STUDIENZENTRUM GERZENSEE
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

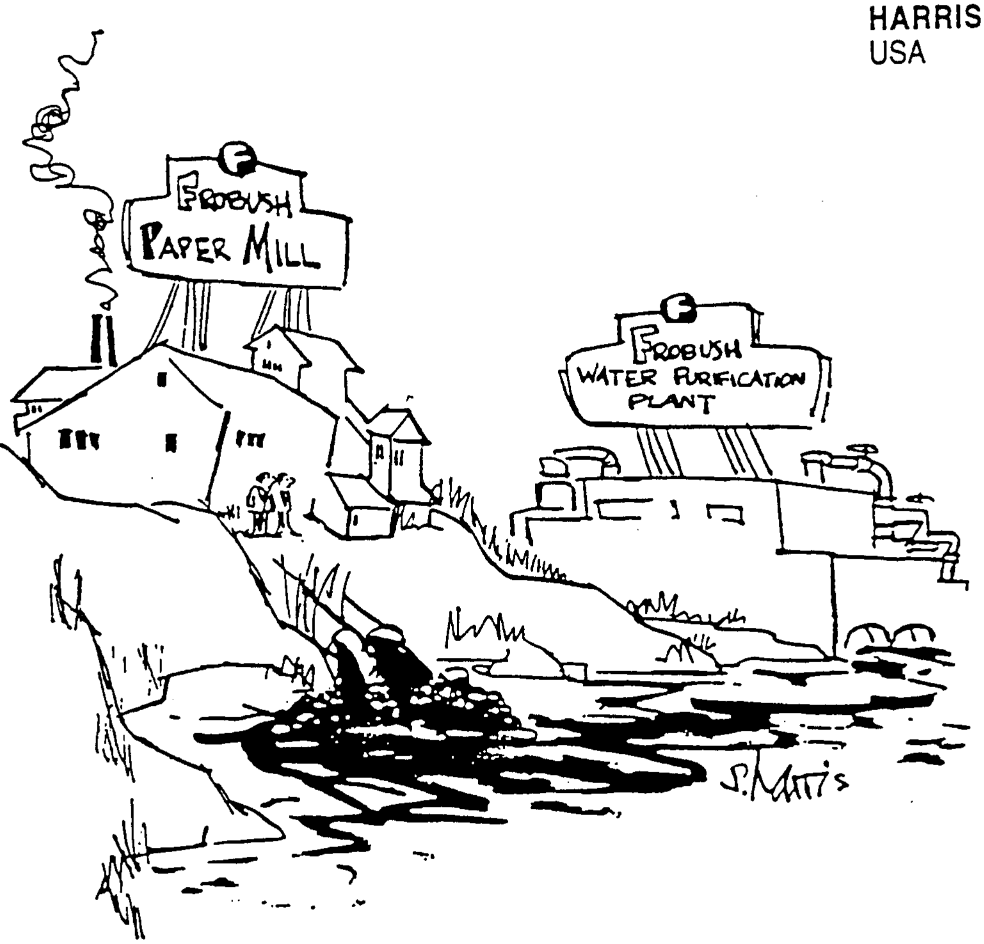
**Program for Beginning Doctoral Students in Economics 1995/96**

**Exam in Microeconomics**

**Monday, October 14, 1996**

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the exception of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Good luck!

HARRIS
USA




Cartoonists & Writers Syndicate

"We create it, we clean it up—business couldn't be better."

# PART 1

**A** - Explain intuitively why in an economy without aggregate risk, where agents are strictly risk averse and have the same beliefs on the probabilities of the states of nature, the Pareto optima are such that all agents are perfectly insured.

**B** - An agent with a Von Neumann - Morgenstern utility function $u(.)$ with $u' > 0$ has an initial resource $W$ and can invest a share $a$ $(0 \le a \le W)$ in a risky asset with random return $\tilde{r}$ such that $E\tilde{r} > 0$, and keep $W - a$. His final wealth will be $W + a\tilde{r}$. The agent is only interested in his final wealth. Show that the optimal amount invested in the risky asset is strictly positive even if the agent is risk lover.

**C** - Consider a risk averse agent with a Von Neumann - Morgenstern utility function $u(.)$, $u' > 0$, $u'' < 0$.

The agent can be in state 1 with probability $p$ and have a wealth $W - l$, or in state 2 with probability $1 - p$ and have a wealth $W$.

An insurance company offers him to pay a premium $q$ per unit of reimbursement in the state 1.

Show that, if the insurance is not fair $(q > p)$, the agent will not choose complete insurance.

**D** - We consider an economy subject to risk at date 1. There are four states of nature possible at date 1, the first three with probability 1/6 and the fourth with probability 1/2. The total initial endowments of the economy at date 1 are such that $W_1 < W_2 < W_3 < W_4$, where the index $i$ in $W_i$ indicates the state of nature. Exchange between agents occurs with two assets at date 0.

Asset 1 delivers at date 1 the incomes $a^1 = (9, 6, 6, 1)$.

Asset 2 delivers $a^2 = (1, 1, 1, 3)$.

What is your expectation of the relative price of the assets $(q^1 / q^2)$ ?

# PART II

We consider a monopoly facing a continuum [0,1] of consumers. Each consumer is characterized by a utility function :

$$\theta \log q + x$$

where $x$ is his consumption of good 1 (which is chosen as numeraire), $q$ is his consumption of good 2 produced by the monopoly.

The parameter $\theta \in [\underline{\theta}, \bar{\theta}]$ with $\bar{\theta} - \underline{\theta} = 1$ specifies the tastes of a consumer (this parameter will be his private information). The distribution of tastes is characterized by the uniform distribution on the interval $[\underline{\theta}, \bar{\theta}]$.

Consumers have large initial resources in good one, denoted $\bar{x}$, so that their behavior is always characterized by the first order conditions of their maximization program.

The monopoly has a variable cost function $C(q) = cq$ and a fixed cost $K$.

**1.** Explain why the profile of consumption $q^*(\theta)$, $\theta \in [\underline{\theta}, \bar{\theta}]$, which corresponds to an interior Pareto optimal (complete information) allocation, is the solution of the program :

$$\max \int_{\underline{\theta}}^{\bar{\theta}} [\theta \log q(\theta) + x(\theta)]\, d\theta$$

$$\text{such that : } \int_{\underline{\theta}}^{\bar{\theta}} x(\theta)\, d\theta = \bar{x} - c \int_{\underline{\theta}}^{\bar{\theta}} q(\theta)\, d\theta - K$$

Determine $q^*(\theta)$. Explain the informational problems of this solution.

**2.** Let $(q(\theta), t(\theta))$ a differentiable revelation machanism which elicits the consumer's characteristic $\theta$. More precisely, if a consumer announces the characteristic $\tilde{\theta}$, he receives a quantity $q(\tilde{\theta})$ of good 2 and must pay $t(\tilde{\theta})$ in good 1.

Characterize the revelation mechanisms which lead all consumers to reveal their true characteristic (first order and second order conditions).

Explain the role of the Spence-Mirrlees conditions in this characterization.

**3.** Write the maximization program of a monopolist who maximizes his profits under the constraint of a non negative utility gain of each consumer, that is to say :

$$\theta \log q(\theta) - t(\theta) \geq 0 \quad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}].$$

To solve this program, the change of variables :

$U(\theta), q(\theta)$ instead of $t(\theta), q(\theta)$ with $U(\theta) = \theta \log q(\theta) - t(\theta)$

is suggested.

Show that the new maximization program can be written :

$$\max \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta \log q(\theta) - cq(\theta) - U(\theta)\right] d\theta$$

$$\text{such that : } \frac{dU}{d\theta} = \log q(\theta)$$

$$U(\underline{\theta}) \geq 0.$$

Explain why $U(\underline{\theta}) = 0$ in the optimal solution.

Solve then the differential equation $\frac{dU}{d\theta} = \log q(\theta)$ with this boundary condition.

Substitute the solution in the objective function and determine the optimal consumption profile.

Choose $\bar{\theta} = 3$, $\underline{\theta} = 2$. Compare with the result of question 1.

**4.** Determine the transfert $t(\theta)$ associated with the solution obtained in question 3. Interpret the result as a non linear «price» $t(q)$.

**5.** We suppose now that the governement uses a linear tax $\tau$ on the consumption of good 2 to control the monopoly.

Explain why the optimal consumption profile for the monopoly, with a tax $\tau$ is characterized by : $\dfrac{2\theta - \bar{\theta}}{q(\theta)} = c + \tau$.

We assume that the government maximizes a weighted average of the consumers' utilities (weight 1), the monopoly's profits (weight $\sigma$), and taxes (weight $\lambda$) with $\sigma \geq \lambda$.

Determine the optimal tax $\tau$, assuming an interior maximum. Discussion.

# Gerzensee exam - October 1996

M. Dewatripont

1. Take the Spence signalling model with two types of workers, H and L, with proportions 1 - $\alpha$ and $\alpha$ in the population. Their respective utility functions are $w - c_H e^2$ and $w - c_L e^2$, where e is the education level, w the wage, and $0 < c_H < c_L$. Worker productivity is L + e for type L and H + e for type H, where 0 < L < H.

a. Show graphically the equilibrium if worker types are publicly observed.

b. Show graphically situations where unobservability of types changes the outcome.

c. In such a case, describe the set of Bayesian-perfect equilibria that involves pooling, and the set that involves complete separation of types.

d. Describe a Bayesian-perfect equilibrium with semi-separation.

e. Which Bayesian-perfect equilibrium satisfies the Cho-Kreps criterion ? Explain.

2. Consider the Rubinstein alternating offer bargaining model with identical discount factor $\delta < 1$ for both parties :

a. Describe the subgame-perfect equilibrium. Prove uniqueness.

b. What is the set of Nash equilibria of this game ?

c. Truncate the game so that it stops after 4 periods. Describe the subgame-perfect equilibrium. Prove uniqueness. What happens when $\delta \to 1$ ? Compare with the infinite-horizon game.

d. What would happen, when $\delta \to 1$, both in the 4 period and the infinite-horizon cases, if only the seller could make offers ?