

STUDIENZENTRUM GERZENSEE  
STIFTUNG DER SCHWEIZERISCHEN NATIONALBANK

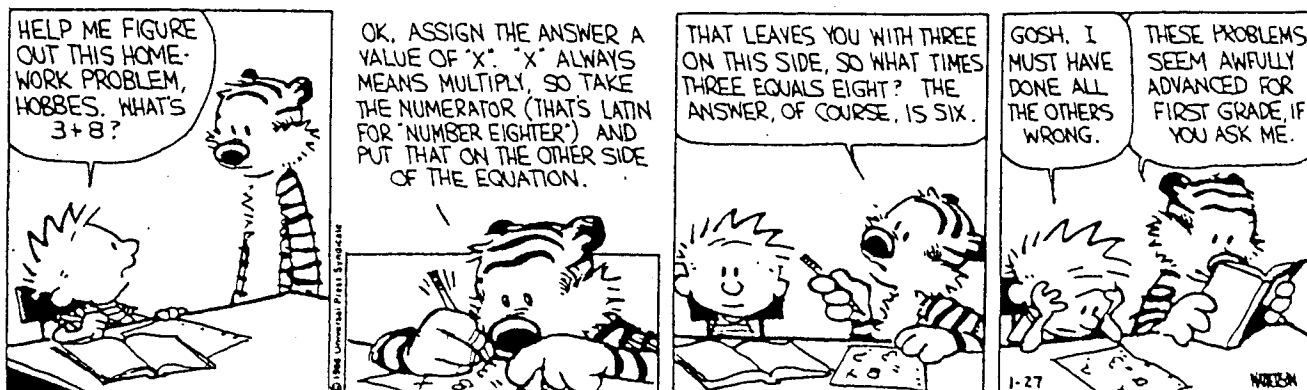
## Program for Beginning Doctoral Students in Economics 1997

### Exam in Macroeconomics

Tuesday, March 10, 1998, 08.30 - 11.30

1. You are allowed to use all material that you want (lecture notes, books, etc.) with the expectations of PC's.
2. Please **do not** mention your name on top of the pages, but use your identification number from the enclosed list. The reason is that the exams will be graded anonymously.
3. Good luck!

## CALVIN AND HOBBS



**Studienzentrum Gerzensee**  
**Macroeconomics Doctoral Course 1997**  
 Final Examination

There are four questions on this examination, which are worth 25 points each. You have three hours to complete the examination. Please budget your time carefully and write legibly in the space provided. Do *not* write on the reverse side of the exam pages.

- (1) *Intertemporal Models of the Current Account.* In this question, we consider a small country in a world economy. The small country and the world economy face no uncertainty. However, the small country does face time variation in the nature of its productive opportunities. The small country's representative agent chooses consumption and leisure so as to maximize:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

with  $u(c_t, l_t) = \log(c_t) + \theta \log(l_t)$ , subject to the intertemporal budget constraint,

$$\sum_{t=0}^{\infty} p_t [c_t + w_t l_t] \leq \sum_{t=0}^{\infty} p_t [\pi_t + w_t T].$$

In these expressions,  $c_t$  is the flow of consumption,  $l_t$  is the flow of leisure,  $\pi_t$  is the flow of nonwage income at date  $t$ ,  $T$  is the endowment of time which is constant at all dates,  $w_t$  is the small country's real wage rate at date  $t$  and  $p_t$  is a discount factor for date  $t$  cash flows, which is determined in the world economy. The small country's labor input is  $n_t = T - l_t$  and its output is  $y_t = a_t n_t$ , with  $a_t$  being a time-varying productivity level. There is no investment, capital or government purchases.

- (a) What would the current account be in this model? Why?

Suppose that this country faces a known seasonal pattern in its productivity. In particular, productivity is low in the first two quarters of the year and high in the last two quarters of the year.

- (b) What is the seasonal pattern of the wage rate?

- (c) Assuming that  $p_t = \beta^t$ , what is the cyclical pattern of labor, consumption, and the current account? Why?

Suppose now that the production function is  $y_t = a_t f(k_t, n_t) = a_t k_t^{1-\alpha} n_t^\alpha$ , where  $k_t$  is the capital stock. Suppose further that firms can purchase the single final good, which can be used for either consumption or investment, from either domestic or international producers.

- (d) Characterize the form of the small country's optimal demand for capital.

- (e) What is the level of the wage rate and how does this move with a seasonal variation in productivity, given that capital is chosen efficiently?

- (f) How is investment determined (assuming that  $k_{t+1} - k_t = i_t - \delta k_t$ )? Does it lead or lag output?

- (g) How is the current account determined in this economy? During which periods will there be a positive associations between current account deficits and output? Between current account deficits and investment?

- (2) *Taxes, Productivity Shocks and Economic Activity.* Consider the standard closed-economy real business cycle model, except that taxes—rather than productivity shocks—are the driving process. In particular, assume that the production function takes the form  $y_t = f(k_t, n_t) = ak_t^{1-\alpha}n_t^\alpha$  and the accumulation equation for capital takes the form  $k_{t+1} - k_t = i_t - \delta k_t$ . Further, assume throughout this whole question that the tax part of the fiscal rule takes the form

$$\tau_t - \tau = \rho(\tau_{t-1} - \tau) + e_{\tau t}$$

where  $\tau_t$  is a tax on gross output,  $\tau$  is its normal level,  $e_{\tau t}$  is a white noise random shock and  $\rho$  is a parameter that controls the persistence of tax rates.

- (a) In this part, assume that the government's fiscal rule is to purchase the economy's single final good whatever resources it has, i.e.,

$$g_t = \tau_t y_t$$

where  $y_t$  is output and  $g_t$  is the level of real government purchases. Under this fiscal policy process, describe how the model's outcomes would compare to those arising from a standard productivity shock, specified as  $a_t - a = \rho(a_{t-1} - a) + e_{at}$ , if  $(1 - \tau_t) = a_t$  at all dates. Of consumption, investment, output, employment, and the real wage, which variables would be the same and why? Which variables would be different and why?

- (b) In this part, assume that the government's fiscal rule is to rebate as lump-sum transfer payments whatever resources it collects as taxes. That is, transfer payments are

$$T_t = \tau_t y_t,$$

where  $y_t$  is output and  $T_t$  is the level of real transfer payments. Under this fiscal policy process, describe how the model's outcomes would compare to those arising from a standard productivity shock using the same constraint as above. Of consumption, investment, output, employment, and the real wage, which variables would be the same and why? Which variables would be different and why?

- (3) *Sticky prices and disinflation:* The Calvo model of sticky prices implies that there is a "Phillips curve" of the form:

$$\pi_t = \beta E_t \pi_{t+1} + \varphi (y_t - \bar{y}_t)$$

where  $\pi_t$  is the inflation rate at date  $t$ ,  $E_t \pi_{t+1}$  is the expected future inflation rate,  $(y_t - \bar{y}_t)$  is the gap between current output and the level that would prevail with imperfect competition but flexible prices and  $\varphi$  is a parameter.

- (a) Briefly describe how this relationship derives from sticky prices.

- (b) Assuming that  $\beta$  is effectively unity, demonstrate that this model implies that there is no cost of an unexpected and permanent disinflation.

- (c) Suppose that the monetary authority takes actions that lead the inflation process to obey the first-order stochastic difference equation,  $\pi_t - \pi = \rho(\pi_{t-1} - \pi) + e_t$  where  $e_t$  is white noise. What will be the statistical behavior of the output gap? How will it be altered by  $\rho$ ? by  $\pi$ ? What will be the correlation between the output gap and inflation?

- (d) Suppose that the monetary authority plans a gradual deflation,

$$\begin{aligned}\pi_t &= \pi - \left(\frac{\pi}{T}\right)t \text{ for } t = 0, 1, \dots, T-1 \\ \pi_t &= 0 \text{ for } t > T\end{aligned}$$

If this deflation is fully credible, then what are the implications for the path of output?

Alternatively, suppose that the plan is imperfectly credible, in the following manner. At each date, if the inflation plan is continued in the current period, then inflation will be generated according to the above equation. However, with a probability  $\alpha$  then the inflation plan will be stopped and inflation will remain at  $\pi$  forever. Finally, if the inflation plan is completed, then inflation will stay forever at 0.

- (e) What will be the output dynamics under a successful plan, i.e., one that actually succeeds in lowering inflation to zero?

- (4) *Linear difference systems under rational expectations.* Consider the following very simple macroeconomic model, which includes just two equations. First, there is the Phillips curve studied above

$$\pi_t = \beta E_t \pi_{t+1} + \varphi z_t + \eta_t$$

but there now is a stationary random shock,  $\eta$ , which may be serially correlated. (For simplicity in the analysis below, we have adopted a simpler notation for the gap, but  $z_t$  is  $(y_t - \bar{y}_t)$  in the previous question). Further, the monetary authority takes monetary policy actions which make inflation evolve according to

$$\pi_t = \gamma E_t \pi_{t+1} + \theta E_t z_{t+1} + m z_t.$$

Under this specification, which we call its policy rule, the central bank adjusts inflation partly in response to expected future inflation  $E_t \pi_{t+1}$ , partly in response to its forecast of the business cycle  $E_t z_{t+1}$ , and partly in response to the current output gap  $z_t$ .

- (a) If  $y_t$  is a two element vector containing  $\pi_t$  and  $z_t$ , then this specification can be placed in the form:

$$A E_t y_{t+1} = B y_t + C x_t$$

What are the matrices  $A, B, C$ ?

- (b) Under what conditions on the parameters is  $A$  nonsingular?

(c) Under what conditions is  $A$  singular?

Suppose now that  $\theta = 0$  (the central bank does not adjust inflation in response to its expectation of the output gap), that  $\varphi$  is positive but small (a small Phillips curve effect of output on inflation), that  $\gamma$  is positive (the central bank allows current inflation to rise when it thinks that future inflation will be higher) and that  $m$  is positive (the central bank allows inflation to rise in booms and fall in recessions). Finally, suppose that  $\varphi/m < 1$ .

(d) What additional restriction(s) must be imposed if there is to be a unique stationary inflation process. How does the inflation process depend on the statistical properties of  $\eta_t$ ?