west coast of Scotland, only half the workers speak Gaelic (everyone speaks English). Firms are risk neutral and have additive technologies. Workers' productivities θ are uniformly distributed on (0,6). A worker privately knows his own θ, and has an opportunity cost of working equal to $g\theta$ if he speaks Gaelic and equal to $n\theta$ otherwise. Although workers look identical, a firm can ask someone to speak Gaelic (it is impossible to fake).

(a) Find the (Akerlof) equilibrium wage and employment patterns if $g = 1/3$ and $n = 5/6$.

(b) What happens if $g = 5/6$ and $n = 1/3$?

(c) If $g = 5/6$, what is the highest value of n for which the labour market doesn't collapse?

Question 6

Consider the following Spence signalling model. Each worker has a fixed labour supply, and privately learns her own type θ before deciding how much education $e \geq 0$ to acquire at cost e^2/θ. Competitive firms earn revenue $\theta + e$ from employing a worker of type θ who has acquired education e (i.e., education has intrinsic value). The distribution of types is continuous and has support $[\ 0, \overline{\theta}\ ]$. Find the most efficient separating equilibrium. (*Hint*: You may resort to using a guess-and-verify method. But try to be more ambitious by writing down and solving the equations that the equilibrium must satisfy.)

Question 7

Consider a competitive screening model with many firms and workers. There are two types of workers, respectively with productivities $\theta = \theta_L$ and $\theta = \theta_H$, where $\theta_H > \theta_L > 0$. A worker with productivity θ has a cost t^2/θ of undertaking some task $t \geq 0$, and the return from such a worker to a firm is $\theta + t$ (i.e., the task is intrinsically useful). The timing of the model is as follows. First, each worker privately learns her own type θ. Then, the firms offer contracts comprising a wage $w(t)$ for undertaking some task t. Finally, each worker chooses which contract, if any, to accept. The fraction of type θ_H workers is λ. Discuss the effect increasing λ has on equilibrium behaviour if (i) $\theta_H/\theta_L = 5$; and (ii) $\theta_H/\theta_L = 2$.

Question 8

A risk-neutral Principal contracts to hire a risk-neutral Agent, whose type θ is drawn from a continuous distribution with support $[\ 0, \bar{\theta}\ ]$. The Agent privately knows which type he is at the time of being hired, when his outside opportunity is $\bar{u}(\theta)$. If he accepts the Principal's contract, the Agent then exerts a publicly-verifiable effort e, at a (privately-observed) cost e/θ. The Principal's revenue is $2\sqrt{e}$. The Principal offers a menu of wage-effort pairs $\{w(\theta),e(\theta) \mid 0 \leq \theta \leq \bar{\theta}\}$, from which the Agent selects.

(a) Find the functional form $\bar{u}(\theta)$ for which there is no distortion in effort.

(b) Discuss circumstances in which this first-best outcome might arise.

Question 9

A risk-neutral Principal contracts to hire an Agent on a project. The project either succeeds and yields $\pi > 0$ dollars revenue, or fails and yields nothing. The Agent is risk-neutral with respect to non-negative income, but has no money of his own with which to pay anything out (i.e. negative wages aren't feasible). The Agent has a reservation utility (an outside opportunity) equivalent to 1 dollar. If the Agent accepts the Principal's contract, he then chooses an effort level which determines the probability f that the project succeeds: by exerting $4f^2$ dollars worth of effort (a privately-observed non-cash expenditure, netted from his wage to calculate his utility), the Agent can choose any f between 0 and 1. For each possible value of π, find the optimal contract – that is, the wage w that the Principal pays the Agent if the project succeeds.