ID Number

Questions by Bob King

1. Forward-looking expectations models (15 points)

Consider an asset that has a finite life: It yields payments $\{d_t\}$ in dates $t = 0, 1, ..., T$ and zero payments in all subsequent periods. Suppose further that the asset's price (after the payment is distributed) is

$$p_t = \frac{1}{1+r}[p_{t+1} + d_{t+1}]$$

Derive the rational expectations (perfect foresight) solution for this asset's value at each of the following dates: $t = 0, 1, 2, ..., T-1, T, T+1$.

2. Intertemporal consumption decisions (15 points)

Consider an individual that has preferences over consumption flows that take the form

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, c_{t-1})$$

where $u(c_t, c_{t-1})$ is strictly increasing and concave in all its arguments. Suppose that the individual's assets a accumulate according to

$$a_{t+1} = (1+r)(a_t + y - c_t).$$

Exogenous income is positive, $y > 0$; initial levels a_0 and c_{-1} are given; and the individual must satisfy a no-Ponzi game condition.

(a) Display a Bellman equation for this problem, carefully explaining what are the arguments of the value function and the equations of motion for state variables.

(b) Find an efficiency condition that describes optimal consumption at a given date and discuss its determinants.

3. Optimal taxation and public good provision (60 points)

Consider an economy in which all individuals have the utility function

$$u(c, g, n) = \log(c) + \theta \log(g) - \chi \frac{1}{1+\gamma} n^{1+\gamma},$$

where c is consumption, $\gamma > 0$, g is the amount of a public good and n is the amount of time spent in market work.

(a) At the social level, it is the case that

$$c + g = an$$

where a is labor productivity. Determine the "first best" level of consumption, the government good, and work that would be chosen by a social planner.

(b) Suppose that the government levies a labor income tax at rate τ to pay for public good provision. Each agent takes the quantity of public goods as well as the tax rate as given when choosing his optimal consumption and labor supply. That is, each agent maximizes $u(c, g, n)$ taking as given g and τ subject to the budget constraint

$$c = (1 - \tau)an.$$

Determine the competitive equilibrium values of c, g, and n. Contrast these to the levels in part (a).

(c) For a social planner that must finance purchases of public goods with a distorting tax rate, find the optimal tax rate and the "second best" level of public good provision.

Question by Sergio Rebelo (20 points)

Suppose that output (Y_t) is produced with value added ($aK_t^{1-\alpha}N^\alpha$) and energy (E_t) combined in fixed proportions. In other words, output is a Leontieff function of value added and energy:

$$Y_t = \min(aK_t^{1-\alpha}N^\alpha, bE_t),$$

where a and b are productivity parameters, K_t and N are capital and labor respectively, and E_t represents the amount of energy used in production. Suppose that energy is imported and that its price in units of output, p, is constant. Assume that $0 < p < b$. The labor supply, N, is exogenous.

The planner's problem for this economy consists of maximizing lifetime utility

$$U = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma}$$
$$0 < \beta < 1, \ \sigma > 0$$

subject to

$$Y_t = \min(aK_t^{1-\alpha}N^\alpha, bE_t),$$
$$Y_t = C_t + I_t + pE_t,$$
$$K_{t+1} = I_t + (1-\delta)K_t.$$

(a) Compute the steady-state capital stock for this economy.

(b) What is the effect on the steady-state capital stock of a permanent increase in the price of energy, p?

Questions by Jordi Galí

1. Optimal Monetary Policy with Wages Set in Advance (42 points)

The representative firm is perfectly competitive and has access to a technology described by

$$y_t = a_t + n_t$$

where y, n, a denote the logs of output, employment, and productivity, respectively. Prices are flexible. We assume

$$a_t = \rho_a \, a_{t-1} + \varepsilon_t^a.$$

The optimal labor supply satisfies

$$w_t - p_t = \varphi \, n_t$$

where w and p denote the log of the (nominal) wage and price levels, respectively.

Aggregate demand is given by the dynamic IS equation:

$$y_t = -\frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + E_t\{y_{t+1}\}$$

where i_t denotes the nominal interest rate and $\pi_t \equiv p_t - p_{t-1}$ is the inflation rate.

(a) Derive the equilibrium behavior of employment, output, and the real interest rate under the assumption of flexible wages and prices. Can one determine the corresponding equilibrium values for the nominal rate and inflation? Explain why.

(b) Next we introduce wage stickiness by assuming that nominal wages are set in advance (i.e., at the end of the previous period), according to the rule

$$w_t = E_{t-1}\{p_t\} + \varphi E_{t-1}\{n_t\}.$$

Characterize the equilibrium behavior of output, employment, inflation, and the real wage under the assumption that the central bank follows the simple rule

$$i_t = \rho + \phi_\pi \pi_t.$$

(c) Characterize the optimal policy and its associated equilibrium in the presence of sticky wages, and suggest an interest rate rule that would implement it (note: we assume efficiency of the equilibrium allocation in the absence of sticky wages).

2. Inflation Persistence and the New Keynesian Model (28 points)

Postwar inflation in most industrialized economies displays a high, positive autocorrelation (i.e., inflation is "persistent"). Some authors have argued that such behavior is inconsistent with the purely forward-looking inflation equation that characterizes the baseline New Keynesian model. Others have argued that the observed inflation persistence cannot be reconciled with the central bank following an optimal policy, in the context of the same model.

Explain as rigorously as possible why the two previous assertions are generally incorrect (e.g., by providing some counterexamples). Feel free to use both verbal or algebraic arguments.