**ID Number**

(18) 1. A researcher carries out a QLR test using **25%** trimming and there are $q = 5$ restrictions. Answer the following questions using the values in the attached table of critical values for the QLR Statistic with **15%** Trimming, and the attached Table of Critical values for the $F_{m,\infty}$ Distribution).

(6) a. The QLR $F$-statistic is 4.2. Should she reject the null at the 5% level?

(6) b. The QLR $F$-statistic is 2.1. Should she reject the null at the 5% level?

(6) c. The QLR $F$-statistic is 3.5. Should she reject the null at the 5% level?

**Critical Values of the QLR *F*-Statistic With 15% Trimming**

| Number of Restrictions (q) | 10% | 5% | 1% |
|---|---|---|---|
| 1 | 7.12 | 8.68 | 12.16 |
| 2 | 5.00 | 5.86 | 7.78 |
| 3 | 4.09 | 4.71 | 6.02 |
| 4 | 3.59 | 4.09 | 5.12 |
| 5 | 3.26 | 3.66 | 4.53 |
| 6 | 3.02 | 3.37 | 4.12 |
| 7 | 2.84 | 3.15 | 3.82 |
| 8 | 2.69 | 2.98 | 3.57 |
| 9 | 2.58 | 2.84 | 3.38 |
| 10 | 2.48 | 2.71 | 3.23 |
| 11 | 2.40 | 2.62 | 3.09 |
| 12 | 2.33 | 2.54 | 2.97 |
| 13 | 2.27 | 2.46 | 2.87 |
| 14 | 2.21 | 2.40 | 2.78 |
| 15 | 2.16 | 2.34 | 2.71 |
| 16 | 2.12 | 2.29 | 2.64 |
| 17 | 2.08 | 2.25 | 2.58 |
| 18 | 2.05 | 2.20 | 2.53 |
| 19 | 2.01 | 2.17 | 2.48 |
| 20 | 1.99 | 2.13 | 2.43 |

Large-Sample Critical Values for the $F$-statistic from the $F_{m,\infty}$ Distribution

| Degrees of Freedom ($m$) | Significance Level | | |
|---|---|---|---|
| | 10% | 5% | 1% |
| 1 | 2.71 | 3.84 | 6.63 |
| 2 | 2.30 | 3.00 | 4.61 |
| 3 | 2.08 | 2.60 | 3.78 |
| 4 | 1.94 | 2.37 | 3.32 |
| 5 | 1.85 | 2.21 | 3.02 |
| 6 | 1.77 | 2.10 | 2.80 |
| 7 | 1.72 | 2.01 | 2.64 |
| 8 | 1.67 | 1.94 | 2.51 |
| 9 | 1.63 | 1.88 | 2.41 |
| 10 | 1.60 | 1.83 | 2.32 |
| 11 | 1.57 | 1.79 | 2.25 |
| 12 | 1.55 | 1.75 | 2.18 |
| 13 | 1.52 | 1.72 | 2.13 |
| 14 | 1.50 | 1.69 | 2.08 |
| 15 | 1.49 | 1.67 | 2.04 |
| 16 | 1.47 | 1.64 | 2.00 |
| 17 | 1.46 | 1.62 | 1.97 |
| 18 | 1.44 | 1.60 | 1.93 |
| 19 | 1.43 | 1.59 | 1.90 |
| 20 | 1.42 | 1.57 | 1.88 |
| 21 | 1.41 | 1.56 | 1.85 |
| 22 | 1.40 | 1.54 | 1.83 |
| 23 | 1.39 | 1.53 | 1.81 |
| 24 | 1.38 | 1.52 | 1.79 |
| 25 | 1.38 | 1.51 | 1.77 |
| 26 | 1.37 | 1.50 | 1.76 |
| 27 | 1.36 | 1.49 | 1.74 |
| 28 | 1.35 | 1.48 | 1.72 |
| 29 | 1.35 | 1.47 | 1.71 |
| 30 | 1.34 | 1.46 | 1.70 |

(20) 2. Let $y_t = \mu + \varepsilon_t$, where $\varepsilon_t \sim$ iidN(0,1), and let $\bar{y} = T^{-1}\sum_{t=1}^{T} y_t$

(10) (a) Show that $T^{-1/2}\sum_{t=1}^{[sT]}(y_t - \bar{y}) \Rightarrow W(s) - sW(1)$, where $W(s)$ is a Wiener process.

(10) (b) Find the limiting distribution of $A_T = \sum_{t=1}^{T}\sum_{i=1}^{t}(y_i - \bar{y})^2$ (in terms of $W(s)$), after dividing $A_T$ by the appropriate power of $T$, that is, derive the asymptotic distribution of $T^{-q}A_T$ for the appropriate value of $q$.

(12) 3. Suppose $u_t$ follows the stationary ARMA(1,1) process $u_t = \phi u_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1}$,

and let $\lambda_k = E(u_t u_{t+k}) = E(u_t u_{t-k})$.

(6) (a) Derive the moving average representation for $u_t$. (That is, find the values of $c_i$ in the representation $y_t = c_0\varepsilon_t + c_1\varepsilon_{t-1} + c_2\varepsilon_{t-2} + c_3\varepsilon_{t-3} + \ldots$ )

(6) (b) Show that $\lambda_k = \phi\lambda_{k-1}$ for $k \geq 2$.

(20) 4. Suppose $y_t = \mu + u_t$ where $u_t$ follows the stationary ARMA(1,1) process $u_t = \phi u_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1}$, where $\varepsilon_t \sim$ iid $(0, \sigma^2)$.

(10) (a). Show that $\sqrt{T}(\overline{y} - \mu) \xrightarrow{d} N(0, V)$ and derive an expression for $V$.

(10) (b) In a sample with $T = 400$, $\overline{y} = 5.4$. A researcher uses the data $y_t - \overline{y}$ to estimate the ARMA parameters and finds $\hat{\phi} = 0.66$, $\hat{\theta} = 0.85$, and $\hat{\sigma}_\varepsilon = 1.2$. Construct a 95% confidence interval for $\mu$.

(20) 5. Suppose $u_t = 0.8u_{t-1} + \varepsilon_t$, and $v_t = \varepsilon_t + e_t$, where $\{\varepsilon_t\}$ and $\{e_t\}$ are mutually independent white noise sequences with variances $\sigma_\varepsilon^2 = 4$ and $\sigma_e^2 = 6$.

(10) (a) Suppose you know $u_t = 10$ and $v_{t+1} = 7$. What is the value of your forecast for $u_{t+1}$.

(10) (b) Suppose you have data on $v_{t+1}$, $v_t$, $v_{t-1}$, ..., but do not have data on lagged values of $u_t$. Explain how you would forecast the value of $u_{t+1}$.

**Problem 6 (20 points)**

Let $X$ be a non–negative, bounded random variable. Suppose that you are interested in the value of $\mu$ that solves

$$0 = E\left[X + \mu + \exp\left(X\mu\right)\right].$$

It is natural to estimate $\mu$ by

$$0 = \frac{1}{n}\sum_{i=1}^{n} X_i + \widehat{\mu} + \exp\left(X_i\widehat{\mu}\right).$$

(a) (15 points) Find the asymptotic distribution of $\widehat{\mu}$ under the assumption that you have a random sample of observations $X_1, ..., X_n$.

**(b)** (5 points) How would you construct a 95% confidence interval for $\mu$?

**Problem 7 (25 points)**

You have a sample of $n$ independent observations. For each observation, you have a discrete variable, $y$, which takes values 0 and 1. You also have a regressor, $x$, and you estimate a logit model in order to characterize the relationship between $y$ and $x$ (you use using $x$ and a constant as explanatory variables). Let $\alpha$ be the constant and let $\beta$ be the coefficient on $x$.

Suppose you estimate $\alpha$ to be 2 and $\beta$ to be 1. Suppose further that you have estimated the covariance matrix of $\widehat{\alpha}$ and $\widehat{\beta}$ to be $\begin{pmatrix} 0.04 & 0.02 \\ 0.02 & 0.06 \end{pmatrix}$

(a) (6 points) What is the estimate of $P(y = 1|x = 1)$?

**(b)** (10 points) Test whether $P(y = 1|x = 1) = 0.75$ at a 5% level of significance.

**(c)** (9 points) Estimate the marginal effect of $x$ at $x = 1$

$$\frac{dP\left(y = 1|\, x = 1\right)}{dx}.$$

**Problem 8 (25 points)**

This problem is concerned with bounding treatment effects. Suppose that $D$ is a random variable that indicates whether an individual has been "treated". If $D = 1$, the individual has received treatment and we observe a random variable $Y_1$. If $D = 0$, the individual has not received treatment and we observe a random variable $Y_0$. We do not observe $Y_0$ if $D = 1$ and we do not observe $Y_1$ if $D = 0$. This is the standard notation in this literature.

Suppose that $P(D = 1) = 0.3$, $E[Y_1 | D = 1] = 60$ and $E[Y_0 | D = 0] = 20$.

(a) (10 points) Suppose that it is known that $0 \leq Y_0 \leq 100$ and $0 \leq Y_1 \leq 100$. Use this to construct bounds on the average treatment effect, $E[Y_1 - Y_0]$.

(b) (15 points) Suppose that it is known that $0 \leq Y_0 \leq \frac{1}{2}Y_1 \leq 100$. Use this to construct bounds on $E[Y_1 - Y_0]$.

**Problem 9 (20 points)**

Suppose you are interested in examining the effect of gun ownership on violent crimes committed using firearms. You assemble a data set on the 50 states for 10 years and use the following regression

$$crime_{it} = \beta_0 + \beta_1 gunrate_{it} + \text{other factors} + u_{it} \tag{1}$$

where $crime_{it}$ is the number of crimes committed with firearms in state $i$ in year $t$, $gunrate_{it}$ is the rate of gun ownership in state $i$ in year $t$, and "other factors" include local economic conditions, geographic indicators, and other observed variables.

(a) *(5 points)* Somebody suggests that you run a fixed effect regression. Specify the regression that controls for state fixed effects. What do these fixed effects control for?

(b) *(5 points)* Suppose one of the variables in "other factors" is whether the state is in the North or the South. Would the fixed effects estimator provide a reasonable estimate of the effect of the geographic location of the state on the crime rate? Explain.

(c) *(10 points)* It seems reasonable to assume that an unexpected increase in crime will lead to an increase in gun ownership in future years. What effect would that have on how you estimate $\beta_1$ in (1)?