ID Number

Question by Prof. Schmidt

(20 points) A consumer has *homothetic preferences* if his utility function is homothetic. A function $f(x)$ is *homothetic* if $f(x) = g(h(x))$, where $g(\cdot)$ is a strictly increasing function and $h(\cdot)$ is a function that is homogeneous of degree 1 in the vector x. Show the following (you may use result (b) to answer parts (c) and (d) of the problem):

(a) If preferences are homothetic, then they can be represented by a utility function that is homogenous of degree 1.

(b) If preferences are homothetic, then the expenditure function can be written as $e(p, u) = e(p) \cdot u$.

(c) If preferences are homothetic, then the indirect utility function can be written as $v(p, w) = v(p) \cdot w$.

(d) If preferences are homothetic, then the Marshallian demand functions take the form $x_i(p, w) = x_i(p) \cdot w$, i.e., they are linear functions of income.

Question by Prof. Rochet

Consider an exchange economy with 2 goods ($\ell = 1, 2$) and 2 consumers (i = 1, 2) with the following utility functions:

$$u_1 = \ln x_{11} + 3\ln x_{21}, u_2 = \ln x_{12} + \ln x_{22},$$

where $x_{\ell i}$ represents the quantity of good ℓ consumed by consumer i and ln denotes the natural logarithm. The initial endowments are $\omega_1 = (0.5, 0)$ for consumer 1, and $\omega_2 = (1.5, 2)$ for consumer 2. Good 1 is taken as a numéraire.

1°) Compute the demand functions of the two consumers.[4 marks]

2°) Determine the excess demand function of the economy, show that there is a unique competitive equilibrium and compute its characteristics (price, consumption bundles, utility levels). [8 marks]

3°) Explain why an equal sharing of the total allocation (egalitarian allocation) is not Pareto Optimal in this economy.[4 marks]

4°) Compute the general equilibrium when the initial endowments correspond to the egalitarian allocation. [4 marks]

Questions by Prof. Dewatripont

Question 1 (35 points)

Consider a pure Bertrand duopoly problem where demand is given by: $Q = 3 - p$, if p is the lower of the two prices charged by the firms, which have constant marginal costs $c_1 = c_2 = 1$.

Assume firm 1 can spend a fixed cost F to raise firm 2's marginal cost to 1.4 before firm 2 chooses its price (example: lobby to impose a tariff if firm 2 is a foreign firm; assume firm 1's lobbying is successful with probability 1).

1. Draw the extensive form of the game.

2. Derive the unique subgame-perfect equilibrium of the game.

3. What is the level of F above which it is not profitable for firm 1 to spend this fixed cost ? Discuss.

4. Explain in words how firm 1's reasoning in 1-3 above would change if firms competed in quantities instead of prices. Is the difference the same as when F serves to decrease firm 1's marginal cost ? (max. 15 lines).

5. Returning to Bertrand competition, assume now, in contrast to 1-3 above, that firm 2 does not observe whether firm 1 spends F, and has to decide on its price level before learning whether its marginal cost will be 1 or 1.4. What is the (pure-strategy) equilibrium set for $F > 0$? And for $F = 0$? Discuss.

Question 2 : Bargaining under Incomplete Information (35 points)

Consider a seller with zero valuation for a good who makes offers to a buyer who can have valuation b_1 or b_2 for this good, where both valuations are equiprobable and $0 < b_1 < b_2 < 2b_1$.

1. If the seller can only make one take-it-or-leave-it offer, what is the unique perfect Bayesian equilibrium ?

2. Assume the buyer has discount factor $\delta_B < 1$ and the seller has discount factor $\delta_S < 1$, and the seller can make one offer in period 1, and a second offer in period 2 if the first offer has been rejected: (i) Prove that the unique perfect Bayesian equilibrium involves immediate trade with probability 1 if $\delta_B = \delta_S$. (ii) What is the unique perfect Bayesian equilibrium if $\delta_B < \delta_S = 1$ instead ?

3. Briefly compare the results under 2 above with the results of: (i) the Rubinstein model of bargaining under complete information, and (ii) Spence's signalling model.

Questions by Prof. Moore

ANSWER **TWO OUT OF 5** QUESTIONS FOR A TOTAL OF 70 POINTS

(Put your answer to the two questions on the sheets after question 5.)

1. Consider an Akerlof model in which firms are risk neutral and have additive technologies. Workers have individual productivities θ that are uniformly distributed on [0, 6]. A worker of type θ has an opportunity cost $6 - \theta$ of being employed by a firm. Each worker privately observes his own type.

(a) Find the equilibrium wage and allocation of labour.

(b) Relative to first-best, how is the equilibrium inefficient?

(c) How might the government improve welfare?

2. Consider the following Spence signalling model. Each worker has a fixed labour supply, and privately learns her own type θ before deciding how much education $e \geq 0$ to acquire at cost e^2/θ. The distribution of types is continuous and has support $[0, \overline{\theta}]$. Competitive firms earn revenue $\theta + f(e)$ from employing a worker of type θ who has acquired education e, where f(e) is an increasing function of e (so that education has intrinsic value). If there is a separating equilibrium in which a worker of type θ chooses to acquire education $k\theta$, where k is a constant, then what must the function f(e) be, and what is the lowest value that k can take? [Hint. You can begin by considering in isolation the problem of what level of e is chosen by a worker of type θ. This will allow you to determine the wage w(e) that firms offer a worker with education e.]

3. Consider a competitive screening model with many firms and workers. There are two types of worker, respectively with productivities $\theta = \theta_L$ and $\theta = \theta_H$, where $\theta_H = 2\theta_L > 0$. A worker with productivity θ has a cost t^2/θ of undertaking some task $t \geq 0$, and the return from such a worker to a firm is $\theta + t$ (i.e., the task is intrinsically useful). The timing of the model is as follows. First, each worker privately learns her own type θ. Then, the firms offer contracts comprising a wage w(t) for undertaking some task t. Finally, each worker chooses which contract, if any, to accept.

(a) If a pure strategy, separating equilibrium exists, then: first, what task and wage are offered to a type θ_L worker; second, what task and wage are offered to a type θ_H worker?
(b) Find the maximum fraction λ of type θ_H workers for which this equilibrium exists.

4. A monopolist serves two types of customer, $i = 1, 2$, and there are N customers of each type. A type i customer derives a monetary benefit of v_i per unit purchased, up to a capacity of X units (X is the same for both types). Assume $0 < v_1 < v_2 < 5v_1/4$. The good is divisible, and the monopolist's cost function is $k(Nx_1 + Nx_2)^2$, where x_i is the amount supplied to each customer of type i, and the constant k is given by $v_1/4NX$.

(a) Assume the monopolist can distinguish the type 2 consumers from type 1 consumers. What is her optimal selling policy?
(b) From now on, assume that the monopolist cannot distinguish the two types. What is her optimal selling policy if she is restricted to linear pricing (i.e. she charges a common price p per unit)?
(c) What is the monopolist's optimal selling policy if she is unrestricted (i.e. she offers a menu of contracts, (m_i, x_i), where a customer pays a total amount m_i for x_i units of the good)?

5. What additional assumptions do Hellwig and Schmidt import to the Holmstrom-Milgrom analysis of linear incentive schemes? Taking each of these additional assumptions in turn, suggest a counter-example for what might happen if the assumption is dropped.